Chapter 2

Computational Work
ANALYSIS AND EQUATION OF MOTION

Differential equation of motion and for visco-elastic square plate of variable thickness in Cartesian coordinate is [1]:

\[
[D_1\left(W_{xxx} + 2W_{xxy} + W_{yyyy}\right) + 2D_{1x}\left(W_{xxx} + W_{xxy}\right) + 2D_{1y}\left(W_{yyy} + W_{xyy}\right) + \rho \phi p^2 W = 0
\]

which is a differential equation of transverse motion for non-homogeneous plate of variable thickness.

Here, \(D_1\) is the flexural rigidity of plate i.e.

\[
D_1 = E h^3 / 12(1 - \nu^2)
\]

Assuming that the square plate of engineering material has a steady two-dimensional temperature distribution i.e.

\[
\tau = \tau_0 (1 - x/a)(1 - y/a)
\]

where \(\tau\) denotes the temperature excess above the reference temperature at any point on the plate and \(\tau_0\) denotes the temperature at any point on the boundary of plate.

The temperature dependance of the modulus of elasticity for most of engineering materials can be expressed in this form,

\[
E = E_0 \left(1 - \gamma \tau \right)
\]
where $E_0$ is the value of the Young's modulus at reference temperature i.e. $\tau = 0$ and $\gamma$ is the slope of the variation of $E$ with $\tau$. The modulus variation (5) become

$$E = E_0 [1 - \alpha (1 - x/a)(1 - y/a)]$$

(5)

where $\alpha = \gamma \tau_0 (0 \leq \alpha < 1)$

It is assumed that thickness varies linearly in one direction and parabolic in other direction i.e.

$$h = h_0 (1 + \beta_1 x/a)(1 + \beta_2 y^2/a^2)$$

(6)

Fig A: - Visco-elastic Square plate

where $a$ is length of a side of square plate and $\beta_1$ & $\beta_2$ are taper parameters in $x$- & $y$- directions respectively and $h = h_0$ at $x = y = 0$
Put the value of $E$ & $h$ from equation (5) & (6) in the expression of $D_1$, one obtain

$$D_1 = \frac{E_t}{12(1-v^2)} \left[ 1 - \alpha \frac{(1-x/a)(1-y/a)}{a^2} \right] \frac{1}{h^2} \left[ 1 + \frac{\beta_1 x/a}{a} \right] \left[ 1 + \frac{\beta_2 y/a}{a} \right] \frac{1}{12(1-v^2)}$$

Rayleigh-Ritz technique is applied to solve the frequency equation. In this method, one requires maximum strain energy must be equal to the maximum kinetic energy. So it is necessary for the problem under consideration that

$$\delta (V^* - T^*) = 0$$

for arbitrary variations of $W$ satisfying relevant geometrical boundary conditions.
Since the plate is assumed as clamped at all the four edges, so the boundary conditions are

\[
W = W_{,x} = 0, \quad x = 0, a \\
W = W_{,y} = 0, \quad y = 0, a
\]

(9)

and the corresponding two-term deflection function is taken as

\[
W = [(x/a)(y/a)(1-x/a)(1-y/a)^2[A_1 + A_2(x/a)(y/a)(1-x/a)(1-y/a)]
\]

(10)

Now assuming the non-dimensional variables as

\[
X = x / a, \quad Y = y / a, \quad \overline{W} = W / a, \quad \overline{h} = h / a
\]

(11)

The expressions for kinetic energy \( T^* \) and strain energy \( V^* \) are [2]

\[
T^* = (1/2)\rho p \overline{h}^2 \overline{a}^2 \int_0^1 \int_0^1 [(1+\beta_1 X)(1+\beta_2 Y)^2\overline{W}^2]dYdX
\]

(12)

and

\[
V^* = Q \int_0^1 \int_0^1 \frac{[1-\alpha(1-X)(1-Y)](1+\beta_1 X)^3(1+\beta_2 Y)^3}{(\overline{W}_{,XX})^2 + 2\nu\overline{W}_{,XX} \overline{W}_{,YY} + 2(1-\nu)(\overline{W}_{,XY})^2}dYdX
\]

