Reliability Optimization Model for Redundant Systems with Multiple Constraints

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Abstract- The Reliability Optimization Models for Redundant Systems with multiple constraints for one mathematical function is established by applying Lagrangean approach, the related case problem is presented to find the component reliabilities \((r_i)\), the number of components in each stage \((x_i)\), stage reliability \((R_i)\) and the System Reliability \((R_s)\). The purpose of this Paper is to optimize a class of Integrated Reliability Model for Redundant Systems with weight as additional constraint apart from basic cost constraint. By taking the mathematical function \(c_j = a_j . \exp \left[ \frac{b_j}{(1-r_j)} \right]\) the optimum component reliability, stage reliability, the number of components in each stage and the system reliability are determined after taking the pre-determined values of Cost and Weight. In this work, an attempt is made to develop an integrated reliability redundant model for a Series – Parallel configuration subject to the multiple constraints. Generally reliability is treated as the function of Cost but in any given practical situation apart from Cost other constraint like Weight will have hidden impact on the reliability of the system. In this model the Lagrangean technique is implemented to determine the Cost and Weight as constraints. The model has yielded very encouraging results and it can be applied to any type of system, simple or complex. The advantage of this model is very flexible and requires little processing time.

Keywords: Reliability Optimization Model; Redundant system; System Reliability; Multiple constraints;

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1. INTRODUCTION

It is increasingly necessary to design reliable systems as there is a great demand for products that offer quality and safety. Another way of improving the reliability of a system is to use redundant components or redundant sub-systems. The importance of designing reliable systems, which normally
present high availability, is increasing due to the engineering requirements of products with better quality and a higher safety level.

However an increase in the number of components and sub systems consequently results in project costs, weight and volume of the system and the design parameters increasing. Hence, it is necessary to use optimization techniques in order to obtain and optimum system with in the desired constraints. This Paper deals with reliability optimization using redundant component connected in Series – Parallel configuration.

So far as the literature on maximization of system Reliability problems are concerned, the researchers opine that optimization problems can be handled with multiple constraints also, but to the best of the knowledge of the authors, the optimization of Reliability Models for Redundant systems with multiple constraints are not reported. In this scenario, the authors want to make an attempt to optimize the reliability of a System with multiple constraints.

To study and optimize, the Integrated Reliability Model for Redundant Systems with multiple constraints is considered with cost and weight as constraints for the given known mathematical function.

To establish the results for the above specified mathematical function \( c_j = a_j \cdot \exp \left[ \frac{b_j}{(1-r_j)} \right] \), Lagrangean Multiplier Method is applied to calculate the number of components in each stage, component reliability and corresponding stage reliability in real value numbers. Since, the number of components in each stage cannot be rounded off to nearest integer due to variation in cost and reliability. Finally the result of this work supports the researcher’s statement of the problem concurring that these models are particularly of high application value for any Series – Parallel Redundant Systems with Multiple Constraints.

2. STATEMENT OF THE PROBLEM

The problem considers the component reliabilities and the no. of components in each stage are unknowns for the given constraints to maximize the system reliability. The authors in this work make an attempt to negotiate the impact of the cost and weight as constraints in optimizing the reliability of the redundant systems under consideration for the selected above mathematical function. Though Cost has direct relation in maximizing System Reliability, the indirect impact of Weight as additional constraints in optimizing the Reliability of a redundant system presents a novel beginning in the mentioned area of research. The Series – Parallel Systems are considered with Cost and Weight as constraints to maximize the Reliability of a redundant system as its objective function.

3. ASSUMPTIONS OF THE MODEL

1. All the components in each stage are assumed to be identical.

2. The components are assumed to be statistically independent i.e. the failure of one component does not affect the performance of the other components in the system.

3. A component is either in working condition or non-working condition.
FIG: SERIES-PARALLEL CONFIGURATION

4. NOMENCLATURE

\( R_s = \) System Reliability.

\( R_j = \) Stage Reliability, \( 0 < R_j < 1 \)

\( F= \) Lagrangean function

\( r_j = \) Reliability of each component in stage \( j \), \( 0 < r_j < 1 \).

