CHAPTER 5

SIMULATION MODEL TO DETERMINE FREQUENCY
OF A SINGLE BUS ROUTE WITH SINGLE AND
MULTIPLE HEADWAYS

5.1 INTRODUCTION

In chapter 4, from the evaluation of routes and the sensitive analysis, it is found that the service level influences the route performance. This in turn depends on the operating frequency of the buses. Hence, as an extension to that study, it is proposed to develop a simulation model to determine the headway/frequency.

Timetable setting and bus scheduling have been the fundamental focus of urban transportation. Earlier researchers have used fixed parameters such as projected passenger demand, and average travelling time for bus scheduling. However, in actual practice, passenger demands (arrival of passengers for different time periods) as well as time between stages usually vary. The projected demand may not reflect the actual daily passenger demand due to stochastic nature of the data. Therefore, to find the best frequency, stochastic passenger arrival data and travelling time between the stages have to be taken into account. A cost effective and efficient bus timetable embodies a compromise between passenger comfort and cost of services. Usually passengers have to wait because of bad scheduling, traffic, breakdown and overcrowding. Overcrowding occurs because of not determining the correct frequency for the demand and improper scheduling. Frequency means total number of trips needed to satisfy the passenger
demand during the fixed period. It is determined by dividing the study period (bus operation duration) with the headway. Headway is the time interval between successive departures of the bus.

In metropolitan cities, usually a bus carries more passengers than its capacity and passengers wait for longer time at all the stages and thus cause more discomfort and dissatisfaction to the passengers. The reasons for this situation are many. Out of these some are controllable factors (frequency of buses, traffic signal, road condition, bus condition) and some are uncontrollable factors (traffic jam, breakdown, etc.). Frequency of buses is one of the important issues that have a bearing on the waiting time of passengers as well as on the operating cost of buses.

Lampkin and Saalmans (1967) formulated a constrained optimization problem for frequency determination. The objective was to minimize the total travel time for a given fleet size constraint and employed random search procedure for obtaining solution. Ceder (1984) described four methods for calculating the frequencies. Two methods are based on point-check counting the passengers on board the transit vehicle at certain points, and two on ride-check counting the passengers along the entire transit route. Sinclair and Oudheusden (1997) proposed a minimum cost network flow model to deal with the problem of bus trip scheduling in heavily congested cities.


The model by Han and Wilson (1982) model recognizes passenger route choice behaviour and seeks to minimize a function of passenger wait
time and bus crowding subject to constraints on number of buses available and the provision of enough capacity on each route.

Yan and Chen (2002) recently developed a deterministic scheduling model, with the objective of maximizing the system profit, given a fixed projected passenger demand and the operating constraints. Their model was shown to be more systematic and efficient than the traditional trial-and-error method.

Many researchers in the past have tried biologically motivated optimization techniques like genetic algorithm and artificial neural network for transit network design (Xiong and Schneider (1992), Chakraborty et al. (1995), Pattnaik et al. (1998), Bielli et al. (2002), Ngamchai and Lovell (2003)). Kliewer et al. (2006) proposed a time–space network based exact optimization model for multi-depot bus scheduling.

The models developed for determination of headway and timetabling by the earlier researchers may not be directly useful for application to the metropolitan cities in most of the developing countries like India where the operating and the transit network conditions are entirely different. And also very few researchers have considered stochastic data (passenger arrivals and travelling time). In this study, a simulation model has been developed to determine frequency for a single bus route with constant and multiple headways considering stochastic data. The model has been applied to one of the bus routes of Chennai city.
5.2 TERMINOLOGY

Stage

A stage is where the passenger boards the bus and alights from the bus; a route contains more than one stage.

Route

A route contains ordered sequence of bus stages (stops). This sequence consists of at least two elements. The starting stop represents the origin (terminus) of the trip and the final stage is the destination (terminus).

Headway

Headway is defined as the time between successive departures on each route.

Frequency

Frequency is defined as the possible number of departures on each route in a specified time interval.

Waiting time

The waiting time is the time spent by the passenger from the arrival time to the departure time.

Trip time

Trip time is the time taken to travel between the terminuses.
Passenger alighting

Passenger alighting is the passengers getting down from the bus at each stage or terminus.

Passenger Boarding

Passenger getting into the bus at terminus or stages to reach their destination.

