CHAPTER - III
GENERATION MECHANISM OF ELF HISS
EMISSIONS AT HIGHER LATITUDES
CHAPTER III

At higher latitudes other than the low latitudes two types of ELF hiss emissions are frequently observed; (i) the so-called detached plasma (DP) hiss emissions, and (ii) the plasmaspheric ELF hiss emissions. The DP hiss emissions are observed in the regions outside the plasmapause where the cold plasma density is considerably enhanced compared with the ambient plasma density. On the other hand, the plasmaspheric ELF hiss emissions are observed within the plasmasphere itself. Although there are sufficient observational data on these emissions, their generation mechanisms are not convincingly understood. In part I of this chapter, we discuss the generation mechanism of DP hiss emissions by calculating their wave power spectral densities at \( L = 6.6 \) in the equatorial plane and comparing them with the observed intensities. In part II, we attempt to explain the generation mechanism of the plasmaspheric ELF hiss emissions observed aboard GEOS-1 satellite both at small and large wave normal angles by calculating their magnetic field intensities in the equatorial plane just near the inner edge of the plasmapause and comparing them with the observed intensities.
CHAPTER-III

PART I: GENERATION MECHANISM OF ELF HISS EMISSIONS IN THE DETACHED PLASMA REGIONS OF MAGNETOSPHERE

3.1 INTRODUCTION

ELF hiss that is most commonly observed within the plasmasphere is usually called plasmaspheric hiss (Russell et al., 1969; Thorne et al., 1973; Hayakawa and Tanaka, 1977). In addition to this, there also exists another important and distinct zone for ELF emissions the so called "detached plasma regions of the magnetosphere" which are the regions outside the plasmapause where the cold plasma density is considerably enhanced compared with the ambient plasma density (Chapell, 1972). Scientists believe that they are initially originated in the plasmasphere and then torn off either by the convection electric field (Chapell, 1974; Barfield et al., 1975) or as a consequence of the interchange instability (Lemaire, 1975). The ELF emissions have been reported to occur several times in such detached plasma regions (Chan, 1974; Chan et al., 1974; Chan and Holzer, 1976; Kivelson, 1976; Cornilleau-Wehrlin et al., 1978). These emissions are referred to as detached plasma (DP) hiss emissions.

Cornilleau-Wehrlin et al. (1978) have discussed the morphological characteristics of DP hiss in the equatorial region such as its association with the plasma density enhancements. Chan (1974) and Chan and Holzer (1976) have investigated the wave normals of DP hiss mainly in the off-equatorial regions, but they
have not been able to distinguish between the generation and propagation effects owing to the lack of equatorial observations. Keeping in view the importance of the information on the distribution of wave normal directions in both the equatorial and off-equatorial regions in studying the generation and propagation of magnetospheric emissions, Hayakawa et al. (1986) have determined the wave normal directions of DP hiss observed exactly in the equatorial plane at \( L = 6.6 \), based on data from the geostationary satellite GEOS-2. Chan (1974) and Kivelson (1976) have attempted to test the theoretical model of wave generation of DP hiss by the electron cyclotron instability using direction finding results in the off-equatorial regions. Kivelson (1976) has found experimentally a lower limit in the particle flux for energetic electron distributions that are responsible for the generation of these emissions. Hayakawa et al. (1986) have suggested that DP hiss can be generated in the equatorial plane under quasi-linear electron cyclotron instability mechanism and all the observed characteristics and direction finding results can be interpreted satisfactorily by this mechanism.

In this part of the present chapter, we extend the work of Hayakawa et al. (1986) and carry out detailed intensity calculations for DP hiss in the equatorial plane by employing plasma density and electron gyrofrequency observed experimentally by GEOS-2 satellite. A close agreement between the calculated and observed intensities confirms the suggestion of Hayakawa et
al. (1986) that DP hiss emissions are generated under quasi-linear electron cyclotron instability mechanism in the equatorial plane and propagated down to the ionosphere in the ducted mode.

3.2 THEORETICAL CONSIDERATION AND THE EXPRESSIONS USED

For quantitative study of whistler-mode electromagnetic waves, Kennel and Petschek (1966) have propounded the linear electron cyclotron instability theory which, afterwards, has been extended to propose quasi-linear electron cyclotron instability theory. Several workers (Roux and Solomon, 1971; Etcheto et al., 1973; Sazhin, 1984) have presented self-consistent calculations leading to the estimates of peak power spectral density and corresponding peak frequency. In quasi-linear theory, quasi-longitudinal ($\theta \approx 0^\circ$) or ducted propagation is assumed and a dynamic equilibrium is supposed to be established in which waves are continuously generated and particles are continuously injected and lost by pitch angle diffusion into the loss cone.

