Chapter 4

Mean Response Time of $G/G/1$ to $G/G/1$ Queueing Network Model without Feedback

4.1 Introduction

Now a days time is very precious for everyone. Everyone require the service as fast as possible. In the analysis of queueing network models, the response time plays an important role in studying the various characteristics. The response time is defined as the time elapsed from the instance of job arrival until its completion. It means the time spent by a customer from arrival until it departs. Response time is the total amount of time it takes to respond to a request for service. That service can be anything from a memory fetch, to a disk IO, to a complex database query, or loading a full web page. Ignoring transmission time for a moment, the response time is the sum of the service time and wait time. The service time is the time it takes to do the work you requested. Chu and Ke [14] examined the statistical behavior of the mean response time for the $M/G/1$ queueing system using bootstrapping simulation. Chu and

Ke [15] proposed a consistent and asymptotically normal (CAN) estimator of the mean response time for a G/M/1 queueing system, which is based on the fixed point of empirical Laplace function. Chu and Ke [16] developed a data based recurrence relation to compute a sequence of mean response times and constructed confidence intervals of mean response times for the $G/G/1$ queueing system. In addition, a numerical simulation study is conducted in order to demonstrate performance of the proposed estimator and bootstrap confidence intervals. In this Chapter we consider a two-stage open queueing network model as shown in Figure 1.1.

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4.2 Estimation of Mean Response Time

Let X and Y be a nonnegative continuous random variables representing inter-arrival times and service times of node-1 of a two stage open queueing network model. Let Y and Z be a nonnegative continuous random variables representing inter-arrival times and service times of node-2 of a two stage open queueing network model respectively.

Let $(X_1, X_2, \cdots, X_n)$ be a random sample drawn from a continuous random variable X and let $(Y_1, Y_2, \cdots, Y_n)$ be a random sample drawn from a continuous random variable Y that is independent of X. Let (X_i, Y_i) represent inter-arrival time and service time for the i^{th} customer of node-1 of a two stage open queueing network model.

Let $(Y_1, Y_2, \cdots, Y_n)$ be a random sample drawn from a continuous random variable Y that is independent of Z and let $(Z_1, Z_2, \cdots, Z_n)$ be a random sample drawn from a continuous random variable Z. Let (Y_i, Z_i) represent inter-arrival time and service time for the i^{th} customer of node-2.

Let R_{1i} and R_{2i} represent the response time of i^{th} customer of node-1 and node-2 of a two stage open queueing network and are determined from $(X_1, Y_1), (X_2, Y_2), \cdots, (X_n, Y_n)$ and $(Y_1, Z_1), (Y_2, Z_2), \cdots, (Y_n, Z_n)$ respectively. Let W_{1i} and W_{2i} denote the waiting time of i^{th} customer of node-1 and node-2. Then

$$R_{1i} = W_{1i} + Y_i, \quad i = 1, 2, \cdots, n, \tag{4.1}$$

$$R_{2i} = W_{2i} + Z_i, \quad i = 1, 2, \cdots, n, \tag{4.2}$$

Using analysis by Kleinrock [47] we can calculate waiting time W_{1i} and W_{2i} of a customer in a two stage open queueing network model using recursion relation given by

$$W_{1i} = (R_{1,i-1} - X_i)I(R_{1,i-1} > X_i) \tag{4.3}$$

$$W_{2i} = (R_{2,i-1} - Y_i)I(R_{2,i-1} > Y_i) \tag{4.4}$$

for $i = 2, 3, \cdots, n$, $W_{11} = 0$ and $W_{21} = 0$ also $I(.)$ denote the indicator function. Using equation (4.3) in (4.1) we obtain

$$R_{1i} = (R_{1,i-1} - X_i)I(R_{1,i-1} > X_i) + Y_i \tag{4.5}$$

and using equation (4.4) in (4.2) we obtain

$$R_{2i} = (R_{2,i-1} - Y_i)I(R_{2,i-1} > Y_i) + Z_i \tag{4.6}$$

for $i = 2, 3, \cdots, n$ and $R_{11} = Y_1$ and $R_{21} = Z_1$. Equations (4.5) and (4.6) are the exact data based recursion relation for calculating response times

$R_{11}, R_{12}, \cdots, R_{1n}$ and $R_{21}, R_{22}, \cdots, R_{2n}$ respectively that are exactly as a sequence of customers response times for a two stage open queueing network model. Hence

$$\hat{r}_1 = \frac{1}{n}\sum_{i=1}^{n} R_{1i}$$

and

$$\hat{r}_2 = \frac{1}{n}\sum_{i=1}^{n} R_{2i}$$

Thus $\hat{r}_1$ and $\hat{r}_2$ is the mean of these response times is a estimator of the mean response times r_1 and r_2 for a two stage open queueing network model.Using simulation technique we will show that $\hat{r}_1$ and $\hat{r}_2$ are consistent estimators of r_1 and r_2.

4.3 Confidence Intervals for Mean Response Time

In this section we construct various confidence intervals for mean response time r_1 and r_2. Different bootstrap approaches such as standard bootstrap, bootstrap-t, percentile bootstrap, bias-corrected and accelerated bootstrap, bias corrected percentile bootstrap approaches are applied to develop the confidence intervals for mean response time. Further to improve the coverage accuracy of these confidence intervals we use calibration technique.

4.3.1 Standard Bootstrap Confidence Intervals

Let $x = (x_1, x_2, \cdots, x_n)'$ be a random sample of size n drawn from population X. Let $y = (y_1, y_2, \cdots, y_n)'$ be a random sample of size n drawn from population Y. Let $z = (z_1, z_2, \cdots, z_n)'$ be a random sample of size n drawn from population Z.

According to the bootstrap procedure, a simple random sample $x^* = (x_1^*, x_2^*, \cdots, x_n^*)'$ can be taken from the empirical distribution function of $x = (x_1, x_2, \cdots, x_n)'$, called a bootstrap sample from $x = (x_1, x_2, \cdots, x_n)'$.

Similarly we can draw a bootstrap samples $y^* = (y_1^*, y_2^*, \cdots, y_n^*)'$ and $z^* = (z_1^*, z_2^*, \cdots, z_n^*)'$ from $y = (y_1, y_2, \cdots, y_n)'$ and $z = (z_1, z_2, \cdots, z_n)'$ respectively.

Using equation (4.5)and (4.6) we respectively obtain $r_{11}, r_{12} \cdots, r_{1n}$ and $r_{21}, r_{22} \cdots, r_{2n}$ are sequences of customers response time. Hence the estimator

$$\hat{r}_i = \frac{1}{n} \sum_{j=1}^{n} r_{ij} \ , \quad i = 1, 2.$$

$\hat{r}_i$ is a natural estimate of the mean response time $r_i, \quad i = 1,\ 2$ of a two stage open queueing network model.

Similarly using equation (4.5)and (4.6) we respectively obtain $r_{11}^*, r_{12}^* \cdots, r_{1n}^*$ and $r_{21}^*, r_{22}^* \cdots, r_{2n}^*$. The estimator

$$\hat{r}_i^* = \frac{1}{n} \sum_{j=1}^{n} r_{ij}^* \ , \quad i = 1,\ 2 \tag{4.7}$$

is called bootstrap estimate of $\hat{r}_i$.