Passenger Load

Number of passengers travelling in the bus from one stage to another stage

5.3 MATHEMATICAL MODEL

The mathematical model is developed for finding the best frequency. The details of the model are given below

Min $Z = f_1 + f_2$ \hspace{1cm} (5.1)

where $f_1$ - Operating cost of the bus

$f_2$ - Waiting time cost of the passenger

$f_1 = C_o \sum_{j=1}^{m} \sum_{i=1}^{n} t_{ji}$ \hspace{1cm} (5.2)

where

$C_o$ - Operating cost per unit time
\( t_{t_{ji}} \) - Travel time of \( j^{th} \) bus in the \( i^{th} \) trip; \( j = 1, 2, \ldots, m; \)
\( i = 1, 2, \ldots, n \)

\[
f_2 = C_w \sum_{j=1}^{m} \sum_{i=1}^{n} p_{b_{ji}} \ast awt
\]

(5.3)

where

- \( C_w \) - Waiting time cost of a passenger per unit time
- \( p_{b_{ji}} \) - Number of passengers boarded in \( i^{th} \) trip of \( j^{th} \) bus
- \( awt \) - Average waiting time of the passenger

Subject to

\[
TP_{ji,k-1 \rightarrow k} + B_{jik} - A_{jik} \leq 75 \quad \forall \ i = 1, 2, \ldots, n
\]
\[
\forall \ k = 1, 2, \ldots, L
\]
\[
\forall \ j = 1, 2, \ldots, m;
\]

(5.4)

Equation (5.4) represents load constraint

\( k \) - Number of stages in the trip, \( i = 1, 2, \ldots, L \)

\( TP_{ji,k-1 \rightarrow k} \) - Passengers travelled from \( k-1^{th} \) stage to \( k^{th} \) stage of \( i^{th} \) trip of \( j^{th} \) bus

\( B_{jik} \) - Number of passengers boarded at \( k^{th} \) stage of \( i^{th} \) trip of \( j^{th} \) bus

\( A_{jik} \) - Number of passengers alighted at \( k^{th} \) stage of \( i^{th} \) trip of \( j^{th} \) bus

\[
TT_{k-1 \rightarrow k} \geq t_{min,k-1 \rightarrow k} \quad k = 1, 2, \ldots, L
\]

(5.5)

Equation (5.5) represents minimum travelling time

\( TT_{k-1 \rightarrow k} \) - Travelling time between \( k-1^{th} \) stage to \( k^{th} \) stage

\( t_{min,k-1 \rightarrow k} \) - Minimum travelling time between \( k-1^{th} \) stage to \( k^{th} \) stage

\[
TT_{k-1 \rightarrow k} \leq t_{max,k-1 \rightarrow k} \quad k = 1, 2, \ldots, L
\]

(5.6)

Equation (5.6) represents maximum travelling time

\( t_{max,k-1 \rightarrow k} \) - Maximum travelling time between \( k-1^{th} \) stage to \( k^{th} \) stage
5.3.1 Objective Function

The objective function consists of user (passenger) cost component and operator cost component. The operator cost is the product of sum of the travelling time of all the trips during the study period and the operating cost per unit time. The waiting time cost is the product of number of passengers travelled, the average waiting time of the passenger during the study period and the cost of waiting time per unit time. The objective is to determine the headway that minimizes the vehicle operating cost and the total waiting time cost of the passengers.

5.3.2 Constraints

The constraints are briefed below.

1. Constraint 1 states that the load (number of passengers) at each stage should be less than or equal to the maximum capacity of the bus.
2. Constraint 2 states that the traveling time should be greater than or equal to the minimum traveling time between the stages / terminus to stage / stage to terminus.
3. Constraint 3 states that the traveling time should be less than or equal to the maximum traveling time between the stages / terminus to stage / stage to terminus.
Most of the data required to obtain solution from this model is stochastic in nature. Hence it is proposed to develop a simulation model to obtain the solution.

5.4 DEVELOPMENT OF THE SIMULATION MODEL

Since most of the data are stochastic in nature, simulation has been used to obtain solution. The simulation model is built using the ARENA version 12, a discrete system simulation software package. The simulation model consists of the following three sub-models:

1. Terminus sub-model
2. Travelling time sub-model
3. Stage sub-model

The simulation model for the route is a combination of these three sub-models. Suppose the route has two stages. The simulation model for the forward tip of a route is shown in Figure 5.1. This consists of two terminus models (T1 and T2) (one for the starting of the route and the other for ending of the route), three travelling time sub-models (TT1, TT2, TT3) and two stage (S1 and S2) sub-models.

![Figure 5.1 Schematic models for a single route with two stages](image-url)
5.4.1 Inputs required for the Simulation Model

1. The input required for the terminus and the stage sub-models is the arrival of passengers in the form of a probability distribution.
2. The input required for the traveling time sub-model is the traveling time of buses between stages in the form of a probability distribution.
3. The input required for the stage sub-model is the alighting percentage of passengers at each stage.
4. The time between successive departures of bus (headway time)
5. The replication length (time period to run the simulation)
6. The number of replications (number of days to run the model)

5.4.2 Outputs from the Simulation Model

1. The average waiting time of the passenger
2. The number of passengers traveled during the study period of one shift (i.e., 8 hours)
3. The total traveling time of the buses for the study period of one shift (i.e., 8 hours)
4. Number of trips required for the different headway condition
5. Arrival and departure time of buses at each stage and terminus
6. Queue length at each stage
7. Boarding and alighting time for each stage.