\( x_j = \) No. of components in stage \( j \).

\( c_j = \) Cost coefficient of each component in stage \( j \)

\( w_j = \) Weight coefficient of each component in stage \( j \).

\( C_0 = \) Maximum allowable System Cost.

\( W_0 = \) Maximum allowable System Weight.

\( a_j = \) Scaling factor for stage ‘\( j \)’ used in the function

\( b_j = \) Shaping factor for stage ‘\( j \)’ used in the function

\( p_j = \) Constant used in weight function.

\( q_j = \) Constant used in weight function.

5. MATHEMATICAL MODEL

Consider that there are ‘\( n \)’ statistically independent stages in Series with \( x_j \) statistically independent in each stage.

System Reliability for the given cost function

\[
R_s = \prod_{j=1}^{n} R_j = \prod_{j=1}^{n} \left[ 1 - (1 - R_j)^{x_j} \right] \quad (1)
\]

Subjected to

\[
\sum_{j=1}^{n} c_j \cdot x_j \leq C_0 \quad (2)
\]

\[
\sum_{j=1}^{n} w_j \cdot x_j \leq W_0 \quad (3)
\]

Non negativity restriction \( x_j \) is an integer and \( r_j, R_j > 0 \)
6. MATHEMATICAL FUNCTION

Cost co efficient of each component in stage ‘j’ is derived from the following relationship between Cost and Reliability.

\[ c_j = a_j \cdot \exp \left[ \frac{b_j}{(1-r_j)} \right] \]  \hspace{1cm} (4)

Where \( c_j \) is cost constraint and \( b_j, d_j \) are constants.

7. PROBLEM FORMULATION

System Reliability for the given cost function

\[ R_s = \prod_{j=1}^{n} R_j \]  \hspace{1cm} (5)

Cost coefficient of each unit in stage ‘j’ is derived from the following relationship between cost and reliability

\[ c_j = a_j \cdot \exp \left[ \frac{b_j}{(1-r_j)} \right] \]  \hspace{1cm} (6)

\[ w_j = p_j \cdot \exp \left[ \frac{q_j}{(1-r_j)} \right] \]  \hspace{1cm} (7)

Since cost constraint is linear in \( x_j \)

\[ \sum_{j=1}^{n} c_j \cdot x_j \leq C_0 \]  \hspace{1cm} (8)

Similarly weight constraint is also linear in \( x_j \)

\[ \sum_{j=1}^{n} w_j \cdot x_j \leq W_0 \]  \hspace{1cm} (9)

Substituting equations (6) and (7) in (8) and (9) we get the following relation

\[ \sum_{j=1}^{n} \left[ a_j \cdot \exp \left[ \frac{b_j}{(1-r_j)} \right] \right] x_j - C_0 \leq 0 \]  \hspace{1cm} (10)

\[ \sum_{j=1}^{n} \left[ p_j \cdot \exp \left[ \frac{q_j}{(1-r_j)} \right] \right] x_j - W_0 \leq 0 \]  \hspace{1cm} (11)

The number of components at each stage \( x_j \) is given through the relation
\[ x_j = \frac{\ln(1 - R_j)}{\ln(1 - r_j)} \]  

(12)

Maximize \[ R_s = \prod_{j=1}^{n} [1 - (1 - r_j)^{x_j}] \]  

(13)

Subject to the constraints

\[
\sum_{j=1}^{n} a_j \cdot \exp \left( \frac{b_j}{(1-r_j)} \right) \frac{\ln(1 - R_j)}{\ln(1 - r_j)} - C_0 \leq 0
\]  

(14)

\[
\sum_{j=1}^{n} p_j \cdot \exp \left( \frac{q_j}{(1-r_j)} \right) \frac{\ln(1 - R_j)}{\ln(1 - r_j)} - W_0 \leq 0
\]  

(15)

8. LAGRANGEAN METHOD

Solving the proposed formulation using Lagrangean method.

\[ F = R_s + \lambda_1 \sum_{j=1}^{n} a_j \cdot \exp \left( \frac{b_j}{(1-r_j)} \right) \frac{\ln(1 - R_j)}{\ln(1 - r_j)} - C_0 + \lambda_2 \sum_{j=1}^{n} p_j \cdot \exp \left( \frac{q_j}{(1-r_j)} \right) \frac{\ln(1 - R_j)}{\ln(1 - r_j)} - W_0 = 0 \]  

(16)

where \( \lambda_1 \) and \( \lambda_2 \) are Lagrangean multipliers and \( F \) being Lagrangean function.