The above re-sampling process can be repeated N times. The N bootstrap estimates $\hat{r}_{i1}^*, \hat{r}_{i2}^*, \cdots, \hat{r}_{iN}^*$ can be computed from the bootstrap resamples. Averaging the N bootstrap estimates we get

$$\hat{r}_N(i) = \frac{1}{N} \sum_{j=1}^{N} \hat{r}_{ij}^* \ , \quad i = 1,\ 2 \tag{4.8}$$

the bootstrap estimate of $r_i \ \ i = 1, 2$ and standard deviation of $r_i \ \ i = 1, 2$ can be estimated by

$$sd(\hat{r}_N(i)) = \left[\frac{1}{N-1} \sum_{j=1}^{N} \left[r_{ij}^* - \hat{r}_N(i) \right]^2 \right]^{\frac{1}{2}} \ , \quad i = 1,\ 2. \tag{4.9}$$

By Chu and Ke [16] the distribution of $\hat{r}_1$ and $\hat{r}_2$ are approximately normal. Therefore $100(1-\alpha)\%$ SB confidence intervals for mean response time r_1 and

r_2 are given by

$$\left(\hat{r}_i - z_{a/2} sd(\hat{r}_N(i)) \ , \quad \hat{r}_i + z_{a/2} sd(\hat{r}_N(i))\right) \tag{4.10}$$

4.3.2 Bootstrap-t Confidence Intervals

Consider N bootstrap estimate $\hat{r}^*_{i1}, \hat{r}^*_{i2}, \cdots, \hat{r}^*_{iN}$ computed from the bootstrap resample. We obtain $Z^*_{ij} = \frac{\hat{r}^*_{ij} - \hat{r}_N(i)}{sd(\hat{r}_N(i))}, i = 1, 2, j = 1, 2, \cdots, N$ and sample $Z^*_{i1}, Z^*_{i2}, \cdots, Z^*_{iN}$ considered as an approximate t distribution(Efron and Tibshirani [26]). Thus $100(1-\alpha)\%$ bootstrap-t confidence intervals for mean response time $r_i \ , \quad i = 1, \ 2$ are given by

$$\left(\hat{r}_i - \hat{t}_{a/2} sd(\hat{r}_N(i)) \ , \quad \hat{r}_i + \hat{t}_{a/2} sd(\hat{r}_N(i))\right) \ , \quad i = 1, \ 2. \tag{4.11}$$

where $\hat{t}_{a/2}$ equals the $\alpha/2$ percentile of the random sample $Z^*_{i1}, Z^*_{i2}, \cdots, Z^*_{iN}$.

4.3.3 Percentile Bootstrap Confidence Intervals

Now call $\hat{r}^*_{i1}, \hat{r}^*_{i2}, \cdots, \hat{r}^*_{iN}$ the bootstrap distribution of $\hat{r}_i \ , \quad i = 1, \ 2$. Let $\hat{r}^*_i(1), \hat{r}^*_i(2), \cdots, \hat{r}^*_i(N)$ be the order statistics of $\hat{r}^*_{i1}, \hat{r}^*_{i2}, \cdots, \hat{r}^*_{iN}$. Then utilizing the $100(\alpha/2)^{th}$ and $100(1-\alpha/2)^{th}$ percentage points of the bootstrap distribution, $100(1-\alpha)\%$ PB confidence intervals for mean response time $r_i \ , \quad i = 1, \ 2$ are given by

$$\left(\hat{r}^*_i\left(\left[N\left(\frac{\alpha}{2}\right)\right]\right), \ \hat{r}^*_i\left(\left[N\left(1-\frac{\alpha}{2}\right)\right]\right)\right) \ , \quad i = 1, \ 2. \tag{4.12}$$

where $[x]$ denotes the greatest integer less than or equal to x.

4.3.4 Bias Corrected and Accelerated Bootstrap Confidence Intervals

The bootstrap distribution $\hat{r}^*_{i1}, \hat{r}^*_{i2}, \cdots, \hat{r}^*_{iN}$ may be biased. This method is designed to correct this potential bias and accelerate convergence of the bootstrap distribution. Set

$$p_i = \frac{1}{N}\sum_{j=1}^{N} I(\hat{r}^*_{ij} < \hat{r}_i)\ , \quad i = 1,\ 2$$

where $I(.)$ is the indicator function.Define $\hat{z}_i = \Phi^{-1}(p_i)\ , \quad i = 1,\ 2$ where Φ^{-1} denotes the inverse function of the standard normal distribution Φ. Let $\widetilde{X}_i(k)$ and $\widetilde{Y}_i(k)\ , \quad i = 1,\ 2, \quad k = 1, 2, \cdots n$ denote the original samples with k^{th} observation X_{ik} and Y_{ik} deleted, also $\hat{r}_{ik}$ be the estimator of $r_i\ , \quad i = 1,\ 2$ calculated by using $\widetilde{X}_i(k)$ and $\widetilde{Y}_i(k)\ , \quad i = 1,\ 2$.

Define

$$\widetilde{r}_i = \frac{1}{n}\sum_{k=1}^{n} \hat{r}_{ik}\ , \quad i = 1,\ 2$$

and

$$\hat{a}_i = \frac{\sum_{k=1}^{n}(\widetilde{r}_i - \hat{r}_{ik})^3}{6\left(\sum_{k=1}^{n}(\widetilde{r}_i - \hat{r}_{ik})^2\right)^{\frac{3}{2}}}\ , \quad i = 1,\ 2$$

where $\hat{z}_i$ and $\hat{a}_i\ , \quad i = 1,\ 2$ are denote bias-correction and acceleration respectively. Thus $100(1-\alpha)\%$ BCAB confidence intervals for mean response time $r_i\ , \quad i = 1,\ 2$ are given by

$$\left(\hat{r}^*_i\left([N\alpha_{i1}]\right),\quad \hat{r}^*_i\left([N\alpha_{i2}]\right)\right)\quad , \quad i = 1\ 2. \tag{4.13}$$

where

$$\alpha_{i1} = \Phi\left[\hat{z}_i + \frac{\hat{z}_i - z_{\alpha/2}}{1 - \hat{a}_i\left(\hat{z}_i - z_{\alpha/2}\right)}\right]$$

and

$$\alpha_{i2} = \Phi\left[\hat{z}_i + \frac{\hat{z}_i + z_{\alpha/2}}{1 - \hat{a}_i\left(\hat{z}_i + z_{\alpha/2}\right)}\right]\quad , \quad i = 1,\ 2.$$

4.3.5 Bias Corrected Percentile Bootstrap Confidence Intervals

The bootstrap distribution $\hat{r}^*_{i1}, \hat{r}^*_{i2}, \cdots, \hat{r}^*_{iN}$ may be biased. This method is designed to correct this potential bias of the bootstrap distribution. Let $a_1 = \Phi(2\hat{z}_i - z_{\alpha/2})$ and $a_2 = \Phi(2\hat{z}_i + Z_{\alpha/2})$, $i =, 1\ 2$. Thus $100(1-\alpha)\%$ BCPB confidence intervals for mean response time r_i , $i = 1,\ 2$ are given by

$$(\hat{r}^*_i([Na_1]), \quad \hat{r}^*_i([Na_2])) \quad , \quad i = 1\ 2. \tag{4.14}$$

where $[x]$ denotes the greatest integer less than or equal to x.