The frequency $f_{\text{max}}$ corresponding to peak power spectral density is calculated using the relation (Sazhin, 1984;1987)

$$f_{\text{max}} = \frac{c^2 f_H^3}{v^2 q f_p^2} \quad \ldots(3.1)$$

where $c$ is the velocity of light in vacuum, $v$, the characteristic parallel velocity of the incoming electrons at the L-shell of the wave generation, $f_H$, the electron gyrofrequency, $f_p$, the plasma
frequency, and \( q \) the coefficient depending on the anisotropy of electron density along magnetospheric field lines. The value of \( q \) for a dipole magnetic field lies between 1.3 and 1.5 (Sazhin, 1987), but we consider \( q = 1.4 \) in our present calculations following Hayakawa et al. (1988).

Equation (3.1) can be put in a more convenient form as (Hayakawa et al., 1988).

\[
\nu_{\text{max}} = 1.6 \times 10^9 / L_\text{eq}^9 \nu_{\text{eq}} W \quad \cdots (3.2)
\]

in which \( n_{\text{eq}} \) is equatorial electron density in \( \text{cm}^{-3} \) and \( W \) is the characteristic parallel energy of the incoming electrons in keV. The maximum power spectral density \( < B_f^2 >_{\text{max}} \) at the peak frequency \( \nu_{\text{max}} \) can be estimated from the relation (Sazhin, 1984)

\[
< B_f^2 >_{\text{max}} = 5 \times 10^{-17} \lambda \nu_{\text{eq}}^{1.5} L_\text{eq}^{4.5} \frac{dn_i}{dt} \quad \cdots (3.3)
\]

where \( \lambda = n_{\text{eq}} L_\text{eq}^{5.5} \frac{dn_i}{dt} \) is the rate of influx of electrons, \( \nu_{\text{eq}} \) the electron energy at \( L = L_\text{eq} \) in eV, and \( L_\text{eq} \) can be chosen arbitrarily. Equation (3.3) calculates intensity directly in \( \sqrt{\text{Hz}^{-1}} \) in the equatorial plane. The Gaussian units are being used in the present study.

3.3 Experimental Observations

Hayakawa et al. (1986) have made an extensive study of DP hiss in the equatorial plane, based on the wave normal direction measurements aboard geostationary satellite GEOS-2. Their
experimental results and the morphological characteristics of DP hiss are summarized as follows:

(a) A detached plasma region in the equatorial plane at L=6.6 was detected during the recovery phase of a substorm and in the local sector of ∼2000 hrs, and it was highly correlated with the presence of ELF emissions.

(b) The frequency of maximum intensity lies between 100 Hz and 200 Hz, which is in agreement with that in the off- equatorial region (50 Hz - 200 Hz) (Chan, 1974) and is considerably lower than the corresponding frequency (∼550 Hz) of plasmaspheric ELF hiss. The peak power spectral density of DP hiss is ∼10^{-5} \text{W} \text{Hz}^{-1} at ∼160 Hz.

(c) The wave distribution functions (WDFs) of DP hiss consist of a single peak and the wave normal directions θ make small angles (less than 25°) with the earth's magnetic field.

(d) The WDFs of DP hiss are approximately circular and provide information about the angular width of the unstable cone within which the waves are unstable (or generated), and the half width of the unstable cone increases with wave frequency.