(13)

where

\[
\overline{W}_{,xx} = (2A_1 y^2*(1-y)^2*(1-x)^2+2+y^2-4*x^*(1-x))+3*A_2*y^3*(1-y)
\]

\[
^3*(2x^*(1-x)^2+6x^2*(1-x)^2+2x^3*(1-x));
\]

\[
\overline{W}_{,yy} = (2A_1 x^2*(1-x)^2*(1-y)^2+2+y^2-4*y^*(1-y))+3*A_2*x^3*(1-x)
\]

\[
^3*(2y^*(1-y)^2+6y^2*(1-y)^2+2y^3*(1-y));
\]
\[ W_{xx} = (2^*A_1*(x^*(1-x)^2-x^2*(1-x))*(2^*y^*(1-y)^2-2^*y^2*(1-y))+3^*A_2 \]
\[ *x^2*(1-x)+3^*x^3*(1-x)^2+3^*y^2*(1-y)^3-3^*y^3*(1-y)^2); \]

and

\[ Q = E_0h_0^3a^3 / 24(1-v^2) \]  \hspace{1cm} (14)

Using equations (12) & (13) in equation (8), one get

\[ (V^{**} - \lambda^2 T^{**}) = 0 \]  \hspace{1cm} (15)

where

\[ V^{**} = \int \int \frac{[1-\alpha(1-X)(1-Y)](1+\beta_1X)^3(1+\beta_2Y^2)^3((\overline{W}_{xx})^2}{(\overline{W}_{yy})^2+2\overline{W}_{xx}\overline{W}_{yy}+2(1-v)(\overline{W}_{xy})^2}dYdX \]  \hspace{1cm} (16)

and

\[ T^{**} = \int \int [(1+\beta_1X)(1+\beta_2Y^2)\overline{W}_{xx}]dYdX \]  \hspace{1cm} (17)

Here,

\[ \lambda^2 = 12 \rho (1-v^2)a^2 / E_0h_0^2 \] is a frequency parameter.

Equation (15) consists two unknown constants i.e. \( A_1 \) & \( A_2 \) arising due to the substitution of \( W \). These two constants are to be determined as follows

\[ \partial(V^{**} - \lambda^2 T^{**}) / \partial A_n \hspace{1cm}, \hspace{0.5cm} n=1,2 \]  \hspace{1cm} (18)

On simplifying (18), one gets

\[ bn_1A_1 + bn_2A_2 = 0 \hspace{1cm}, \hspace{0.5cm} n=1,2 \]  \hspace{1cm} (19)
where $b_{n1}$, $b_{n2}$ ($n=1,2$) involve parametric constant and the frequency parameter.

For a non-trivial solution, the determinant of the coefficient of equation (19) must be zero. So one gets, the frequency equation as

$$\begin{vmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{vmatrix} = 0$$

(20)

With the help of equation (20), one can obtain a quadratic equation in $\lambda^2$ from which the two values of $\lambda^2$ can be found. These two values represent the two modes of vibration of frequency i.e. $\lambda_1$ (Mode 1) & $\lambda_2$ (Mode 2) for different values of taper constant and thermal gradient for a clamped plate.

where

$$b_{11}=(-382/4729725*\alpha*\beta_2^3-382/4729725*\alpha*\beta_1^3+376/121275$$

$$*\beta_2^2*\beta_1^2+181/242550*\beta_2^3*\beta_1^2+13/2695*\beta_2*\beta_1^2-23/1455300*\beta_2^3*\alpha*\beta_1*v-1/242550*\alpha*\beta_2^3*v+32/40425*\beta_1^3$$

$$+6/1225*\beta_2-1/22050*\alpha*\beta_2^2*\beta_1^2+2/11025*\beta_1^2*v-97/121275*$$

$$\alpha*\beta_2^2*\beta_1-97/242550*\beta_2^2*\alpha*\beta_1+181/242550*\beta_2^2*\beta_1^3+9/1225$$

$$*\beta_2*\beta_1^2+16/13475*\beta_2^2*\beta_1^3-67/859950*\beta_2^3*\alpha*\beta_1^2-1/8820*\beta_2^3$$

$$*\beta_1^2*v-17/40425*\alpha*\beta_1^2-34/40425*\alpha*\beta_2+13/2695*\beta_2^2*\beta_1^2-17/40425*\alpha*\beta_2^2+16/13475*\beta_2^3*\beta_1^2-1/8820*\beta_2^3*\beta_1^3$$

$$v-1/8820*\beta_2^2*\alpha*\beta_1^2+4/1225-1/11025*\beta_1^3*v-1/11025*\alpha*\beta_1^3*v-1/242550*\alpha*\beta_1^3*v-1/22050*\alpha*\beta_1^3$$

$$v-67/859950*\alpha*\beta_1^3*\beta_2-97/242550*\beta_2*\alpha*\beta_1^2-23/1455300*\alpha*\beta_1^3*\beta_2*v-1/8820*\beta_2^2*\alpha*\beta_1^3$$

$$v-2/11025*\beta_2^2*\alpha*\beta_1^2+26/8085*\beta_1^2-1/3150*\beta_2^2*\beta_1^2*v-1/1455300*d*\beta_2-1/793800*d*\beta_1\beta_1^2*$$