4.4 Simulation Study

In order to evaluate performance of the natural estimator $r_i, i = 1, 2$ and different confidence intervals constructed for mean response time $r_i, i = 1, 2$ using bootstrap methods discussed in Section 4.3 we conduct a numerical simulation study. The consistency of $r_i, i = 1, 2$ is examined by comparing true value of $r_i, i = 1, 2$ with the average simulated estimates $\hat{r}_i, i = 1, 2$, whereas the different confidence intervals are assessed in terms of their coverage percentages, average lengths, relative coverages and relative average lengths. Relative coverage is defined as the ratio of coverage percentage to average length of confidence interval. Larger relative coverage implies the better performances of the corresponding confidence interval. Relative average length is defined as the ratio of average length to the true value of $r_i, i = 1, 2$. For a given confidence level shorter relative average length implies the better performances of the corresponding confidence interval. In order to achieve these goals, in simulation study we select following queueing network models:-

1. $E_4/H_4^{Pe}/1$ to $H_4^{Pe}/H_4^{Po}/1$,

2. $E_4/H_4^{Po}/1$ to $H_4^{Po}/H_4^{Pe}/1$,

3. $H_4^{Pe}/H_4^{Po}/1$ to $H_4^{Po}/E_4/1$

where (E_4) denotes a 4-stage Erlang distribution, (H_4^{Pe}) represents a 4-stage hyperexponential distribution and (H_4^{Po}) is a 4-stage hypoexponential distribution on arrival times and service times at two nodes.

There is no theoretical formula for the true value of $r_i, i = 1, 2$ with regard to queueing network models 1 to 3. Using strong law of large numbers (Ross [61]), we have estimated the true value of $r_i, i = 1, 2$ by the simulated sample values of $\hat{r}_i, i = 1, 2$ for sample size $n = 10^7$ and are shown in Table 4.1 and 4.2.

Table 4.1: Simulation analysis of queueing network model without feedback for consistency of $\hat{r}_i, i = 1, 2$ for $r_1 < r_2$

Queueing network model	The true value of $r_i, i = 1, 2$	The average of 1000 simulated $\hat{r}_i, i = 1, 2$					
		$n = 5$	$n = 10$	$n = 20$	$n = 30$	$n = 50$	$n = 100$
$E_4/H_4^{Pe}/1$ to	$r_1 = 1.01223$	0.99740	1.01507	1.01164	1.00669	1.00790	1.01537
$H_4^{Pe}/H_4^{Po}/1$	$r_2 = 1.78596$	1.07058	1.21998	1.40911	1.51203	1.61064	1.68165
$E_4/H_4^{Po}/1$ to	$r_1 = 1.01214$	1.00994	1.01199	1.00774	1.01591	1.01021	1.00888
$H_4^{Po}/H_4^{Pe}/1$	$r_2 = 1.78273$	1.05210	1.21362	1.44207	1.48992	1.62394	1.70282
$H_4^{Pe}/H_4^{Po}/1$ to	$r_1 = 1.01834$	0.99698	1.01361	1.02138	1.01486	1.01395	1.01489
$H_4^{Po}/E_4/1$	$r_2 = 1.62863$	1.04359	1.18239	1.32274	1.40433	1.48179	1.55797

For queueing network models 1 to 3, a random sample of sample size n (= 5, 10, 20, 30) is drawn from the original samples. Using Equations (4.5) and (4.6), the natural estimates of $\hat{r}_i, i = 1, 2$ are calculated. Further N =1000 bootstrap resamples are drawn from the original samples. By formula (4.7), N bootstrap estimates $\hat{r}_{i1}^*, \hat{r}_{i2}^*, \cdots, \hat{r}_{iN}^*, i = 1, 2$ are calculated from the bootstrap resamples. Using equations (4.8) and (4.9), the estimated standard deviation of $\hat{r}_i, i = 1, 2$ are computed as $sd(\hat{r}_N(i)), i = 1, 2$. From equations (4.10) to (4.14) we obtain the SB, the Boot-t, the PB , the BCPB and the BCAB

Table 4.2: Simulation analysis of queueing network model without feedback for consistency of $\hat{r}_i, i = 1, 2$ for $r_1 > r_2$

Queueing network model	The true value of $r_i, i = 1, 2$	The average of 1000 simulated $\hat{r}_i, i = 1, 2$					
		$n = 5$	$n = 10$	$n = 20$	$n = 30$	$n = 50$	$n = 100$
$E_4/H_4^{Pe}/1$ to	$r_1 = 2.02946$	1.30631	1.46536	1.69428	1.77118	1.87930	1.95221
$H_4^{Pe}/H_4^{Po}/1$	$r_2 = 0.20361$	0.20537	0.20314	0.20576	0.20387	0.20260	0.20314
$E_4/H_4^{Po}/1$ to	$r_1 = 2.03175$	1.30944	1.48727	1.69305	1.73542	1.87706	1.93905
$H_4^{Po}/H_4^{Pe}/1$	$r_2 = 0.20370$	0.20767	0.20192	0.20265	0.20375	0.20339	0.20374
$H_4^{Pe}/H_4^{Po}/1$ to	$r_1 = 2.23246$	1.35047	1.54602	1.75725	1.86175	1.93205	2.10865
$H_4^{Po}/E_4/1$	$r_2 = 0.20254$	0.20015	0.20217	0.20423	0.20348	0.20194	0.20219

confidence intervals for mean response times with confidence level 90%. The above simulation process is replicated N =1000 times and we average the N simulated estimates $\hat{r}_i, i = 1, 2$. The consistency property of the natural estimators of $r_i, i = 1, 2$ are shown in Tables 4.1 and 4.2. Also we comupte coverage percentage, average length, relative coverage and relative average length of bootstrap confidence intervals.

Further calibration technique discussed in Section 1.4 is used to improve the coverage percentage of confidence intervals obtained in (4.10) to (4.14). Based on the different interval estimation approaches coverage percentage, average length, relative coverage and relative average length of $r_i, i = 1, 2$ before calibration and after calibration are shown in Tables 4.5 to 4.10 for queueing network models 1 to 3. Also some interesting results are shown in Tables 4.11 and 4.12, which queueing network model or estimation approaches give greater relative coverage and shortest relative average length.

In Tables 4.1 and 4.2 we note down the mean of N simulated $\hat{r}_i, i = 1, 2$ for various values of n. Tables 4.1 and 4.2 gives the sample mean of $\hat{r}_i, i = 1, 2$ converges to the true value of $r_i, i = 1, 2$ as the sample size becomes large enough under any specified queueing network model as we expect.

