Chapter-IV
CHAPTER 4
OPTIMIZATION METHODS

4.1 Introduction

This chapter deals with the optimization methods used for selection of optimal parameter combination for development of Aluminium Metal Matrix Composite. In this work, Grey Relational analysis, Desirability Function Analysis, Taguchi-Fuzzy logic, Grey-Fuzzy and Desirability -Fuzzy methods are used to determine optimal parameters combinations. These methods are explained in the following sections.

4.2 Grey Relational Analysis

Grey Relational analysis (GRA) uses a specific concept of information. It defines situations with no information as black, and those with perfect information as white. However, neither of these idealized situations ever occurs in real world problems. In fact, situations between these extremes are described as being grey, hazy or fuzzy. Therefore, a grey system means that a system in which part of information is known and part of information is unknown. With this definition, information quantity and quality form a continuum from a total lack of information to complete information – from black through grey to white. Since uncertainty always exists, one is always somewhere in the middle, somewhere between the extremes, somewhere in the grey area. Procedure of the steps for grey-relational analysis is as follows

4.2.1 Data pre-processing

Data pre-processing (Data Normalization) is normally required since the range and unit in one data sequence may differ from the others. Data preprocessing is also necessary when the sequence scatter range is too large, or when the directions of the target in the sequences are different. Data pre-processing is a means of transferring the original sequence to a comparable sequence. Depending on the characteristics of a data sequence, there are various methodologies of data pre-processing available for the grey relational analysis.

If the target value of original sequence is infinite, then it has a characteristic of the “larger the better”. The original sequence can be normalized as follows:
When the “smaller the better” is a characteristic of the original sequence, then the original sequence should be normalized as follows:

\[
X^*_i(k) = \frac{x^o_i(k) - \min x^o_i(k)}{\max x^o_i(k) - \min x^o_i(k)}
\] (4.2.1)

However, if there is a definite target value “nominal the best” to be achieved, the original sequence will be normalized in form:

\[
X^*_i(k) = 1 - \frac{|x^o_i(k) - x^o|}{\max x^o_i(k) - x^o}
\] (4.2.2)

Or, the original sequence can be simply normalized by the most basic methodology, i.e. let the values of original sequence are divided by the first value of the sequence:

\[
X^*_i(k) = \frac{x^o_i(k)}{x^o_i(1)}
\] (4.2.4)

Where \(i = 1 \ldots, m; k = 1 \ldots, n. \) \(m\) is the number of experimental data items and \(n\) is the number of parameters. \(x^o_i(k)\) Denotes the original sequence, \(x^*_i(k)\) the sequence after the data pre-processing, \(\max x^o_i(k)\) the largest value of \(x^o_i(k)\), \(\min x^o_i(k)\) the smallest value of \(x^o_i(k)\), and \(x^o\) is the desired value.

### 4.2.2 Grey relational coefficient and grey relational grade

In grey relational analysis, the measure of the relevancy between two systems or two sequences is defined as the grey relational grade. When only one sequence, \(x_o(k)\), is available as the reference sequence, and all other sequences serve as comparison sequences, it is called a local grey relation measurement. After data pre-processing is carried out, the grey relation coefficient \(\xi_l(k)\) for the \(k^{th}\) performance characteristics in the \(i^{th}\) experiment can be expressed as

\[
\xi_l(k) = \frac{\Delta_{\min} + \zeta \Delta_{\max}}{\Delta_{ol}(k) + \zeta \Delta_{\max}}
\] (4.2.5)

Where, \(\Delta_{ol}\) is the Deviation sequence.
\[ \Delta_{ai} = \| x^*_{o}(k) - x^*_{i}(k) \| \]
\[ \Delta_{min} = \min_{j \in i} \min_{k} \| x^*_{o}(k) - x^*_{j}(k) \| \]
\[ \Delta_{max} = \max_{j \in i} \max_{k} \| x^*_{o}(k) - x^*_{j}(k) \| \]  \hspace{1cm} (4.2.6)

\( x^*_{o}(k) \) Denotes the reference sequence and \( x^*_{i}(k) \) denotes the comparability sequence. 
\( \zeta \) is distinguishing or identification coefficient: \( \zeta \in [0, 1] \) (the value may be adjusted based on the actual system requirements). A value of \( \zeta \) is the smaller and the distinguished ability is the larger. \( \zeta = 0.5 \) is generally used.