$$7350*\beta_2^3*\beta_1^3*v-1/29400$$

$$*\beta_2^3*\beta_1^3*v+26/8085*\beta_2^2*\beta_1^2-1/7640*\beta_2^2*\alpha*\beta_1^2*v-1/2910600$$

$$d*\beta_2*\beta_1+17/97020*\beta_2^3*\beta_1^3-1/7350*\beta_2^3*\beta_1^3*v-67/1719900$$

$$\alpha*\beta_2^3*\beta_1^2-67/1719900*\alpha*\beta_1^3*\beta_2^2-97/485100*\beta_2^2*\alpha*\beta_1^2-$$
\[ \frac{1}{3675} \beta_2^* \beta_1^* 2^* v - \frac{1}{11025} \beta_2^* 3^* v - \frac{1}{11025} \alpha^* \beta_2^* v + \frac{6}{1225} \beta_1^* - \frac{1}{1225} \alpha^* - \frac{1}{396900} \beta_2^* 2^* \beta_1^* v - \frac{1}{23910600} \alpha^* \beta_1^* 3^* \beta_2^* 2^* v - \frac{31}{32016000} \alpha^* \beta_1^* 3^* \beta_2^* 3^* v - \frac{23}{29106000} \alpha^* \beta_2^* 3^* \beta_1^* 2^* v - \frac{677}{89189100} \alpha^* \beta_1^* 3^* \beta_2^* 3^* + \frac{32}{40425} \beta_2^* 3^* - \frac{1}{14410} \alpha^* \beta_2^* \beta_1^* v - \frac{34}{40425} \alpha^* \beta_1^* ; \\
\]

\[ b_{12} = b_{21} = \left( -\frac{1257}{3046803400} \alpha^* \beta_2^* 3^* + 149/111561500 \right) + 29/11561550 \alpha^* \beta_2^* 3^* + 53/30830800 \beta_2^* 2^* \beta_1^* - \frac{1}{254100} \alpha^* \beta_2^* - 3^* 8016000 \beta_2^* 2^* \beta_1^* 3^* v - 3^* 6166160 \beta_2^* 2^* \beta_1^* 3^* v - 19/24924000 \alpha^* \beta_1^* 3^* v - 3^* 6166160 \alpha^* \beta_1^* 2^* v + 29/7707700 \beta_2^* 3^* - 1^* 508200 \alpha^* \beta_2^* 2^* - 19/7214407200 \alpha^* \beta_1^* 3^* \beta_2^* 3^* v - 1^* 369969600 \alpha^* \beta_2^* 2^* \beta_1^* 3^* v - 1^* 14580576 \alpha^* \beta_2^* 2^* \alpha^* \beta_1^* 2^* + 2^* 254/25050025 \beta_2^* 2^* \beta_1^* 2^* v + 24^* 109309200 \beta_2^* 2^* \beta_1^* 3^* + 149/7707700 \beta_2^* 3^* - 3^* 30830800 \beta_2^* 2^* \beta_1^* v - 953/20040200 \alpha^* \beta_2^* \beta_1^* 1^* 30830800 \alpha^* \beta_2^* v - 1^* 18498480 \alpha^* \beta_1^* 3^* \beta_2^* 3^* - 2^* 277/619418800 \alpha^* \beta_2^* 2^* - 1^* 2290288 \beta_2^* 2^* \alpha^* \beta_1^* 2^* v - 1^* 2290288 \alpha^* \beta_2^* 2^* - 1^* 1451544 \alpha^* \beta_2^* 2^* - 7^* 619418800 \beta_2^* 3^* \alpha^* \beta_1^* 1^* 1^* 2290288 \alpha^* \beta_2^* 2^* v - 1^* 1451544 \alpha^* \beta_2^* 2^* - 1^* 109309200 \alpha^* \beta_2^* 2^* v + 2^* 24^* 109309200 \beta_2^* 3^* \beta_1^* 2^* 4^* 3^* 40080040 \beta_2^* 2^* \alpha^* \beta_1^* - 1^* 6166160 \alpha^* \beta_2^* 2^* v + 29/11561550 \alpha^* \beta_2^* v + 53/30830800 \alpha^* \beta_2^* v + 2^* 24^* 109309200 \beta_2^* 3^* \beta_1^* 2^* 4^* 3^* 30830800 \alpha^* \beta_1^* 3^* \beta_2^* 3^* - 1^* 15415400 \alpha^* \beta_2^* 2^* v - 1^* 541080540 \alpha^* \beta_1^* 2^* d^* \beta_2^* 2^* - 2^* 1^* 2885762880 \alpha^* \beta_1^* v + 1^* 2885762880 \alpha^* \beta_1^* v + 1^* 15415400 \beta_2^* 2^* v - 2^* 254100 \alpha^* \beta_1^* v - 1^* 1442881440 \alpha^* \beta_2^* 3^* \beta_2^* 3^* - 1^* 8016000 \beta_2^* 3^* \beta_2^* 3^* v - 1^* 30830800 \alpha^* \beta_1^* v - 1^* 2290288 \beta_2^* 2^* \alpha^* \beta_1^* v - 5^* 73/13627213600 \alpha^* \beta_1^* 3^* \beta_2^* 3^* - 1^* 1257/3406803400 \alpha^* \beta_1^* 3^* - 1^* 1082161080 \alpha^* \beta_2^* 3^* + 3^* 1^* 801600800 \beta_2^* 3^* \beta_1^* 3^* - 1^* 160320160 \beta_2^* 3^* \beta_2^* 3^* v - 277/1238837600 \alpha^* \beta_2^* 3^* \beta_1^* 2^* - 277/1238837600 \alpha^* \beta_1^* 3^* \beta_2^* 2^* 953/801600800 \beta_2^* 2^* \alpha^* \beta_1^* 2^* 1^* 30830800 \beta_2^* 3^* v + 53/4624620 \beta_1^* 2^* 1^* 924924000 \alpha^* \beta_2^* 3^* v ; \\
\]