Table 4.3: Maximum percentage(%) increase in coverage percentage due to calibration technique

Increase in Coverage Percentages for $r_1 < r_2$				
Queueing Network Model	$n = 5$	$n = 10$	$n = 20$	$n = 30$
$E_4/H_4^{Pe}/1$ to $H_4^{Pe}/H_4^{Po}/1$	10.3	5.7	12.9	8.1
$E_4/H_4^{Po}/1$ to $H_4^{Po}/H_4^{Pe}/1$	9.5	9.8	10.5	8.4
$H_4^{Pe}/H_4^{Po}/1$ to $H_4^{Po}/E_4/1$	7.9	9.9	10.7	9.9
Increase in Coverage Percentages for $r_1 \geq r_2$				
$E_4/H_4^{Pe}/1$ to $H_4^{Pe}/H_4^{Po}/1$	9.5	9.3	11.8	10.0
$E_4/H_4^{Po}/1$ to $H_4^{Po}/H_4^{Pe}/1$	11.5	9.9	7.7	9.7
$H_4^{Pe}/H_4^{Po}/1$ to $H_4^{Po}/E_4/1$	12.2	10.8	11.3	8.6

The frequency of coverage for an accurate confidence interval is binomially distributed with $N = 1000$ and $p = 0.90$. Therefore a 99% confidence range for the coverage percentage is $0.9 \pm 2.58\sqrt{0.9 \times 0.1/1000}$. Hence we 99% expect that true 90% confidence interval would have proportion of coverage between $(0.8755, 0.9245)$. For queueing network models 1 to 3 and four different sample sizes n (= 5, 10, 20, 30), In all there are 24 coverage percentages produced in Tables 4.5 to 4.10 for each of the five bootstrap methods. The chances of coverage percentage is inside the 99% confidence range $(0.8755, 0.9245)$ before and after calibration are sumarized in Table 4.4.

Table 4.4: The chances of coverage percentage inside the 99% confidence range for queueing network model without feedback

Bootstrap methods	SB1	SB2	Boot-$t1$	Boot-$t2$	PB1	PB2	BCPB1	BCPB2	BCAB1	BCAB2
Before Calibration	1/24	0/24	2/24	0/24	0/24	0/24	0/24	0/24	0/24	0/24
After calibration	7/24	12/24	5/24	6/24	6/24	6/24	5/24	6/24	4/24	6/24

4.5 Conclusions

The following conclusions are made based on confidence intervals for mean response time of a two stage queueing network model without feedback:

- From Tables 4.1 and 4.2 we find that the approximated mean response time approaches to the true value of $r_i, i = 1, 2$ when $n \geq 10^7$. Hence according to simulation study we observe that $\hat{r}_i, i = 1, 2$ is a consistent estimator of the mean response time $r_i, i = 1, 2$.

- From Table 4.4 it is observed that before calibration hardly there is any coverage percentage inside the 99% confidence interval range $(0.8755, 0.9245)$ but after calibration the SB method has highest chance that the coverage percentage is inside the 99% confidence interval range $(0.8755, 0.9245)$.

- From Tables 4.5 to 4.10, before and after calibration we observed that if $\rho_i < 0.5$ the bootstrap confidence intervals SB, Boot-t, PB, BCPB and BCAB have increasing coverage percentage and relative coverage as n increases but average length and relative average length are decreasing as n increases to 30. Also we observed that if $\rho_i \geq 0.5$ the bootstrap confidence intervals SB, Boot-t, PB, BCPB and BCAB have increasing coverage percentage,average length and relative average length as n increases but relative coverages are decreasing as n increases to 30. The SB method has the largest coverage percentage among the SB, Boot-t, PB, BCPB and BCAB methods before and after calibration for small samples.

- From Table 4.11 we observed that with respect to relative coverage BCPB method before calibration and SB method after calibration for $r_1 < r_2$ among the SB, Boot-t, PB, BCPB and BCAB methods can build the best confidence intervals for mean response time $r_i, i = 1, 2$. Also BCAB method before calibration and SB method after calibration for $r_1 \geq r_2$ among the SB, Boot-t, PB, BCPB and BCAB methods can construct the best confidence intervals for mean response time $r_i, i = 1, 2$.

- From Table 4.12 we observed that with respect to relative average length BCAB method before calibration and SB method after calibration for $r_1 < r_2$ among the SB, Boot-t, PB, BCPB and BCAB methods can construct the best confidence intervals for mean response time $r_i, i = 1, 2$. Also BCPB or BCAB method before calibration and BCAB method after calibration for $r_1 \geq r_2$ among the SB, Boot-t, PB, BCPB and BCAB methods can construct the best confidence intervals for mean response time $r_i, i = 1, 2$.

- From Table 4.3 we observed that due to calibration, coverage percentage increases between 5.70% to 12.90% for small sample sizes for $E_4/H_4^{Pe}/1$ to $H_4^{Pe}/H_4^{Po}/1$ queueing network model.

4.6 Summary

In this chapter we estimate mean reponse time $r_i, i = 1, 2$ for a queueing network models. Using recursion relation we obtained a sequence of response time for a queueing network model and the arithmetic mean of these response times is used as an estimate of the mean response time $r_i, i = 1, 2$. This estimator $\hat{r}_i, i = 1, 2$ is verified to be consistent by simulation study. Further we evaluated confidence intervals for the mean response time $r_i, i = 1, 2$ based on the bootstrap methods such as SB, Boot-t, PB, BCPB and BCAB. Confidence intervals are assessed in terms of their coverage percentage, average length, relative coverage and relative average length.

The simulation study imply that with respect to relative coverage BCPB and BCAB methods before calibration and SB method after calibration among the SB, Boot-t, PB, BCPB and BCAB methods can construct the best confidence intervals for mean response time $r_i, i = 1, 2$. Further with respect to relative average length BCAB or BCPB methods before calibration and SB or BCAB methods among the SB, Boot-t, PB, BCPB and BCAB methods can construct the best confidence intervals for mean response time $r_i, i = 1, 2$. Coverage percentage increased from 5.70% to 12.90% due to calibration for small samples.

Different estimation approaches SB, Boot-t, PB, BCPB and BCAB are applied to construct various confidence intervals for mean response time $r_i, i = 1, 2$. These approaches are successfully and efficiently applied to practical queueing network models. Also calibration technique is used to improve the coverage percentage of confidence intervals.

Table 4.5: Simulation results of $E_4/H_4^{Pe}/1$ to $H_4^{Pe}/H_4^{Po}/1$ queueing network model without feedback for $r_1 < r_2$