3.4 RESULTS AND DISCUSSION

The equatorial plane is generally supposed to be the most likely source region for electromagnetic waves (Russell et al., 1969; Helliwell, 1965; Tsurutani and Smith, 1977). The DP hiss reported by Hayakawa et al., (1986) and considered in this part of
the chapter was observed on 19 Dec., 1979 during the time interval 1800-1900 hrs UT in the equatorial plane (geomag. lat. ~ 0.9°) at L = 6.6. Figure 3.1 shows the experimentally observed power spectral densities of DP hiss in the equatorial plane at L=6.6. Actually this figure is reproduced from Figs. 3(a) and 4(a) of Hayakawa et al. (1986). Fig 3.1 clearly indicates that the peak power spectral densities of $\sim 4 \times 10^{-6}$, $\sim 1 \times 10^{-5}$ and $\sim 1 \times 10^{-8}$ Hz $^{-1}$ have been observed aboard geostationary satellite GEOS-2 at the wave frequencies of 120, 160 and 750 Hz, respectively and that the intensity of $\sim 1 \times 10^{-8}$ Hz $^{-1}$ corresponds to the secondary maxima at 750 Hz. Assuming that the DP hiss is generated in the equatorial plane at L=6.6, we, therefore, calculate the peak power spectral density $\langle B_f^2 \rangle_{\text{max}}$ at these three selected frequencies of 120, 160 and 750 Hz only. The experimentally observed values of electron gyrofrequency and plasma frequency in the equatorial plane at L=6.6 are $f_H = 2.03$ kHz and $f_p = 57.60$ kHz, respectively. The later value yields an equatorial electron density $n_{\text{eq}}$ of 41 el. cm$^{-3}$. Taking $n_{\text{eq}} = 41$ el. cm$^{-3}$ and considering different values of characteristic energy $W$, we first calculate the peak frequencies $f_{\text{max}}$ using equation (3.2) from which we find that

$$f_{\text{max}}(\text{kHz}) = \frac{1.6423}{W \ (\text{keV})} \quad \ldots(3.4)$$
FIG. 3.1 - WAVE POWER SPECTRAL DENSITY OF DP HISS EMISSIONS OBSERVED ABOARD GEOS-2 SATELLITE IN THE EQUATORIAL PLANE AT L = 6.6 (AFTER HAYAKAWA ET AL., 1986).
The calculated values of $f_{\text{max}}$ are plotted against the characteristic energy $W$ of the incoming electrons in Fig. 3.2. From Fig. 3.2, we find $W = 13.69, 10.25$ and $2.20$ keV corresponding to $f_{\text{max}} = 120, 160$ and $750$ Hz, respectively. Thus the DП hiss observed by the geostationary satellite GEOS-2 seems most likely to be radiated by energetic electrons of $\sim 2-15$ keV at $L = 6.6$ in the equatorial plane.

The calculated values of characteristic energy are then used to estimate the peak power spectral densities at the corresponding peak frequencies. We choose $L = L_0 = 6.6$. With $n_{\text{eq}} = 41$ el. cm$^{-3}$, we get $\Lambda = 1.3191 \times 10^6$. Now, taking $W_0 = 13.60, 10.25$ and $2.20$ keV, we get the values of $\langle B_f^2 \rangle_{\text{max}}$ at $120, 160$ and $750$ Hz, respectively, as shown in Table 3.1 which also shows the observed peak power spectral densities at these frequencies as deduced from Fig 3.1.

**Table 3.1 - Observed and Calculated Peak Power Spectral Densities at L=6.6 in the Equatorial Plane**

<table>
<thead>
<tr>
<th>Peak frequency ($f_{\text{max}}$) in Hz</th>
<th>Characteristic energy (keV)</th>
<th>$\frac{d\eta}{d\Omega}$ (cm$^{-3}$ s$^{-1}$)</th>
<th>Peak intensity in W Hz$^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>observed</td>
<td>Calculated</td>
</tr>
<tr>
<td>120</td>
<td>13.60</td>
<td>$8.0 \times 10^{-6}$</td>
<td>$\sim 4 \times 10^{-6}$</td>
</tr>
<tr>
<td>160</td>
<td>10.25</td>
<td>$3.0 \times 10^{-5}$</td>
<td>$\sim 1 \times 10^{-5}$</td>
</tr>
<tr>
<td>750</td>
<td>2.20</td>
<td>$3.3 \times 10^{-7}$</td>
<td>$\sim 1 \times 10^{-8}$</td>
</tr>
</tbody>
</table>
FIG. 3.2 - VARIATION OF PEAK FREQUENCY $f_{\text{max}}$ WITH CHARACTERISTIC ENERGY $W$. 
Table 3.1 shows a close agreement between the calculated and the observed values of the peak power spectral density. Thus the calculated values of the intensity of DP hiss are found to be quite consistent with the observed intensities.

