INTRODUCTION

Fuzzy set theory and fuzzy topology are approached as generalizations of ordinary set theory and ordinary topology. We consider fuzzy subsets as functions from a non-empty set to a membership lattice. Throughout this work we follow the definition of fuzzy topology given by Chang[3] with membership set as an arbitrary complete and distributive lattice.

Category theory is the branch of mathematics which studies the abstract properties of 'sets with structures' and 'structure preserving functions'. It provides a tool by which many parallel techniques used in several branches of mathematics can be linked and treated in a unified manner.

In this work, we present some applications of category theory in Fuzzy Topology based mainly on two notions 'simple reflection and coreflection'. This thesis is presented in five chapters.
In 1974, C.K. Wong [34] introduced the concept of 'fuzzy point belongs to a fuzzy set'. Later the same concept was defined in different ways by Srivastava, Lal and Srivastava [30]. The definitions of the relation \( \subseteq \) of a fuzzy point belonging to a fuzzy set given independently by these authors seem to be very much alike. But on thorough analysis, they are found to differ in certain aspects. This study is included in chapter I. We arrive at the conclusion that the definition given by Piu and Liu[27] is the most appropriate one for fuzzy set theory. A characterization of fuzzy open set is necessary for the study of fuzzy topology. This leads us to study the fuzzy neighbourhood system of a fuzzy point. Piu and Liu [27], Demitri and Pascali [4] introduced the notion of fuzzy neighbourhood system. Both the definitions do not generalize the corresponding definitions of ordinary topology. To rectify this anomaly we introduce a new definition for fuzzy neighbourhood system by the addition of two more axioms. These axioms are necessary in the fuzzy context. In the case of ordinary topology where \( L = \{0,1\} \), these axioms are trivially satisfied. The basics of fuzzy topology is strengthened in chapter I.
Pelham Thomas [26] introduced the concept of associated regular spaces. Later P.M. Mathew [22] introduced associated completely regular spaces. "What is the speciality of these spaces among all subcategories, say reflective, coreflective"?

Chapter II provides an answer to this question which holds for all those classes for which interesting characterizations of completely regular spaces and regular spaces are known. As a generalization to this, an associated p-space is constructed and their properties are studied. We formulated these concepts in Category theory and obtained a characterization of the simple reflective subcategories of the category of topological spaces.

In the third chapter a fuzzy parallel of associated completely regular spaces is constructed and their properties studied. Fuzzy completely regular space was introduced and studied by Hutton [10,11]. A different version of fuzzy complete regularity is available in [15]. However, we follow the definition given in [11].
The properties of fuzzy completely regular spaces enable us to construct fuzzy associated p-spaces. We obtain this as a generalization of the concepts that we have introduced in the second chapter. In order to widen the range of application we do this in the language of category theory. The results obtained enable us to treat the known theories in an unified manner. Thus we obtained some characterizations of the simple reflective subcategories of the category of fuzzy topological spaces in the fourth chapter.

In the fifth chapter we present some applications of Category theory in Fuzzy Topology based on the notion 'Coreflection'. The coreflective subcategories of the class of fuzzy topological spaces are considered in the works of Lowen and Wuyts [20]. In this chapter we give an internal description of the coreflection. This was motivated by the work of V. Kannan [13]. The notion of topological coreflections are discussed in the paper by Herrlich and Strecker [8]. V. Kannan [13] characterized the smallest coreflective subcategory
Chapter I

of the category of topological spaces \( \text{TOP} \), containing a given subcategory \( \mathcal{C} \) of \( \text{TOP} \). We introduce the class of induced fuzzy topological spaces \( I(\mathcal{F}) \) corresponding an arbitrary family of fuzzy topological spaces \( \mathcal{F} \). The study of induced fuzzy topological spaces coincides with the generation of coreflective subcategories of the category of fuzzy topological spaces. We also characterize coreflection as the lattice meet of all finer fuzzy topologies.

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