Coverage Percentages								
Estimation	Before Calibration				After Calibration			
Approches	$n=5$	$n=10$	$n=20$	$n=30$	$n=5$	$n=10$	$n=20$	$n=30$
SB1	0.739	0.832	0.861	0.873	0.842	0.875	0.906	0.913
SB2	0.738	0.816	0.822	0.841	0.799	0.819	0.860	0.865
Boot-t1	0.740	0.826	0.859	0.882	0.814	0.861	0.905	0.897
Boot-t2	0.700	0.754	0.745	0.735	0.737	0.756	0.774	0.766
PB1	0.734	0.815	0.851	0.848	0.824	0.867	0.894	0.912
PB2	0.699	0.764	0.746	0.758	0.787	0.795	0.837	0.834
BCPB1	0.734	0.809	0.842	0.852	0.832	0.866	0.904	0.909
BCPB2	0.694	0.763	0.726	0.754	0.793	0.794	0.843	0.834
BCAB1	0.724	0.806	0.839	0.859	0.813	0.845	0.900	0.907
BCAB2	0.702	0.760	0.728	0.760	0.796	0.811	0.857	0.841
Average Lengths								
SB1	0.750	0.578	0.438	0.369	0.918	0.668	0.480	0.392
SB2	1.294	1.746	2.082	2.208	1.606	1.956	2.445	2.457
Boot-t1	0.747	0.574	0.436	0.367	0.885	0.665	0.481	0.384
Boot-t2	1.261	1.664	1.944	2.034	1.580	2.034	2.566	2.689
PB1	0.740	0.566	0.431	0.364	0.882	0.671	0.487	0.401
PB2	1.217	1.571	1.863	2.038	1.493	1.880	2.471	2.519
BCPB1	0.732	0.562	0.430	0.363	0.871	0.666	0.486	0.400
BCPB2	1.184	1.559	1.874	2.032	1.463	1.884	2.480	2.540
BCAB1	0.717	0.559	0.428	0.362	0.859	0.662	0.485	0.394
BCAB2	1.221	1.625	1.961	2.142	1.508	1.975	2.615	2.657
Relative Coverages								
SB1	0.985	1.439	1.964	2.367	0.918	**1.310**	**1.887**	2.328
SB2	0.571	0.467	0.395	**0.381**	0.498	0.419	**0.352**	**0.352**
Boot-t1	0.991	1.438	1.971	**2.400**	0.920	1.295	1.883	**2.337**
Boot-t2	0.555	0.453	0.383	0.361	0.466	0.372	0.302	0.285
PB1	0.991	1.440	**1.974**	2.332	0.935	1.293	1.835	2.272
PB2	0.574	0.486	**0.400**	0.372	0.527	**0.423**	0.339	0.331
BCPB1	1.003	1.439	1.960	2.350	**0.955**	1.301	1.861	2.273
BCPB2	**0.586**	**0.489**	0.388	0.371	**0.542**	0.421	0.340	0.328
BCAB1	**1.009**	**1.442**	1.961	2.376	0.946	1.277	1.857	2.305
BCAB2	0.575	0.468	0.371	0.355	0.528	0.411	0.328	0.317
Relative Average Length								
SB1	0.752	0.570	0.433	0.366	0.920	0.658	**0.475**	0.390
SB2	1.208	1.431	1.478	1.460	1.500	1.603	**1.735**	**1.625**
Boot-t1	0.749	0.566	0.431	0.365	0.887	0.655	0.475	**0.381**
Boot-t2	1.178	1.364	1.380	1.345	1.476	1.667	1.821	1.778
PB1	0.742	0.558	0.426	0.361	0.884	0.661	0.482	0.399
PB2	1.137	1.288	**1.322**	1.348	1.394	**1.541**	1.754	1.666
BCPB1	0.734	0.554	0.425	0.360	0.873	0.656	0.480	0.397
BCPB2	**1.106**	**1.278**	1.330	**1.344**	**1.366**	1.545	1.760	1.680
BCAB1	**0.719**	**0.551**	**0.423**	**0.359**	**0.861**	**0.652**	0.479	0.391
BCAB2	1.140	1.332	1.392	1.417	1.408	1.618	1.855	1.757

Table 4.6: Simulation results of $E_4/H_4^{Po}/1$ to $H_4^{Po}/H_4^{Pe}/1$ queueing network model without feedback for $r_1 < r_2$

Coverage Percentages								
Estimation	Before Calibration				After Calibration			
Approches	$n = 5$	$n = 10$	$n = 20$	$n = 30$	$n = 5$	$n = 10$	$n = 20$	$n = 30$
SB1	0.755	0.844	0.870	0.863	0.834	0.845	0.870	0.897
SB2	0.760	0.805	0.805	0.852	0.800	0.850	0.848	0.877
Boot-t1	0.757	0.836	0.865	0.864	0.817	0.841	0.873	0.891
Boot-t2	0.719	0.745	0.709	0.761	0.756	0.786	0.756	0.809
PB1	0.757	0.830	0.843	0.845	0.821	0.853	0.864	0.888
PB2	0.699	0.734	0.754	0.765	0.791	0.821	0.843	0.849
BCPB1	0.749	0.815	0.852	0.849	0.823	0.855	0.865	0.888
BCPB2	0.690	0.745	0.747	0.761	0.785	0.842	0.844	0.835
BCAB1	0.742	0.810	0.845	0.842	0.802	0.850	0.863	0.887
BCAB2	0.691	0.748	0.748	0.773	0.786	0.846	0.853	0.835
Average Lengths								
SB1	0.732	0.595	0.446	0.365	0.905	0.663	0.472	0.397
SB2	1.328	1.699	2.081	2.257	1.634	2.027	2.448	2.569
Boot-t1	0.732	0.590	0.443	0.364	0.870	0.667	0.474	0.394
Boot-t2	1.295	1.621	1.945	2.081	1.619	2.080	2.614	2.785
PB1	0.726	0.583	0.438	0.361	0.874	0.674	0.479	0.401
PB2	1.217	1.531	1.931	2.105	1.531	1.953	2.384	2.629
BCPB1	0.714	0.578	0.436	0.360	0.866	0.666	0.478	0.400
BCPB2	1.203	1.512	1.910	2.086	1.499	1.948	2.361	2.636
BCAB1	0.702	0.574	0.434	0.359	0.855	0.664	0.481	0.403
BCAB2	1.238	1.564	2.000	2.184	1.556	2.028	2.504	2.738
Relative Coverages								
SB1	1.032	1.419	1.951	2.362	0.921	1.274	**1.844**	2.262
SB2	0.572	0.474	0.387	**0.377**	0.490	0.419	0.346	**0.341**
Boot-t1	1.035	1.417	1.952	**2.373**	0.940	1.261	1.841	**2.263**
Boot-t2	0.555	0.460	0.365	0.366	0.467	0.378	0.289	0.291
PB1	1.043	**1.423**	1.926	2.343	0.939	1.266	1.804	2.216
PB2	**0.574**	0.479	0.390	0.363	0.517	0.420	0.354	0.323
BCPB1	1.049	1.410	**1.956**	2.357	**0.951**	**1.284**	1.811	2.222
BCPB2	0.574	**0.493**	**0.391**	0.365	**0.524**	**0.432**	**0.358**	0.317
BCAB1	**1.058**	1.410	1.949	2.345	0.938	1.280	1.796	2.203
BCAB2	0.558	0.478	0.374	0.354	0.505	0.417	0.341	0.305
Relative Average Length								
SB1	0.724	0.588	0.442	0.360	0.896	**0.655**	**0.468**	0.390
SB2	1.262	1.400	1.443	1.515	1.553	1.670	1.697	**1.724**
Boot-t1	0.724	0.583	0.440	0.358	0.861	0.659	0.471	**0.388**
Boot-t2	1.231	1.335	1.348	**1.397**	1.538	1.714	1.813	1.869
PB1	0.719	0.576	0.434	0.355	0.865	0.666	0.475	0.394
PB2	1.157	1.261	1.339	1.413	1.455	1.609	1.653	1.765
BCPB1	0.707	0.571	0.432	0.355	0.857	0.658	0.474	0.393
BCPB2	**1.143**	**1.246**	**1.325**	1.400	**1.425**	**1.605**	**1.637**	1.769
BCAB1	**0.695**	**0.568**	**0.430**	**0.353**	**0.846**	0.656	0.477	0.396
BCAB2	1.177	1.289	1.387	1.466	1.479	1.671	1.737	1.838

Table 4.7: Simulation results of $H_4^{Pe}/H_4^{Po}/1$ to $H_4^{Po}/E_4/1$ queueing network model without feedback for $r_1 < r_2$