Etcheto et al. (1973) have given the following expressions [see their equation (15)] for peak frequency and peak power spectral density;

\[ x_{\text{max}} \sim 2.4 \times 10^2 / n_{\text{eq}} L^3 \]  \hspace{1cm} \ldots (3.5)

\[ \langle B_f^2 \rangle_{\text{max}} (\gamma \text{Hz}^{-1}) \approx 7.6 \times 10^{-7} n_{\text{eq}} L^{5.5} \frac{dn_i}{dt} \]  \hspace{1cm} \ldots (3.6)

where \( x_{\text{max}} \) ( = \( f_{\text{max}} / f_H \)) is the maximum normalized wave frequency. Taking \( L = 6.6 \), \( n_{\text{eq}} = 41 \text{ el. cm}^{-3} \), \( \frac{dn_i}{dt} = 1 \times 10^{-5} \text{ cm}^{-3} \)

and \( f_H = 2.03 \text{ kHz} \), we get from equations (3.5) and (3.6), \( f_{\text{max}} = 41.33 \text{ Hz} \) and \( \langle B_f^2 \rangle_{\text{max}} = 1.0025 \times 10^{-5} \gamma \text{Hz}^{-1} \). Although the intensity obtained by the model of Etcheto et al. (1973) is in good agreement with the observed peak power spectral density of the DP hiss (\( \sim 10^{-5} \gamma \text{Hz}^{-1} \) at 160 Hz), the value \( f_{\text{max}} = 41.33 \text{ Hz} \) does not coincide with any of the three observed values of 120, 160, and 750 Hz. Moreover, the model of Etcheto et al. (1973) gives only a single peak of power spectral density at a given equatorial location contrary to the actual observations. So, the model of Etcheto et al. (1973) does not seem suitable for studying the generation mechanism of DP hiss.

The values of \( \frac{dn_i}{dt} \) taken in the present calculations are \( \sim \)
\[ 10^{-7} - 10^{-5} \text{ cm}^{-3} \text{ s}^{-1} \] which are comparable with the value of source intensity (Etcheto et al., 1973). 

\[ b = \frac{0.3}{L^5} \text{ cm}^{-3} \text{ s}^{-1} \]

\[ = 2.3955 \times 10^{-5} \text{ cm}^{-3} \text{ s}^{-1} \text{ at } L=6.6 \text{ in the equatorial plane.} \]

The wave power spectral density \( \langle B_{r}^{2} \rangle_{\text{max}} \) is directly proportional to \( \frac{dn_{i}}{dt} \) so an order of increase in the value of \( \frac{dn_{i}}{dt} \) will increase the intensity by 10 dB (or and order of magnitude). The value of \( \frac{dn_{i}}{dt} \) generally increases with Kp-index (Hess, 1968). The intensity of DP hiss considered in the present study corresponds to the recovery phase of a substorm during which the injection (influx) of electrons is almost stopped. Hence, the values of \( \frac{dn_{i}}{dt} \) in the range \( 10^{-7} - 10^{-5} \text{ cm}^{-3} \) assumed in the present calculations seem to be reasonable.

In quasi-linear electron cyclotron theory, quasi-longitudinal (\( \theta \simeq 0^\circ \)) or ducted propagation is assumed (Etcheto et al., 1973; Sazhin, 1984). The direction finding results of Hayakawa et al. (1986) indicate that wave normals for the DP hiss make small angles (\( \theta < 25^\circ \)) with the earth's magnetic field. The small wave normal angles at the equator are consistent both with the wave generation as discussed above and with the ducted propagation down to the ionosphere. Chan (1974) has found that the wave normal directions of DP hiss away from the equator at high geomagnetic latitudes (\( 30^\circ - 50^\circ \)) show a considerable scatter over an angular range of \( 2^\circ - 40^\circ \), but they are mostly less than \( 35^\circ \), which are undoubtedly larger than the \( \theta \) values at the
equator. Moreover, Chan (1974) has found no latitudinal dependence of wave normals. The additional scatter in wave normal directions at higher latitudes may be due to the scattering of wave normals by the density irregularities during the course of ducted propagation away from the equatorial source region (Helliwell, 1965). Since the electron density enhancement in the detached plasma regions is usually extremely high compared with the values for normal whistler ducts as given by Angerami (1970), wave trapping at large \( \theta \) values, in agreement with the observations of Chan (1974), can be expected. Thus, the DP hiss emissions seem to remain trapped owing to the density enhancement of the detached plasma region down to the ionosphere. Beghin et al. (1985) have also found a close association between enhancements of local electron density and observation of impulsive ELF emissions of hiss type on the basis of simultaneous high resolution density measurements and direction finding for ELF hiss made on the AUREOL 3 satellite. They (Beghin et al., 1985) have shown that ELF emissions are trapped within the field-aligned plasma density enhancements.