5.4.3 The Logical Flow Chart

The logical flow chart for simulating the terminus sub-model is shown in Figure 5.2. It explains the sequence of processes taking places in
the bus terminus. First the capacity of the bus and the study period is initialised. The headway, the arrival distribution of passengers at the terminus and stages is given as the input. After initialisation the available N buses will arrive at the terminus and wait in the queue for despatch. The passengers at the terminus and at different stages arrive according to the distribution defined and wait for the bus. The important event in the simulation is Next Most Immediate Event (NMIE). The following 6 events occur at the NMIE.

1. Passengers arrive at different stages and join the respective queue and wait for the bus.

2. Bus arrives at different stages based on the travel time from the previous stage.

3. Passengers arrive at the terminal according to the defined distribution and join the queue and wait for the bus.

4. Once the bus arrives from the previous stage, all the passengers alight from the bus and the buses joins the queue and wait for the despatch.

5. Release the bus from the terminus, based on the headway. There will be two bus queues at the terminus; one is the generated bus, wait in the queue (BQ1) for despatch and the other queue is the bus arrived from the other terminus (BQ2). Preference is given to the queue of buses from the other terminus. The Passengers will board the bus based on the seat availability and the excess passengers will wait for the next bus.

6. End of process: once the study period is over, the simulation stops and the statistics are printed.
Figure 5.2 Logical flow chart of simulation model

C = capacity of the bus
Release time of bus determined (headway)
Passenger arrives at the terminus and stages according to Poisson distribution
End of simulation is determined
Replication length

‘N’ buses arrives at the terminus and made available in (BQ₁)

Passengers arrive at the stages (S₁, S₂, ... Sₙ) and join the queue

Update the clock
Update the clock
Update the clock
Update the clock

Bus available in the queue BQ₂
Release the bus from queue BQ₂
Release the bus from queue BQ₁

No
No
Yes
Yes

Passengers in queue > Seat available (C)
All passengers in queue board the bus

A

Buses available in (BQ₁)

Passenger arrives in T₁

NMIE

End of simulation

Passenger arrives at the stages (S₁, S₂, ... Sₙ) and join the queue

Update the clock

Bus arrive from the previous stage

All passenger alight the bus.

Bus joins as the last member of bus queue (BQ₂)

Passenger arrival to S₁/S₂/S₃...Sn

Passenger joins the queue

Bus arrival to S₁/S₂/...Sn

Release of Bus

Stop simulation

Print statistics

Stop

Passenger arrives to the terminus

All passengers in queue board the bus

STOP
Passengers board the bus equal to the seat available from the queue

Update the seat available in the bus

Update the passenger queue

Arrival time of the bus to the next stage is determined based on the travel time

Go to next stage

Figure 5.2 (Continued)
Figure 5.2 (Continued)

1. Update the clock
   Passenger arrives at the stage
   Arrived passenger joins the queue
   3

2. Update the clock
   Bus arrives from the previous stage
   Determine the alighting passenger
   Passengers alight the bus
   Update the seat available in the bus
   Yes
   3
   All the passengers in the queue board the bus
   No
   Seats available >
   Passengers waiting at stage queue
   No
   Passengers boarding the bus are equal to seats available from the queue
   Update the passenger queue
   Update the seats available in the bus
   Arrival time of the bus to the next stage is determined based on the travel time
   Bus leaves to next stage

Figure 5.2 (Continued)
The logical flow chart for the simulation process at the stages is shown in Figure 5.2. Two things take place at the stages

1. Passengers arrive at the stages according to the Poisson distribution, join the queue and wait for the bus.
2. Bus arrives at the stage; the passengers alight from the bus as per the alighting percentage, the passengers waiting in the queue will board the bus if seats are available and the bus leaves for the next stage.

The simulation model is built using the software ARENA -12. The required input is given to all the three sub-models and the simulation is run for 75 replications (days), determined based on stabilisation of results. Using the output obtained from simulation, the total cost is determined.

The simulation model is verified using real time data of a single bus route collected from a metropolitan city. The results show that the number of trips obtained is the same as the existing number of trips for a slightly higher capacity bus (100) and the total number of passengers boarded and alighted are also the same (i.e., no passenger waits at any of the stages). The trip time and the number of passengers traveled obtained from the model are also more or less the same as the actual data. This shows that the developed model is correct.

5.5 CASE STUDY

One of the important bus routes from the metropolitan city of Chennai city has been selected for the study. The route has five stages and two terminuses. From the past data, it is observed that the average traveling time
is 70 minutes and the average fare is INR 5 (MTC website). To determine the headway, it is assumed that the passenger load and the travel time is the same in the forward trip and the return trip since buses are dedicated to the route. The headway of the route is determined for the forward trip and is assumed it as the same for the return trip.

### 5.5.1 Assumptions

The following assumptions have been made to apply the model for the case considered.