Coverage Percentages								
Estimation	Before Calibration				After Calibration			
Approches	$n=5$	$n=10$	$n=20$	$n=30$	$n=5$	$n=10$	$n=20$	$n=30$
SB1	0.757	0.826	0.855	0.883	0.814	0.885	0.885	0.893
SB2	0.772	0.820	0.843	0.865	0.820	0.853	0.883	0.894
Boot-t1	0.758	0.820	0.853	0.881	0.792	0.873	0.877	0.891
Boot-t2	0.739	0.760	0.746	0.775	0.773	0.786	0.801	0.812
PB1	0.747	0.798	0.842	0.861	0.796	0.878	0.881	0.909
PB2	0.732	0.753	0.752	0.774	0.800	0.852	0.859	0.848
BCPB1	0.753	0.809	0.840	0.854	0.801	0.874	0.881	0.895
BCPB2	0.709	0.757	0.758	0.751	0.788	0.838	0.856	0.850
BCAB1	0.738	0.798	0.833	0.856	0.785	0.873	0.864	0.885
BCAB2	0.709	0.762	0.755	0.759	0.787	0.840	0.859	0.854
Average Lengths								
SB1	0.766	0.605	0.454	0.376	0.913	0.696	0.499	0.394
SB2	1.149	1.496	1.745	1.869	1.406	1.697	1.996	2.029
Boot-t1	0.761	0.599	0.451	0.374	0.876	0.690	0.497	0.394
Boot-t2	1.123	1.427	1.629	1.719	1.402	1.762	2.160	2.233
PB1	0.751	0.590	0.446	0.369	0.874	0.687	0.500	0.411
PB2	1.070	1.373	1.617	1.738	1.329	1.690	2.041	2.223
BCPB1	0.738	0.584	0.443	0.367	0.865	0.681	0.499	0.409
BCPB2	1.060	1.362	1.628	1.753	1.300	1.667	2.026	2.216
BCAB1	0.724	0.579	0.441	0.366	0.853	0.680	0.499	0.405
BCAB2	1.086	1.405	1.683	1.814	1.337	1.712	2.115	2.297
Relative Coverages								
SB1	0.988	1.365	1.882	2.350	0.892	1.272	**1.775**	**2.266**
SB2	0.672	0.548	**0.483**	**0.463**	0.583	0.503	**0.442**	**0.441**
Boot-t1	0.996	1.370	1.892	**2.355**	0.904	1.265	1.765	2.263
Boot-t2	0.658	0.532	0.458	0.451	0.551	0.446	0.371	0.364
PB1	0.994	1.353	1.890	2.332	0.911	1.278	1.763	2.213
PB2	**0.684**	0.548	0.465	0.445	0.602	**0.504**	0.421	0.382
BCPB1	**1.021**	**1.386**	**1.895**	2.327	**0.926**	1.283	1.765	2.188
BCPB2	0.669	**0.556**	0.466	0.428	**0.606**	0.503	0.422	0.384
BCAB1	1.019	1.378	1.891	2.340	0.920	**1.284**	1.731	2.184
BCAB2	0.653	0.542	0.449	0.418	0.589	0.491	0.406	0.372
Relative Average Length								
SB1	0.768	0.597	0.445	0.370	0.915	0.687	0.488	0.388
SB2	1.101	1.265	1.319	1.331	1.347	1.435	**1.509**	**1.445**
Boot-t1	0.763	0.591	0.442	0.369	0.878	0.681	**0.487**	**0.388**
Boot-t2	1.076	1.207	1.231	**1.224**	1.344	1.490	1.633	1.590
PB1	0.754	0.582	0.436	0.364	0.877	0.678	0.489	0.405
PB2	1.025	1.161	**1.223**	1.237	1.273	1.430	1.543	1.583
BCPB1	0.740	0.576	0.434	0.362	0.868	0.672	0.489	0.403
BCPB2	**1.016**	**1.152**	1.231	1.248	**1.245**	**1.410**	1.532	1.578
BCAB1	**0.727**	**0.571**	**0.431**	**0.360**	**0.856**	**0.671**	0.489	0.399
BCAB2	1.041	1.188	1.272	1.292	1.281	1.448	1.599	1.635

Table 4.8: Simulation results of $E_4/H_4^{Pe}/1$ to $H_4^{Pe}/H_4^{Po}/1$ queueing network model without feedback for $r_1 \geq r_2$

Coverage Percentages								
Estimation	Before Calibration				After Calibration			
Approches	$n=5$	$n=10$	$n=20$	$n=30$	$n=5$	$n=10$	$n=20$	$n=30$
SB1	0.735	0.773	0.796	0.853	0.783	0.852	0.842	0.852
SB2	0.749	0.815	0.852	0.864	0.817	0.881	0.882	0.893
Boot-t1	0.703	0.711	0.718	0.782	0.734	0.788	0.737	0.766
Boot-t2	0.751	0.813	0.847	0.857	0.797	0.866	0.871	0.894
PB1	0.696	0.727	0.725	0.738	0.777	0.814	0.843	0.835
PB2	0.728	0.790	0.834	0.831	0.809	0.875	0.865	0.884
BCPB1	0.681	0.732	0.720	0.743	0.776	0.807	0.823	0.837
BCPB2	0.728	0.795	0.836	0.833	0.809	0.883	0.861	0.890
BCAB1	0.669	0.715	0.710	0.732	0.760	0.801	0.812	0.832
BCAB2	0.731	0.790	0.845	0.838	0.817	0.883	0.864	0.897
Average Lengths								
SB1	1.566	2.009	2.317	2.562	1.983	2.375	2.863	2.774
SB2	0.153	0.118	0.091	0.075	0.185	0.140	0.101	0.082
Boot-t1	1.524	1.913	2.157	2.353	1.941	2.446	2.993	2.939
Boot-t2	0.153	0.117	0.090	0.075	0.179	0.139	0.101	0.083
PB1	1.440	1.866	2.115	2.335	1.841	2.237	2.834	3.001
PB2	0.151	0.115	0.089	0.074	0.180	0.139	0.102	0.085
BCPB1	1.413	1.864	2.104	2.345	1.817	2.220	2.846	3.003
BCPB2	0.149	0.114	0.089	0.074	0.178	0.139	0.101	0.085
BCAB1	1.342	1.767	2.004	2.246	1.698	2.101	2.701	2.874
BCAB2	0.152	0.117	0.090	0.075	0.182	0.144	0.102	0.085
Relative Coverages								
SB1	0.469	0.385	0.344	**0.333**	0.395	0.359	0.294	**0.307**
SB2	4.885	6.911	9.368	**11.488**	4.409	6.276	**8.701**	**10.879**
Boot-t1	0.461	0.372	0.333	0.332	0.378	0.322	0.246	0.261
Boot-t2	**4.917**	**6.959**	9.384	11.458	4.452	6.251	8.598	10.810
PB1	0.483	0.390	0.343	0.316	0.422	0.364	0.298	0.278
PB2	4.815	6.855	9.364	11.236	4.485	6.301	8.514	10.404
BCPB1	0.482	0.393	0.342	0.317	0.427	0.364	0.289	0.279
BCPB2	4.902	6.955	**9.418**	11.319	**4.555**	**6.370**	8.492	10.529
BCAB1	**0.498**	**0.405**	**0.354**	0.326	**0.448**	**0.381**	**0.301**	0.290
BCAB2	4.796	6.771	9.363	11.221	4.482	6.145	8.460	10.498
Relative Average Length								
SB1	1.199	1.371	1.367	1.446	1.518	1.621	1.690	**1.566**
SB2	0.746	0.581	0.442	0.369	0.902	0.691	0.493	**0.403**
Boot-t1	1.167	1.306	1.273	1.328	1.486	1.669	1.766	1.659
Boot-t2	0.744	0.575	0.439	0.367	0.872	0.682	**0.492**	0.406
PB1	1.102	1.273	1.248	1.318	1.409	1.527	1.672	1.695
PB2	0.736	0.567	0.433	0.363	0.878	0.684	0.494	0.417
BCPB1	1.081	1.272	1.242	1.324	1.391	1.515	1.679	1.696
BCPB2	**0.723**	**0.563**	**0.431**	**0.361**	**0.865**	**0.682**	0.493	0.415
BCAB1	**1.027**	**1.206**	**1.183**	**1.268**	**1.299**	**1.434**	**1.594**	1.622
BCAB2	0.742	0.574	0.439	0.366	0.888	0.707	0.496	0.419