In the present study local generation of DP hiss at \( L=6.6 \) in the equatorial plane is assumed. This assumption is supported by the experimental fact that the WDFs of DP hiss are almost circular (Hayakawa et al., 1986). The circular shapes of WDFs reflect the properties of the source, that is, they are
consistent with the isotropic emission generation (Lefeuvre and Helliwell, 1985). Likhter (1979) has used the measured wave data to locate the source region of the magnetospheric emissions. For this, he (Likhter, 1979) has considered three types of experimental evidence:

1. The relationship between the wave frequency and the local plasma parameters at the source,
2. The enhanced probability of occurrence of emissions or the enhanced amplitudes of the waves in particular magnetospheric regions,
3. The relationship between the wave characteristics and other phenomena in magnetospheric plasma.

Thus a close agreement between the calculated and the observed peak power spectral densities and discussion on wave propagation as above confirm the suggestion of Hayakawa et al. (1986) that the DP hiss emissions observed aboard GEOS-2 satellite are generated locally at L=6.6 in the equatorial plane under quasi-linear electron cyclotron instability mechanism by energetic electrons of energy ~2-15 keV and are propagated down to the ionosphere in the ducted mode.

3.5 CONCLUSION

We have carried out detailed intensity calculations for ELF hiss emissions observed by geostationary satellite GEOS-2 in a detached plasma region of the magnetosphere in order to confirm their generation mechanism. The calculated and the observed
intensities and frequency spectrum (peak frequencies) are found to be in a close agreement which confirms that these emissions are generated under quasi-linear electron cyclotron instability mechanism at $L = 6.6$ in the magnetospheric equatorial plane by energetic electrons of energy ~2-15 keV and are propagated down to the ionosphere in the ducted mode of propagation.
CHAPTER-III

PART II- GENERATION MECHANISM OF PLASMASPERIC ELF HISS EMISSIONS OBSERVED ABOARD GEOS-1 SATELLITE

3.6 INTRODUCTION

Plasmaspheric hiss emissions are broad-band and structureless extremely low frequency (ELF) electromagnetic whistler-mode waves that are almost always present and commonly observed within the plasmasphere (Taylor and Curnett, 1968; Russell et al., 1969; Muzzio and Angerami, 1972; Thorne et al., 1973; Tsurutani et al., 1975; Hayakawa and Tanaka, 1977; Cornilleau-Wehrlin et al., 1978), and are different than the detached plasma (DP) hiss emissions discussed in part I of this chapter. Since these emissions play an important role in the formation of slot region between the inner and outer radiation belts (Lyons et al., 1972; Lyons and Thorne, 1973), a thorough understanding of their generation and propagation mechanisms is highly essential. However, in spite of a large number of observations of these emissions, their generation mechanism is not properly understood. From a detailed analysis of the OGO-5 wave data, Thorne et al. (1973) have suggested that these emissions are generated with their wave normals aligned with the geomagnetic field, just inside the plasmapause by the cyclotron resonance instability due to medium energy (10-100 keV) electrons. The cyclotron resonance instability mechanism, however, is convective rather than absolute, so the generation
mechanism envisioned, whether it be the original Kennel and Petschek (1966) mechanism or that developed by Etcheto et al. (1973), requires at the equator the existence of a continuous source of longitudinal waves i.e. waves with their normals close to the direction of the earth's magnetic field. Oblique waves are not amplified because they suffer too much Landau damping. Obviously the mechanism can be maintained by amplified waves that return to the source region after being reflected at the base of the ionosphere (ducted waves) or after a magnetospheric reflections (non-ducted waves), so long as they return with their normals sufficiently close to the field to allow further amplification.

Recently, Hayakawa et al. (1987) have carried out direction finding studies for plasmaspheric ELF hiss emissions and determined the wave distribution functions of these emissions at the equatorial region inside the plasmapause, by applying the maximum entropy method to the data observed by GEOS-1 satellite. They have found that just inside the plasmapause, the wave normal direction of ELF hiss emissions was nearly aligned with the magnetic field, and when the observing position was 0.3–0.5 Re inside from the plasmapause, there were two different groups of wave normal angles; one of medium wave normal angles ranging from 20 - 60°, and the other of large wave normal angles ranging from 70 - 80°. Hayakawa et al. (1987) have suggested that the emissions observed with small wave normal angles are generated by
the electron cyclotron instability such as that discussed by Thorne et al. (1973) and those observed with large wave normal angles might be generated by Cerenkov or Landau resonance instability mechanism.