\[ b_{22} = \left( \frac{41}{121275} - \frac{155}{16648632} \alpha^* \beta_2^* 3^* + \frac{589}{92492400} \beta_2^* 2^* \beta_1^* v - \frac{61}{34684650} \beta_2^* 3^* \alpha^* \beta_1^* v - \frac{131}{10672200} \beta_2^* 3^* \beta_1^* 2^* v - 19/3557400 \alpha^* \beta_2^* 3^* \beta_1^* 2^* - 19/3557400 \alpha^* \beta_1^* 3^* \beta_2^* 2^* - 31/1101100 \beta_2^* 2^* \alpha^* \beta_1^* 2^* 2^* 148225 \beta_2^* 2^* \alpha^* \beta_1^* v - 1^* 2522520 \alpha^* \beta_2^* 3^* v - 1^* 97020 \alpha^* \beta_1^* v + 4^* 539000 \beta_2^* \beta_1^* - 69/700700 \alpha^* \beta_2^* - 69/700700} \\
\]

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**INTRODUCTION TO MATLAB**

**MATLAB** is a computer program for people doing numerical computation, especially linear algebra (matrices). It began as a "MATRIX LABORATORY" program, intended to provide interactive access to the libraries Linpack and Eispack. It has since grown well beyond these libraries, to become a powerful tool for visualization, programming, research, engineering, and communication.

Matlab is a tool for mathematical calculations. First, it can be used as a scientific calculator. Next, it allows you to plot or visualize data in many different ways, perform matrix algebra, work with polynomials or integrate functions. Like in a programmable calculator, you can create, execute and save a sequence of commands in order to make your computational process automatic. In the end, Matlab can also be treated as a user-friendly programming language, which gives the possibility to handle mathematical calculations in an easy way. In summary, as a computing/programming environment, Matlab is especially designed to work with data sets as a whole such as vectors, matrices and images.

Matlab, i.e. it is used to enter commands and display text results. The string `>>` is the Matlab prompt (or `EDU>>` for the Student Edition).
When the Command Window is active, a cursor appears after the prompt, indicating that Matlab is waiting for your command. Matlab responds by printing text in the Command Window or by creating a Figure Window for graphics. To exit Matlab use the command exit or quit. MATLAB is a high-performance language for technical computing. It integrates computation, visualization, and programming in an easy-to-use environment where problems and solutions are expressed in familiar mathematical notation.

MATLAB stands for matrix laboratory. MATLAB was originally written to provide easy access to matrix software developed by the LINPACK and EISPACK projects. Today, MATLAB engines incorporate the LAPACK and BLAS libraries, embedding the state of the art in software for matrix computation. MATLAB has evolved over a period of years with input from many users. In university environments, it is the standard instructional tool for introductory and advanced courses in mathematics, engineering, and science. In industry, MATLAB is the tool of choice for high-productivity research, development and analysis. MATLAB features a family of add-on application-specific solutions called Tool boxes. Very important to most users of MATLAB, toolboxes allow you to learn and apply specialized technology.
Operators used in MATLAB

Some operators used in expressions:

- +  Addition
- -  Subtraction
- *  Multiplication
- /  Division
- \  Left division
- ^  Power
- '  Complex conjugate transpose
- () Specify evaluation order