Table 4.9: Simulation results of $E_4/H_4^{Po}/1$ to $H_4^{Po}/H_4^{Pe}/1$ queueing network model without feedback for $r_1 \geq r_2$

Coverage Percentages								
Estimation	Before Calibration				After Calibration			
Approches	$n=5$	$n=10$	$n=20$	$n=30$	$n=5$	$n=10$	$n=20$	$n=30$
SB1	0.722	0.804	0.831	0.847	0.794	0.827	0.847	0.884
SB2	0.721	0.824	0.850	0.863	0.817	0.881	0.902	0.895
Boot-t1	0.681	0.730	0.735	0.750	0.738	0.760	0.769	0.820
Boot-t2	0.718	0.821	0.845	0.853	0.792	0.875	0.902	0.892
PB1	0.703	0.729	0.764	0.776	0.797	0.825	0.841	0.843
PB2	0.702	0.812	0.834	0.849	0.817	0.866	0.888	0.899
BCPB1	0.685	0.714	0.751	0.753	0.769	0.813	0.806	0.850
BCPB2	0.703	0.809	0.841	0.836	0.812	0.874	0.890	0.889
BCAB1	0.661	0.709	0.740	0.743	0.755	0.798	0.797	0.840
BCAB2	0.712	0.817	0.835	0.846	0.821	0.872	0.897	0.889
Average Lengths								
SB1	1.542	1.989	2.440	2.512	1.988	2.385	2.673	2.921
SB2	0.150	0.123	0.090	0.076	0.200	0.137	0.102	0.082
Boot-t1	1.503	1.890	2.273	2.311	1.952	2.473	2.874	3.183
Boot-t2	0.149	0.121	0.090	0.076	0.192	0.136	0.102	0.083
PB1	1.424	1.779	2.255	2.357	1.838	2.317	2.639	3.040
PB2	0.148	0.119	0.089	0.075	0.194	0.136	0.101	0.084
BCPB1	1.395	1.767	2.240	2.370	1.781	2.300	2.671	3.039
BCPB2	0.145	0.118	0.088	0.074	0.192	0.134	0.101	0.084
BCAB1	1.326	1.677	2.139	2.266	1.671	2.156	2.549	2.911
BCAB2	0.149	0.121	0.090	0.075	0.198	0.136	0.104	0.084
Relative Coverages								
SB1	0.468	0.404	0.341	0.**337**	0.399	0.347	0.317	**0.303**
SB2	4.818	6.709	9.402	11.349	4.088	6.411	8.855	**10.921**
Boot-t1	0.453	0.386	0.323	0.325	0.378	0.307	0.268	0.258
Boot-t2	4.812	6.766	9.425	11.294	4.125	6.431	8.829	10.754
PB1	0.494	0.410	0.339	0.329	0.434	0.356	**0.319**	0.277
PB2	4.754	6.808	9.406	**11.390**	4.219	6.378	8.767	10.718
BCPB1	0.491	0.404	0.335	0.318	0.432	0.353	0.302	0.280
BCPB2	**4.850**	**6.856**	**9.547**	11.244	**4.222**	**6.529**	**8.860**	10.635
BCAB1	**0.499**	**0.423**	**0.346**	0.328	**0.452**	**0.370**	0.313	0.289
BCAB2	4.793	6.764	9.329	11.229	4.140	6.391	8.639	10.600
Relative Average Length								
SB1	1.177	1.337	1.441	1.447	1.518	1.603	1.579	1.683
SB2	0.721	0.608	0.446	0.373	0.962	0.681	0.503	**0.402**
Boot-t1	1.148	1.271	1.343	1.332	1.491	1.663	1.698	1.834
Boot-t2	0.719	0.601	0.442	0.371	**0.924**	0.674	0.504	0.407
PB1	1.088	1.196	1.332	1.358	1.404	1.558	1.559	1.752
PB2	0.711	0.591	0.438	0.366	0.933	0.672	0.500	0.412
BCPB1	1.066	1.188	1.323	1.366	1.360	1.547	1.577	1.751
BCPB2	**0.698**	**0.584**	**0.435**	**0.365**	0.926	**0.663**	**0.496**	0.410
BCAB1	**1.012**	**1.128**	**1.263**	**1.305**	**1.276**	**1.449**	**1.505**	**1.677**
BCAB2	0.715	0.598	0.442	0.370	0.955	0.676	0.512	0.412

Table 4.10: Simulation results of $H_4^{Pe}/H_4^{Po}/1$ to $H_4^{Po}/E_4/1$ queueing network model without feedback for $r_1 \geq r_2$