After the grey relational coefficient is derived, it is usual to take the average value of the grey relational coefficients as the grey relational grade. The grey relational grade is defined as

\[ y_i = \frac{1}{n} \sum_{k=1}^{n} \xi_i(k) \]  \hspace{1cm} (4.2.7)

The grey relational grade \( y_i \) represents the level of correlation between the reference sequence and the comparability sequence. If the two sequences are identical, then the value of grey relational grade is equal to 1. The grey relational grade also indicates the degree of influence that the comparability sequence could exert over the reference sequence. Therefore, if a particular comparability sequence is more important than the other comparability sequences to the reference sequence, then the grey relational grade for that comparability sequence and reference sequence will be higher than other grey relational grades.

The grey relational grade values are calculated for each factor at each level and the optimal level for each factor is identified based on their individual grey relational grade values. The optimal level of any influential factor has highest grey relational grade value among their considered levels.

### 4.3 Desirability Function Analysis

One more useful approach for optimization of multiple responses is to use the simultaneous optimization technique popularized by Derringer and Suich. Their procedure introduces the concept of Desirability Function Analysis (DFA). The
method makes use of an objective function, \( D(X) \), called the desirability function and transforms an estimated response into a scale free value \( d_i \) called desirability. The Desirability ranges are from zero to one (least to most Desirability, respectively). The factor settings with maximum total desirability are considered to be the optimal parameter conditions.

**4.3.1 Optimization steps using desirability functional analysis**

Calculate the individual desirability \( d_i \) for the corresponding responses using the formula proposed by Derringer and Suich. There are three forms of the desirability functions according to the response characteristics.

(a) **The nominal-the-best**: The value of \( \hat{y} \) is required to achieve a particular target \( T \). When the \( \hat{y} \) equals to \( T \), the desirability value equals to 1; if the departure of \( \hat{y} \) exceeds a particular range from the target, the desirability value equals to 0, and such situation represents the worst case. The desirability function of the nominal-the-best can be written as

\[
d_i = \begin{cases} 
\frac{(\hat{y} - y_{\text{min}})}{(T - y_{\text{min}})}^s, & y_{\text{min}} \leq \hat{y} \leq T, \quad s \geq 0 \\
\frac{(\hat{y} - y_{\text{min}})}{(T - y_{\text{min}})}^t, & T \leq \hat{y} \leq y_{\text{min}}, \quad t \geq 0
\end{cases} \quad (4.3.1)
\]

Where the \( y_{\text{max}} \) and \( y_{\text{min}} \) represent the upper and lower tolerance limits of \( \hat{y} \) and \( s \) and \( t \) represent the indices.

(b) **The larger-the-better**: The value of \( \hat{y} \) is expected to be the larger the better. When the \( \hat{y} \) exceeds a particular criteria value, which can be viewed as the requirement, the desirability value equals to 1; if the \( \hat{y} \) is less than a particular criteria value, which is unacceptable, the desirability equals to 0. The desirability function of the larger-the-better can be written as
Where the $y_{min}$ represents the lower tolerance limit of $\hat{y}$, the $y_{max}$ represents the upper tolerance limit of $\hat{y}$ and $r$ represents index.

(c)\textit{The smaller-the-better}: The value of $\hat{y}$ is expected to be the smaller the better. When the $\hat{y}$ is less than a particular criteria value, the desirability value equals to 1; if the $\hat{y}$ exceeds a particular criteria value, the desirability value equals to 0. The desirability function of the smaller-the-better can be written as

$$d_i = \begin{cases} 
  0, & \hat{y} \leq y_{min} \\
  \left(\frac{\hat{y} - y_{min}}{y_{max} - y_{min}}\right)^r, & y_{min} \leq \hat{y} \leq y_{max}, \ r \geq 0 \\
  1, & \hat{y} \geq y_{max} 
\end{cases}$$

Where the $y_{min}$ represents the lower tolerance limit of $\hat{y}$, the $y_{max}$ represents the upper tolerance limit of $\hat{y}$ and $r$ represents the weight. The $s$, $t$ and $r$ in Eq.3.18, 3.19, and 3.20 indicate the weights and are defined according to the requirement of the user. If the corresponding response is expected to be closer to the target, the weight can be set to the larger value; otherwise, the weight can be set to the smaller value.

The individual desirability values have been accumulated to calculate the overall desirability, using the following equation (k). Here $D$ is the overall or composite desirability value, $d_i$ is the individual desirability value of $i$th quality characteristic and $n$ is the total number of responses.

$$D = (d_1 \ast d_2 \ast d_2 \ldots \ldots \ast d_n)^{1/n} \quad (4.3.4)$$

The composite desirability values are calculated for each factor at each level (Table 6) and the optimal level for each factor is identified based on their individual composite desirability values. The optimal level of any influential factor has highest composite desirability value among their considered levels.
4.4 Fuzzy Method

Using Fuzzy logic, the test results are analyzed and optimum influential factor combination is identified as follows