1. **Time period:** The MTC operates the bus for 480 minutes per shift. The total time is divided into three time periods based on the average demand. The three time periods, representing below normal ($T_{bn}$) for lean demand, Normal ($T_n$) for medium demand, and peak ($T_p$) for heavy demand. The bus operation starts at time $t = 0$.

2. **Fleet size:** There are $N$ vehicles available for despatching in the bus route. Each vehicle has the same passenger capacity ($C$)

3. **Relationship between demand and headway:** Passenger arrivals are functionally independent of the bus headway.

4. Headway remains constant throughout each time period (presently MTC Chennai follows constant headway throughout the shift).

5. **Passenger alighting:** Let $N$ denote the number of passengers in the bus when it arrives at stage $i$. The number $V_i$ of passengers alighting at stage $i$, conditioned on $N_i$, is assumed to be binomial ($N_i$, $P_i$), written as $B (N_i, P_i)$, where $P_i$ is the
probability that an individual will alight. That is, the conditional distribution of $V_i \mid N_i$ is $B(N_i, P_i)$.

6. Time required boarding and alighting: The time required to board or alight is assumed as a constant of 3 seconds

7. Queue capacity: The queue at each bus stop has infinite capacity. Passengers waiting will board the bus on a FCFS basis.

5.6 SIMULATION MODEL FOR A SINGLE ROUTE

The route selected for study has five stages in the forward trip and five stages in the return trip. The simulation model is built for the selected route using the simulation software ARENA-12 which is shown in Figure 5.3. This model consists of five bus stages in the forward trip (S_1 to S_5) and five stages in the reverse trip (S_6 to S_10) with a terminus one each at the start and at the end of the route. In Figure 5.3, TT represents the traveling time between stages. Passengers may board and alight at any stage or terminus.
Figure 5.3 Simulation model for the selected route

As already discussed the simulation model for the route has three sub-models: one for the Terminus, the second one for Travel time and the third for the Stages. These are explained below.

5.6.1 Terminus Sub-Model

The terminus sub-model is shown in Figure 5.4. The functions of the terminus model are:

1. To generate the passengers
2. To generate the buses and make ready for dispatch and
3. To generate the signal for the bus dispatch based on the headway.
The Passenger generation segment generates the passengers at the terminus as per the distribution and arrival rate. The generated passenger joins the queue and waits for boarding the bus. The buses are generated according to the requirement. The generated bus joins the queue (BQ₁) and waits for departure. Buses which arrive from the previous stage unload the passengers, join the queue (BQ₂) and wait at the terminus for the departure. The third function of the terminus model is to give a signal for bus despatch as per the headway. Priority for despatch is given for the buses in the queue BQ₂.

![Figure 5.4 Simulation model of a terminus](image)

**Figure 5.4 Simulation model of a terminus**

### 5.6.2 Travelling Time Sub-Model

Once the bus departs from the terminus, it enters into the travelling time sub-model, as shown in Figure 5.5. This model determines the travelling time based on the travel time distribution for each time period (Below normal
time, Normal time and peak time). The buses will travel as per the determined travel time. In the simulation model the travelling time is nothing but delay time. Based on the travel time the buses will be delayed between the stages. Once the travel time is completed, the buses will be allowed to go to the next stage.

![Simulation model for travelling time sub-model](image)

**Figure 5.5 Simulation model for travelling time sub-model**

5.6.3 Stage Sub-Model

Figure 5.6 shows the simulation model for stages. The following three activities are performed by this model.

1. Passenger generated (Passenger arrived) at the stages as per the arrival time distribution joins the queue and waits for the bus.

2. When the bus arrives at the stage, passengers alight as per the determined passenger alighting percentage and the remaining passengers stay in the bus. The number of passengers alighting
vary for different time periods. This number is calculated as per the alighting percentage of passengers travelled for the different time periods.

3. After alighting, the passengers who are already in the queue will board the bus provided seats are available, else wait for another bus.

![Diagram of Simulation model for stages](image)

**Figure 5.6 Simulation model for stages**

After the passengers board the bus, the bus departs from the stage and goes to the travel time model. Similar events take place at all the stages.

5.7 **DATA COLLECTION**

Relevant data required for running the simulation model are collected from the Metropolitan Transport Corporation (MTC) of Chennai city.
1. Cost of operating \((C_o)\) the bus = INR 5.75/minute

2. Cost of waiting time \((C_w) = INR 0.20/minute. This is based on the average income of residents in the route selected and working hours (Verma and Dhingra 2006)

3. The interarrival time between passengers has been computed from the data sheets available in the MTC for the terminus and stages for different time periods (Table 5.1).

4. The alighting percentage of passengers at each stage and the terminus for all the time periods computed from data available with MTC is shown in Table 5.2.

5. Traveling time follows uniform distribution and is different for different time periods. This has been verified from the data collected (Table 5.3). This data has been collected by actually travelling in the bus on all days of a week.