Coverage Percentages								
Estimation	Before Calibration				After Calibration			
Approches	$n=5$	$n=10$	$n=20$	$n=30$	$n=5$	$n=10$	$n=20$	$n=30$
SB1	0.731	0.817	0.815	0.841	0.784	0.845	0.868	0.866
SB2	0.774	0.819	0.859	0.864	0.847	0.886	0.881	0.889
Boot-t1	0.692	0.738	0.730	0.747	0.725	0.774	0.801	0.783
Boot-t2	0.772	0.820	0.855	0.865	0.835	0.878	0.876	0.884
PB1	0.675	0.738	0.765	0.755	0.797	0.843	0.839	0.841
PB2	0.767	0.802	0.845	0.835	0.831	0.878	0.880	0.899
BCPB1	0.663	0.733	0.745	0.747	0.773	0.833	0.850	0.824
BCPB2	0.762	0.799	0.836	0.833	0.820	0.885	0.873	0.900
BCAB1	0.647	0.707	0.729	0.739	0.747	0.815	0.842	0.818
BCAB2	0.766	0.808	0.844	0.841	0.823	0.877	0.874	0.900
Average Lengths								
SB1	1.667	2.127	2.596	2.740	2.025	2.534	3.054	3.105
SB2	0.127	0.100	0.074	0.062	0.159	0.117	0.082	0.067
Boot-t1	1.624	2.026	2.424	2.530	2.007	2.660	3.222	3.349
Boot-t2	0.127	0.099	0.074	0.062	0.155	0.116	0.082	0.067
PB1	1.521	1.965	2.384	2.508	1.888	2.474	3.048	3.235
PB2	0.126	0.098	0.073	0.062	0.156	0.117	0.084	0.070
BCPB1	1.496	1.938	2.355	2.534	1.869	2.467	2.986	3.266
BCPB2	0.124	0.097	0.073	0.062	0.153	0.117	0.083	0.070
BCAB1	1.423	1.838	2.248	2.434	1.753	2.349	2.861	3.135
BCAB2	0.127	0.099	0.074	0.062	0.156	0.118	0.083	0.070
Relative Coverages								
SB1	0.438	0.384	0.314	**0.307**	0.387	0.333	0.284	**0.279**
SB2	6.083	8.221	**11.588**	13.837	5.328	7.558	**10.771**	**13.211**
Boot-t1	0.426	0.364	0.301	0.295	0.361	0.291	**0.249**	0.234
Boot-t2	6.066	**8.281**	11.584	**13.903**	**5.387**	7.575	10.646	13.190
PB1	0.444	0.376	0.321	0.301	0.422	0.341	0.275	0.260
PB2	6.092	8.175	11.536	13.527	5.338	7.476	10.520	12.848
BCPB1	0.443	0.378	0.316	0.295	0.414	0.338	0.285	0.252
BCPB2	**6.138**	8.202	11.450	13.506	5.359	**7.582**	10.477	12.891
BCAB1	**0.455**	**0.385**	**0.324**	0.304	**0.426**	**0.347**	0.294	0.261
BCAB2	6.049	8.185	11.444	13.505	5.263	7.416	10.477	12.904
Relative Average Length								
SB1	1.235	1.376	1.477	1.472	1.499	1.639	1.738	**1.668**
SB2	0.636	0.493	0.363	0.307	0.794	0.580	**0.400**	0.331
Boot-t1	1.203	1.311	1.380	1.359	1.486	1.721	1.834	1.799
Boot-t2	0.636	0.490	0.361	0.306	0.774	**0.573**	0.403	**0.329**
PB1	1.127	1.271	1.357	1.347	1.398	1.600	1.735	1.738
PB2	0.629	0.485	0.359	0.303	0.778	0.581	0.410	0.344
BCPB1	1.108	1.254	1.340	1.361	1.384	1.596	1.699	1.754
BCPB2	**0.620**	**0.482**	**0.357**	**0.303**	**0.764**	0.577	0.408	0.343
BCAB1	**1.054**	**1.189**	**1.279**	**1.307**	**1.298**	**1.519**	**1.628**	1.684
BCAB2	0.633	0.488	0.361	0.306	0.781	0.585	0.408	0.343

Table 4.11: Performances of the estimation approaches with greatest relative coverage to mean response time under various queueing network model without feedback :

Estimation approach with greatest relative coverage of confidence intervals for $r_1 < r_2$								
Estimation	Before Calibration				After Calibration			
Approches	$n = 5$	$n = 10$	$n = 20$	$n = 30$	$n = 5$	$n = 10$	$n = 20$	$n = 30$
$E_4/H_4^{Pe}/1$ to	BCAB1	BCAB1	PB1	Boot-t1	BCPB1	SB1	SB1	Boot-t1
$H_4^{Pe}/H_4^{Po}/1$	BCPB2	BCPB2	PB2	SB2	BCPB2	PB2	SB2	SB2
$E_4/H_4^{Po}/1$ to	BCAB1	PB1	BCPB1	Boot-t1	BCPB1	BCPB1	SB1	Boot-t1
$H_4^{Po}/H_4^{Pe}/1$	PB2	BCPB2	BCPB2	SB2	BCPB2	BCPB2	BCPB2	SB2
$H_4^{Pe}/H_4^{Po}/1$ to	BCPB1	BCPB1	BCPB1	Boot-t1	BCPB1	BCAB1	SB1	SB1
$H_4^{Po}/E_4/1$	PB2	BCPB2	SB2	SB2	BCPB2	PB2	SB2	SB2
Estimation approach with greatest relative coverage of confidence intervals for $r_1 \geq r_2$								
Estimation	Before Calibration				After Calibration			
Approches	$n = 5$	$n = 10$	$n = 20$	$n = 30$	$n = 5$	$n = 10$	$n = 20$	$n = 30$
$E_4/H_4^{Pe}/1$ to	BCAB1	BCAB1	BCAB1	SB1	BCAB1	BCAB1	BCAB1	SB1
$H_4^{Pe}/H_4^{Po}/1$	Boot-t2	Boot-t2	BCPB2	SB2	BCPB2	BCPB2	SB2	SB2
$E_4/H_4^{Po}/1$ to	BCAB1	BCAB1	BCAB1	SB1	BCAB1	BCAB1	PB1	SB1
$H_4^{Po}/H_4^{Pe}/1$	BCPB2	BCPB2	BCPB2	PB2	BCPB2	BCPB2	BCPB2	SB2
$H_4^{Pe}/H_4^{Po}/1$ to	BCAB1	BCAB1	BCAB1	SB1	BCAB1	BCAB1	Boot-t1	SB1
$H_4^{Po}/E_4/1$	BCPB2	Boot-t2	SB2	Boot-t2	Boot-t2	BCPB2	SB2	SB2

Table 4.12: Performances of the estimation approaches with shortest relative average length to mean response time under various queueing network model without feedback :

Estimation approach with shortest relative average length of confidence intervals for $r_1 < r_2$								
$E_4/H_4^{Pe}/1$ to	BCAB1	BCAB1	BCAB1	BCAB1	BCAB1	BCAB1	SB1	Boot-t1
$H_4^{Pe}/H_4^{Po}/1$	BCPB2	BCPB2	PB2	BCPB2	BCPB2	PB2	SB2	SB2
$E_4/H_4^{Po}/1$ to	BCAB1	BCAB1	BCAB1	BCAB1	BCAB1	SB1	SB1	Boot-t1
$H_4^{Po}/H_4^{Pe}/1$	BCPB2	BCPB2	BCPB2	Boot-t2	BCPB2	BCPB2	BCPB2	SB2
$H_4^{Pe}/H_4^{Po}/1$ to	BCAB1	BCAB1	BCAB1	BCAB1	BCAB1	BCAB1	Boot-t1	Boot-t1
$H_4^{Po}/E_4/1$	BCPB2	BCPB2	PB2	Boot-t2	BCPB2	BCPB2	SB2	SB2
Estimation approach with shortest relative average length of confidence intervals for $r_1 \geq r_2$								
$E_4/H_4^{Pe}/1$ to	BCAB1	BCAB1	BCAB1	BCAB1	BCAB1	BCAB1	BCAB1	SB1
$H_4^{Pe}/H_4^{Po}/1$	BCPB2	BCPB2	BCPB2	BCPB2	BCPB2	BCPB2	Boot-t2	SB2
$E_4/H_4^{Po}/1$ to	BCAB1	BCAB1	BCAB1	BCAB1	BCAB1	BCAB1	BCAB1	BCAB1
$H_4^{Po}/H_4^{Pe}/1$	BCPB2	BCPB2	BCPB2	BCPB2	Boot-t2	BCPB2	BCPB2	SB2
$H_4^{Pe}/H_4^{Po}/1$ to	BCAB1	BCAB1	BCAB1	BCAB1	BCAB1	BCAB1	BCAB1	SB1
$H_4^{Po}/E_4/1$	BCPB2	BCPB2	BCPB2	BCPB2	BCPB2	Boot-t2	SB2	Boot-t2