The number of components in each stage \( (x_j) \), optimum component reliability \( (r_j) \), stage reliability \( (R_s) \) and the system reliability \( (R_s) \) are derived from the Lagrangean method. The method provides real valued solution with reference to cost and weight.

The stationary point can be obtained by differentiating the Lagrangean function with respect to \( R_j, r_j, \lambda_1, \lambda_2 \) and \( \lambda_3 \).

\[
\frac{\partial F}{\partial r_j} = \lambda_1 \left[ \sum_{j=1}^{n} \ln(1 - R_j) \cdot a_j \cdot \exp \left[ \frac{b_j}{(1-r_j)} \right] \frac{b_j}{(1-r_j)^2} \cdot \frac{1}{\ln(1 - r_j)} \right] + \lambda_2 \left[ \sum_{j=1}^{n} p_j \cdot \exp \left[ \frac{q_j}{(1-r_j)} \right] \frac{q_j}{(1-r_j)^2} \cdot \frac{1}{\ln(1 - r_j)} \right] = 0
\]  

(17)

\[
\frac{\partial F}{\partial R_j} = 1 + \lambda_1 \left[ \sum_{j=1}^{n} \ln(1 - R_j) \cdot a_j \cdot \exp \left[ \frac{b_j}{(1-r_j)} \right] \frac{(-1)}{\ln(1 - R_j)} \right] + \lambda_2 \left[ \sum_{j=1}^{n} p_j \cdot \exp \left[ \frac{q_j}{(1-r_j)} \right] \frac{(-1)}{\ln(1 - R_j)} \right] = 0
\]  

(18)

\[
\frac{\partial F}{\partial \lambda_1} = \sum_{j=1}^{n} a_j \cdot \exp \left[ \frac{b_j}{(1-r_j)} \right] \ln(1 - R_j) - C_0 = 0
\]  

(19)
\[
\frac{\partial F}{\partial \lambda_2} = \sum_{j=1}^{n} p_j \cdot \exp \left[ \frac{q_j}{(1 - r_j)} \ln \left( \frac{1 - R_j}{1 - r_j} \right) \right] \cdot W_0 = 0
\]

(20)

9. RESULTS AND DISCUSSIONS

The following reliability design tables related to cost and weight are calculated by using the component reliabilities and the number of components derived from Lagrangean method.

9.1 Case Study:

Consider the case of a Mechanical system with three stages for which the component Reliability and the Stage Reliability to be determined.

To determine the optimum component reliability, stage reliability, number of components in each stage and the System Reliability not to exceed the system cost Rs.6000, Weight of the system 7000kg. The component Reliabilities, Stage Reliabilities, Number of components in each stage and the System Reliability are determined by solving the above mathematical function by using MATLAB Version 7.10 and are presented in the following tables. Inputs in the MATLAB given as (0.68;0.7;0.8;0.84;0.94;0.98;-0.021;-0.0342).

9.2 Cost and Weight as constraints:

9.2.1 Reliability Design Without \(x_j\) rounding off:

<table>
<thead>
<tr>
<th>Stage</th>
<th>(r_i)</th>
<th>(R_i)</th>
<th>(x_j)</th>
<th>(c_j)</th>
<th>(c_j x_j)</th>
</tr>
</thead>
<tbody>
<tr>
<td>01</td>
<td>0.7161</td>
<td>0.8277</td>
<td>1.40</td>
<td>1533.45</td>
<td>2146.83</td>
</tr>
<tr>
<td>02</td>
<td>0.6339</td>
<td>0.9158</td>
<td>2.46</td>
<td>1288.68</td>
<td>3170.15</td>
</tr>
<tr>
<td>03</td>
<td>0.6232</td>
<td>0.8431</td>
<td>1.89</td>
<td>339.26</td>
<td>641.20</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>5958.18</td>
</tr>
</tbody>
</table>