<table>
<thead>
<tr>
<th>Time</th>
<th>5 to 6 am</th>
<th>6 to 7 am</th>
<th>7 to 8 am</th>
<th>8 to 9 am</th>
<th>9 to 10</th>
<th>10 to 11 am</th>
<th>11 to 12 am</th>
<th>12 to 1 pm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Terminus1</td>
<td>0.74</td>
<td>0.31</td>
<td>0.19</td>
<td>0.15</td>
<td>0.22</td>
<td>0.33</td>
<td>0.35</td>
<td>0.51</td>
</tr>
<tr>
<td>Stage1</td>
<td>2.48</td>
<td>0.99</td>
<td>0.6</td>
<td>0.2</td>
<td>0.29</td>
<td>0.43</td>
<td>0.67</td>
<td>0.72</td>
</tr>
<tr>
<td>Stage2</td>
<td>1.63</td>
<td>0.69</td>
<td>0.35</td>
<td>0.28</td>
<td>0.31</td>
<td>0.46</td>
<td>0.56</td>
<td>0.81</td>
</tr>
<tr>
<td>Stage3</td>
<td>1.34</td>
<td>0.62</td>
<td>0.33</td>
<td>0.24</td>
<td>0.36</td>
<td>0.5</td>
<td>0.6</td>
<td>1.38</td>
</tr>
<tr>
<td>Stage4</td>
<td>5</td>
<td>2.22</td>
<td>1.29</td>
<td>0.66</td>
<td>1.05</td>
<td>1.53</td>
<td>1.8</td>
<td>1.94</td>
</tr>
<tr>
<td>Stage5</td>
<td>2.03</td>
<td>0.95</td>
<td>0.63</td>
<td>0.36</td>
<td>0.72</td>
<td>0.68</td>
<td>1.1</td>
<td>1.7</td>
</tr>
</tbody>
</table>

Table 5.1 Mean interarrival time between passengers for different time periods for stages and the terminus
Table 5.2  Alighting percentage of passengers

<table>
<thead>
<tr>
<th>Time</th>
<th>5 to 6 am</th>
<th>6 to 7am</th>
<th>7 to 8 am</th>
<th>8 to 9 am</th>
<th>9 to 10</th>
<th>10 to 11 am</th>
<th>11 to 12am</th>
<th>12 to 1 pm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stage1</td>
<td>0.5</td>
<td>0.3</td>
<td>0.3</td>
<td>0.4</td>
<td>0.5</td>
<td>0.3</td>
<td>0.3</td>
<td>0.5</td>
</tr>
<tr>
<td>Stage2</td>
<td>0.4</td>
<td>0.6</td>
<td>0.5</td>
<td>0.3</td>
<td>0.5</td>
<td>0.5</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>Stage3</td>
<td>0.3</td>
<td>0.2</td>
<td>0.4</td>
<td>0.3</td>
<td>0.5</td>
<td>0.5</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>Stage4</td>
<td>0.4</td>
<td>0.4</td>
<td>0.4</td>
<td>0.3</td>
<td>0.5</td>
<td>0.5</td>
<td>0.4</td>
<td></td>
</tr>
<tr>
<td>Stage5</td>
<td>0.7</td>
<td>0.6</td>
<td>0.7</td>
<td>0.7</td>
<td>0.6</td>
<td>0.6</td>
<td>0.7</td>
<td></td>
</tr>
<tr>
<td>Terminus 2</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Table 5.3  Traveling time for different time periods between stages

<table>
<thead>
<tr>
<th></th>
<th>Peak (Minutes)</th>
<th>Normal (Minutes)</th>
<th>Below Normal (Minutes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Terminus1-Stage1</td>
<td>23</td>
<td>20</td>
<td>29</td>
</tr>
<tr>
<td>Stage1-Stage2</td>
<td>15</td>
<td>13</td>
<td>19</td>
</tr>
<tr>
<td>Stage2-Stage3</td>
<td>25</td>
<td>20</td>
<td>28</td>
</tr>
<tr>
<td>Stage3-Stage4</td>
<td>16</td>
<td>10</td>
<td>18</td>
</tr>
<tr>
<td>Stage4-Stage5</td>
<td>12</td>
<td>9</td>
<td>16</td>
</tr>
<tr>
<td>Stage5-Terminus2</td>
<td>7</td>
<td>3</td>
<td>7</td>
</tr>
</tbody>
</table>

5.8  CASE 1: SINGLE BUS ROUTE (SINGLE HEADWAY)

With the input of the above data, the simulation model is run and found that the results are consistent beyond 75 replications. So, the simulation is stopped after 75 replications. In this case, the headway is kept constant for all the three time periods. The simulation was started with a minimum
headway of 5 minutes. If the headway is less than 5 minutes, the total cost will be more because the operating cost will be high (high frequency). The maximum headway was fixed at 13 minutes. If the headway is more than 13 minutes, the number of trips (frequency) will be less and the passenger demand shall not meet (i.e., some passengers will be waiting at the stages even after the end of the simulation). Hence, the headway is varied from 5 minutes to 13 minutes and at each headway the simulation is run for 75 replications and the output is recorded. Based on these outputs, the operating cost, waiting time cost and total cost are computed, which is shown in Table 5.4.