4.4.1 Calculating S//N ratios for experimental results

For different data sequences the data should be normalized and is depends upon the quality of the response, whether it is to be minimized or maximized or a nominal value (smaller the better or larger the better or Nominal the better).

i) The Smaller-The-Better

The Signal-To-Noise ratio for the Smaller-The-Better is:

\[ \frac{S}{N} = -10 \log_{10} \left( \frac{\sum y^2}{n} \right) \]  \hspace{1cm} (4.4.1)

ii) The Larger-The-Better

The Signal-To-Noise ratio for the Larger-the-better is:

\[ \frac{S}{N} = -10 \log_{10} \left( \frac{1}{n} \sum \frac{1}{y^2} \right) \]  \hspace{1cm} (4.4.2)

iii) Nominal-the-best

The Signal-To-Noise ratio for the Nominal-the-best is:

\[ \frac{S}{N} = 10 \log_{10} \left( \frac{y^2}{s^2} \right) \]  \hspace{1cm} (4.4.3)

4.4.2 Determination of the Comprehensive Output Measure with fuzzy logic

The fuzzy logic unit consist a fuzzifier, membership functions, a fuzzy rule base, an inference engine and a defuzzifier. In the fuzzy logic analysis, the fuzzifier uses membership functions to fuzzify the normalized values first. Next, the inference
The input variables of the fuzzy logic are converted into linguistic fuzzy subsets using membership functions of a triangle form, and are uniformly assigned. The fuzzy rule base consists of a group of if-then control rules to express the inference relationship between input and output. A typical linguistic fuzzy rule called Mamdani is described as

Rule 1: if x1 is A1, x2 is B1, x3 is C1, x4 is D1, x5 is E1, x6 is F1, x7 is G1, x8 is H1, x9 is I1, x10 is J1, and x11 is K1 then y is E1 else

Rule 2: if x1 is A2, x2 is B2, x3 is C2, x4 is D2, x5 is E2, x6 is F2, x7 is G2, x8 is H2, x9 is I2, x10 is J2, and x11 is K2 then y is E2 else

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Rule n: if x1 is An, x2 is Bn, x3 is Cn, x4 is Dn, x5 is En, x6 is Fn, x7 is Gn, x8 is Hn, x9 is In, x10 is Jn, and x11 is Kn then y is En else

In above Ai, Bi, Ci, Di, Ei, Fi, Gi, Hi, Ii, Ji, Ki are fuzzy subsets for input variables defined by the corresponding membership functions i.e., α/Ai, α/Bi, α/Ci, α/Di, α/Ei, α/Fi, α/Gi, α/Hi, α/Ii, α/Ji, α/Ki and the output variable ‘yo’ is the COM, and is converted into linguistic fuzzy subsets using membership functions of a triangle form. Unlike the input variables, the output variable is assigned into some number subsets. Then, considering the conformity of performance characteristics for input variables, ‘n’ number fuzzy rules are defined. The fuzzy inference engine is the kernel of a fuzzy system. It can solve a problem by simulating the thinking and decision pattern of human being using approximate or fuzzy reasoning. For this research, the max-min compositional operation of Mamdani is adopted to perform calculation of COM.
The COM values are calculated for each factor at each level and the optimal level for each factor is identified based on their individual COM values. The optimal level of any influential factor has highest COM value among their considered levels.

### 4.5 Grey-Fuzzy Method

Grey-Fuzzy Method is a hybrid approach obtained by combining the grey relational analysis and Fuzzy logic. Here the input values for the fuzzy logic is grey relational coefficients. First in Grey-Fuzzy analysis, grey relational coefficients are fuzzyfied and inference engine generates fuzzy value by performing fuzzy reasoning on fuzzy rule base in which 2048 rules are created and entered. Then the de-fuzzyfier gives Grey Fuzzy Grade by converting fuzzy value.

The Grey-Fuzzy grade values are calculated for each factor at each level and the optimal level for each factor is identified based on their individual Grey-Fuzzy grade values. The optimal level of any influential factor has highest Grey-Fuzzy grade value among their considered levels.

### 4.6 Desirability-Fuzzy Method

Desirability-Fuzzy Method is a hybrid approach obtained by combining the desirability function analysis and Fuzzy logic. Here the input values for the fuzzy logic is individual desirability values. First in Desirability-Fuzzy analysis, individual desirability values are fuzzyfied and inference engine generates fuzzy value by performing fuzzy reasoning on fuzzy rule base in which 2048 rules are created and entered. Then the de-fuzzyfier gives Desirability-Fuzzy Grade by converting fuzzy value.

The Desirability-Fuzzy grade values are calculated for each factor at each level and the optimal level for each factor is identified based on their individual Desirability-Fuzzy grade values. The optimal level of any influential factor has highest Desirability-Fuzzy grade value among their considered levels.