Table II. Reliability design relating to Weight (Kgs):

<table>
<thead>
<tr>
<th>Stage</th>
<th>(r_i)</th>
<th>(R_i)</th>
<th>(X_i)</th>
<th>(W_i)</th>
<th>(W_i X_i)</th>
</tr>
</thead>
<tbody>
<tr>
<td>01</td>
<td>0.7161</td>
<td>0.8277</td>
<td>1.40</td>
<td>2008.55</td>
<td>2811.97</td>
</tr>
<tr>
<td>02</td>
<td>0.6339</td>
<td>0.9158</td>
<td>2.46</td>
<td>1333.85</td>
<td>3281.27</td>
</tr>
<tr>
<td>03</td>
<td>0.6232</td>
<td>0.8431</td>
<td>1.89</td>
<td>479.13</td>
<td>905.55</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>6998.79</td>
</tr>
</tbody>
</table>

System Reliability (\(R_x\)) = 0.6390

9.2.2 Reliability Design with \(x_j\) rounding off:

The reliability design is reestablished by considering the values of \(x_j\) to be integers (by rounding off the value of \(x_j\) to the nearest integer) and the relevant results relating to cost and weight are presented in the following table, further giving the information by calculating the variation due to cost and weight and the system reliability (after rounding off \(x_j\)).

<table>
<thead>
<tr>
<th>Stage</th>
<th>(r_i)</th>
<th>(R_i)</th>
<th>(X_i)</th>
<th>(c_j)</th>
<th>(c_j x_i)</th>
</tr>
</thead>
<tbody>
<tr>
<td>01</td>
<td>0.7161</td>
<td>0.9194</td>
<td>2</td>
<td>1533.45</td>
<td>3066.90</td>
</tr>
<tr>
<td>02</td>
<td>0.6339</td>
<td>0.8659</td>
<td>3</td>
<td>1288.68</td>
<td>3866.04</td>
</tr>
<tr>
<td>03</td>
<td>0.3327</td>
<td>0.8580</td>
<td>2</td>
<td>190.39</td>
<td>678.52</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>7611.46</td>
</tr>
</tbody>
</table>
Table IV. Reliability design relating to Weight (Kgs):

<table>
<thead>
<tr>
<th>Stage</th>
<th>$r_i$</th>
<th>$R_i$</th>
<th>$X_i$</th>
<th>$W_i$</th>
<th>$W_i\cdot X_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>01</td>
<td>0.7161</td>
<td>0.9194</td>
<td>2</td>
<td>2008.55</td>
<td>1817.10</td>
</tr>
<tr>
<td>02</td>
<td>0.6339</td>
<td>0.8659</td>
<td>3</td>
<td>1333.85</td>
<td>1142.27</td>
</tr>
<tr>
<td>03</td>
<td>0.6232</td>
<td>0.8580</td>
<td>2</td>
<td>479.13</td>
<td>397.26</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td></td>
<td>8976.91</td>
<td></td>
</tr>
</tbody>
</table>

System Reliability ($R_i$) = 0.6830
Variation in total Cost = 27.74%
Variation in total Weight = 28.26%
Variation in System Reliability 6.44%

10. CONCLUSION

All most all the models that are presented primarily considered Cost as the basic constraint. In this scenario, the authors proposed as a class of Integrated Reliability Models for Redundant Systems with multiple constraints as a novel beginning in the mentioned area of research and the optimizing the system reliability for the said model, and the results reported are highly useful for Reliability/Design Engineers. The Lagrangean approach has given the Reliability of three stage system is $0.6830$, where the number of components are real.

This model can also be further investigated for different mathematical functions of interest and also can be applied for Parallel – Series configuration systems, where the application of these models for such systems will be feasible only when the cost of the system is very low.

REFERENCES