Table 5.4 Total cost with constant headway for all time periods

<table>
<thead>
<tr>
<th>Headway (minutes)</th>
<th>Average Waiting time (minutes)</th>
<th>Total Trips</th>
<th>Total Passenger travelled</th>
<th>Trip Time (minutes)</th>
<th>Total Waiting time (minutes)</th>
<th>Operating cost (INR)</th>
<th>Waiting time cost (INR)</th>
<th>Total cost (INR)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>5</td>
<td>192</td>
<td>10587</td>
<td>13440</td>
<td>52903</td>
<td>73920</td>
<td>10581</td>
<td>84501</td>
</tr>
<tr>
<td>6</td>
<td>5.6</td>
<td>174</td>
<td>10587</td>
<td>12180</td>
<td>59224</td>
<td>66990</td>
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<td>79741</td>
<td>47740</td>
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<td>9</td>
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<td>276850</td>
<td>27720</td>
<td>55370</td>
<td>83090</td>
</tr>
</tbody>
</table>

5.8.1 Results and Discussion

For each headway, the average passenger waiting time, the number of trips covered by the bus to satisfy the demand and the trip time are obtained from the simulation model. The operating cost, the waiting time cost and the total cost are computed and tabulated in Table 5.4. The objective of
the problem is to determine the optimal (best) headway so as to minimize the total cost. Figure 5.7 shows the graphical representation of the costs versus headway. From the graph, it is observed that the operating cost decreases as headway increases. And the waiting time cost increases as the headway increases. The trade off between the waiting time cost of the passengers and operating cost is found at headway of 10 minutes (Table 5.4). Also the average waiting time of the passengers is 12.2 minutes, the number of trips required is 96 and the total cost is INR 62792. Thus the optimal (best) headway is found to be 10 minutes. With this headway, starting from 5.00 AM, the timetable has been prepared and shown in Table 5.5. In this case, it is to be noted that the headway is the same for all the time periods.

![Figure 5.7 Headway versus Total cost](image)

**Figure 5.7 Headway versus Total cost**
Table 5.5 Sample Timetable for the single route

<table>
<thead>
<tr>
<th>Terminus</th>
<th>Stage1</th>
<th>Stage2</th>
<th>Stage3</th>
<th>Stage 4</th>
<th>Stage 5</th>
<th>Terminus2</th>
</tr>
</thead>
<tbody>
<tr>
<td>5:10 AM</td>
<td>5:25 AM</td>
<td>5:36 AM</td>
<td>5:55 AM</td>
<td>6:03 AM</td>
<td>6:08 AM</td>
<td>6:11 AM</td>
</tr>
<tr>
<td>5:30 AM</td>
<td>5:45 AM</td>
<td>5:56 AM</td>
<td>6:15 AM</td>
<td>6:24 AM</td>
<td>6:30 AM</td>
<td>6:33 AM</td>
</tr>
<tr>
<td>5:40 AM</td>
<td>5:55 AM</td>
<td>6:06 AM</td>
<td>6:25 AM</td>
<td>6:34 AM</td>
<td>6:40 AM</td>
<td>6:43 AM</td>
</tr>
<tr>
<td>6:00 AM</td>
<td>6:16 AM</td>
<td>6:27 AM</td>
<td>6:46 AM</td>
<td>6:55 AM</td>
<td>7:01 AM</td>
<td>7:07 AM</td>
</tr>
<tr>
<td>6:10 AM</td>
<td>6:26 AM</td>
<td>6:36 AM</td>
<td>6:55 AM</td>
<td>7:05 AM</td>
<td>7:16 AM</td>
<td>7:23 AM</td>
</tr>
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<td>6:36 AM</td>
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<td>6:30 AM</td>
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<td>6:40 AM</td>
<td>6:56 AM</td>
<td>7:08 AM</td>
<td>7:32 AM</td>
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<td>7:57 AM</td>
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<td>6:50 AM</td>
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<td>7:47 AM</td>
<td>8:01 AM</td>
<td>8:16 AM</td>
<td>8:22 AM</td>
</tr>
</tbody>
</table>

5.9 CASE 2: SINGLE BUS ROUTE (MULTIPLE HEADWAYS)

In case 1, the headway is considered as the same for all the three time periods. However, in practice it is essential to have different headway for different time periods in order to meet the changing traffic demand during peak and slack periods. Usually, in a real situation, the passenger demand is lean during below normal time period, during the peak hours the demand will be high and during the normal time period the demand would be medium. Accordingly, it is required to operate the buses with different headway to meet the change in demand. This is termed multiple headway. However, the headway is constant during a given time period.

The simulation model developed for case 1 has been slightly modified for the multiple headway case. The stage sub-model and travelling time sub-model remain the same as in case 1. But there is slight change in the terminus sub-model which is shown in Figure 5.8, where bus despatch will be different for different time periods (headway is more for below normal period,
for the normal period it is slightly reduced and for the peak period the headway is further reduced). The working of the model for bus despatch is explained through the following algorithm.

**Figure 5.8 Terminus simulation model for case 2**

### 5.9.1 Algorithm for bus despatch

- **BNT**: Below normal Time (BNT<sub>1</sub> = 5 to 7 am (120 minutes) & BNT<sub>2</sub> = 12 to 1 pm (480 minutes)
- **NT**: Normal Time NT<sub>1</sub> = 7 to 8 am (180 minutes) & NT<sub>2</sub> = 10 to 12 am (400 minutes)
- **PT**: Peak Time 8 AM to 10 AM (300 minutes)
- **BNTH**: Below Normal Time Headway
- **NTH**: Normal Time Headway
- **PTH**: Peak time Headway
- **T<sub>c</sub>**: Simulation time
$T_n$ : End of simulation time

$T_B$ : Bus release time

Step 1: Initialise $BNT_1$, $BNT_2$, $NT_1$, $NT_2$, $PT$, $BNTH$, $NTH$, $PTH$ , $T_C$, $T_n$

$T_B=0$

Step 2: Update the clock ($T_C$)

Step 3: If ($T_C \leq T_n$) go to step 4; else go to step 18

Step 4: If ($T_C \leq BNT_1$) go to step 5; else go to step 8

Step 5: If ($T_C - T_B = BNTH$) go to step 6; else go to step 2

Step 6: Give signal for the bus despatch

Step 7: Record time $T_B$ and go to step 2

Step 8: If ($BNT_1 \leq T_C \leq NT_1$) go to step 9; else go to step 12

Step 9: If ($T_C - T_B = NTH$) go to step 10; else go to step 2

Step 10: Give the signal for the bus despatch

Step 11: Record time $T_B$ and go to step 2

Step 12: If ($NT_1 \leq T_C \leq PT$) go to step 13; else go to step 16

Step 13: If ($T_C - T_B = PTH$) go to step 14; else go to step 2

Step 14: Give the signal for the bus despatch

Step 15: Record time $T_B$ and go to step 2

Step 16: If ($PT \leq T_C \leq NT_2$) go to step 17; else go to step 5

Step 17: Go to step 9

Step 18: Stop simulation and record the statistics

The simulation is run for the study period (8 hours) with 75 replications. The input parameters for the model are the same as case 1. The simulation model is run by using different headways for different time periods; the headway for the peak period varies from 5 to 10 minutes and for the medium period the headway varies from 10 to 15 minutes and for the below normal time period 10 to 19 minutes. For each combination of
headway the simulation is run for 75 replications. A sample output of simulation is shown in Table 5.6.

**Table 5.6 Sample simulation output for multiple headway**

<table>
<thead>
<tr>
<th>Headway (Minutes)</th>
<th>Average waiting time (Minutes)</th>
<th>Total trips (No’s)</th>
<th>Passenger Travelled (No’)</th>
<th>Operation Time (Minutes)</th>
<th>Waiting time (Minutes)</th>
<th>Operating cost (Minutes)</th>
<th>Waiting time cost (Minutes)</th>
<th>Operating total cost (Minutes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>17,10,8</td>
<td>16</td>
<td>80</td>
<td>10587</td>
<td>5600</td>
<td>165369</td>
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<td>33074</td>
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</tr>
<tr>
<td>18,10,8</td>
<td>15</td>
<td>80</td>
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<td>32200</td>
<td>32290</td>
<td>64490</td>
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<td>30590</td>
<td>33370</td>
<td>63960</td>
</tr>
<tr>
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<td>80</td>
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<td><strong>5600</strong></td>
<td><strong>140881</strong></td>
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<td><strong>60376</strong></td>
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<td>5880</td>
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<td>5600</td>
<td>172992</td>
<td>32200</td>
<td>34598</td>
<td>66798</td>
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</table>

### 5.9.2 Results and Discussion

For each combination of headway, the average waiting time of passengers, number of trips required to satisfy the passenger demand and
average travelling time are recorded. The operating cost, waiting time cost and the total cost are computed for each combination as shown in Table 5.6. The headway combination corresponding to the minimum total cost is selected as the best headway. The minimum total cost is INR.60376. The cost data versus the headway is shown graphically in Figure 5.9. The headways corresponding to the minimum total cost are 19 minutes for below normal time period, 10 minutes for normal time and 7 minutes for the peak time period. And the number of trips required to satisfy the passenger demand are 80 with an average passenger waiting time of 13 minutes. With these three headways the timetable is prepared for the selected route.

![Figure 5.9 Headway versus total cost](image)

In case 1, with constant (same) headway for all time periods, the timetable preparation was easy. In case 2, the headway is different for each time period. So, preparing the timetable is not simple as in case 1. Here, the problem is to set the departure times in the transition between one time periods to another time period. A common headway smoothing rule for the transition between time periods is used (Ceder 1986). This rule may result in either overcrowding or under utilization.
For example, consider two time periods 5.00 to 5.59 and another 6.00 to 7.59. Suppose the headway for the first time period is 19 minutes and for the second period is 10 minutes. According to the common average headway rule, the transition headway will be \((19+10)/2 = 15\) minutes. Thereby the departures are set at 5.00, 5.19, 5.38, 5.57, and 6.12 AM. The timetable thus prepared for the case of multiple headway is shown in Table 5.7.

Table 5.7 Timetable of Single bus route (Multiple headway: case 2)

<table>
<thead>
<tr>
<th>Terminus</th>
<th>Stage 1</th>
<th>Stage 2</th>
<th>Stage 3</th>
<th>Stage 4</th>
<th>Stage 5</th>
<th>Terminus 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>5:00 AM</td>
<td>5:15 AM</td>
<td>5:26 AM</td>
<td>5:45 AM</td>
<td>5:54 AM</td>
<td>5:59 AM</td>
<td>6:02 AM</td>
</tr>
<tr>
<td>5:19 AM</td>
<td>5:34 AM</td>
<td>5:46 AM</td>
<td>6:03 AM</td>
<td>6:13 AM</td>
<td>6:17 AM</td>
<td>6:20 AM</td>
</tr>
<tr>
<td>6:16 AM</td>
<td>6:32 AM</td>
<td>6:44 AM</td>
<td>7:01 AM</td>
<td>7:16 AM</td>
<td>7:27 AM</td>
<td>7:32 AM</td>
</tr>
<tr>
<td>6:35 AM</td>
<td>6:50 AM</td>
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<td>7:27 AM</td>
<td>7:41 AM</td>
<td>7:52 AM</td>
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</tr>
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<td>7:56 AM</td>
</tr>
<tr>
<td>7:09 AM</td>
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<td>7:29 AM</td>
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<td>8:28 AM</td>
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</tr>
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<td>7:39 AM</td>
<td>7:54 AM</td>
<td>8:05 AM</td>
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<td>8:33 AM</td>
<td>8:38 AM</td>
<td>8:41 AM</td>
</tr>
<tr>
<td>7:49 AM</td>
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<td>8:53 AM</td>
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<td>9:01 AM</td>
</tr>
<tr>
<td>8:06 AM</td>
<td>8:21 AM</td>
<td>8:32 AM</td>
<td>8:51 AM</td>
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<td>8:13 AM</td>
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<td>8:58 AM</td>
<td>9:07 AM</td>
<td>9:12 AM</td>
<td>9:15 AM</td>
</tr>
</tbody>
</table>

5.10 CONCLUSION

A simulation model has been developed to determine the best headway for a single bus route. The model has been employed to a practical
case and determined the best headway so as to minimize the total cost (operating cost of the bus plus waiting time cost of passengers) considering constant headway (case 1) and multiple headways (case 2). The comparative results obtained by the simulation model for case 1 and case 2 are shown in Table 5.8. It is observed that the total cost for the case of multiple headways is less than the single headway. Further, there is a reduction in the number of trips by 16 with a marginal increase of 1 minute of passenger waiting time. Thus, the multiple headways are more beneficial to the transport corporation.

The recommended headway for below normal time period is 19 minutes, for normal period it is 10 minutes and for the peak period, the headway is 7 minutes. In both the models, the bus is despatched based on headway. In practical the operational issues are, difficult in preparing the crew scheduling and less utilisation of fleets.

Table 5.8  Comparison between single headway and multiple headways

<table>
<thead>
<tr>
<th>Description</th>
<th>Single headway</th>
<th>Multiple headway</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of trips (Nos)</td>
<td>96</td>
<td>80</td>
</tr>
<tr>
<td>Average Waiting time of the passenger (Minutes)</td>
<td>12.2</td>
<td>13</td>
</tr>
<tr>
<td>Waiting time cost (INR)</td>
<td>25408</td>
<td>28176</td>
</tr>
<tr>
<td>Operation cost (INR)</td>
<td>36960</td>
<td>32200</td>
</tr>
<tr>
<td>Total cost (INR)</td>
<td>62792</td>
<td>60376</td>
</tr>
<tr>
<td>Headway (Minutes)</td>
<td>10 minutes (for all time periods)</td>
<td>19 minutes (Below normal), 10 minutes (Normal), 7 minutes (peak)</td>
</tr>
</tbody>
</table>