In this part of the chapter, we study the generation mechanism of the plasmaspheric ELF hiss emissions observed aboard GEOS-1 satellite in the equatorial region by calculating the equatorial magnetic field intensities of these emissions in terms of both the Cerenkov and cyclotron resonance instability mechanisms. The results show that the Cerenkov instability mechanism is not capable of explaining the generation of oblique wave normals.

3.7 THEORETICAL CONSIDERATION AND EXPRESSIONS USED

In order to calculate the intensity of plasmaspheric ELF hiss observed at large wave normal angles (oblique wave normals), we consider the expression derived by Mansfield (1964) which has been used previously by Jorgensen (1960), Lim and Leaspere (1972) and Taylor and Shawhan (1974) to interpret the auroral latitude emissions and by Prakash et al. (1979) and Prakash and Singh (1981) to interpret low latitude VLF hiss emissions. According to Mansfield (1964), the power (in W Hz\(^{-1}\)) produced by a particle radiating in the Cerenkov mode is given by
\[
\frac{dp}{df} = \sum_{i=1}^{2} \frac{q^2 \omega |B_i^2| T_{ii} J_i^2(L_0) + B_{ii}^2 T_{33} J_0^2(L_0) - 2B_{ii} B_{il} T_{13} J_i(L_0) J_0(L_0)}{2 \epsilon_0 \epsilon_\parallel \left( \frac{B_n^2}{\epsilon_1} - 4 C_n \epsilon_1 \epsilon_1 \epsilon_2 \right)^{1/2}}
\]

where

\[
\beta_\parallel = \frac{V_\parallel}{c} \quad \text{and} \quad \beta_\perp = \frac{V_\perp}{c}
\]

\[
B_n = \left( \frac{1}{\beta_\parallel} \right)^2 (\epsilon_3 - \epsilon_1) + \epsilon_2^2 - \epsilon_1^2 - \epsilon_1 \epsilon_3
\]

\[
C_n = \left( \frac{1}{\beta_\parallel} \right)^2 (\epsilon_2^2 - \epsilon_3^2 - \epsilon_1 \epsilon_3) + \epsilon_3 (\epsilon_1^2 - \epsilon_2^2)
\]

\[
n^2 = \left[ -B_n \pm \left( B_n^2 - 4 C_n \epsilon_1 \epsilon_1 \epsilon_2 \right)^{1/2} \right] / 2 \epsilon_1
\]

\[
T_{11} = \epsilon_1 \epsilon_3 - \epsilon_1 n^2 \sin^2 \theta - \epsilon_3 n^2 \cos^2 \theta
\]

\[
T_{13} = \epsilon_2 n^2 \sin \theta \cos \theta
\]

\[
T_{33} = \epsilon_1^2 - \epsilon_2^2 - \epsilon_1 n^2 + (n^4 - \epsilon_1 n^2) \cos^2 \theta
\]

\[
L_0 = (\omega / \Omega_0) \beta_\perp n \sin \theta
\]

In these expressions, \( q \) is the charge of the particle, \( w \), the angular frequency of the whistler-mode Cerenkov wave, \( V_\parallel \) and \( V_\perp \), the components of the particle velocity perpendicular to and along the geomagnetic field direction respectively, \( c \), the velocity of light in vacuum, \( \theta \), the wave normal angle relative to the magnetic field direction, \( \epsilon_1, \epsilon_2 \) and \( \epsilon_3 \) the dielectric tensor elements with the ions included, \( J_0 \) and \( J_1 \), the Bessel's functions of first kind and zeroth and first order, respectively, and \( L_0 \) is the argument of the Bessel's functions. The dielectric
tensor elements $\varepsilon_1$, $\varepsilon_2$, and $\varepsilon_3$ for a hydrogen plasma are given as follows (Mansfield, 1967):

$$
\varepsilon_1 = 1 + \frac{w_p^2}{\Omega_e^2 - w^2} + \frac{1836 w_p^2}{\Omega_e^2 - (1836 w)^2}
$$

$$
\varepsilon_2 = \frac{w_p^2 \Omega_e}{w(w^2 - \Omega_e^2)} + \frac{w_p^2 \Omega_e}{w(\Omega_e^2 - (1836 w)^2)}
$$

$$
\varepsilon_3 = 1 - \frac{w_p^2}{w^2} - \frac{w^2}{1836 w^2}
\tag{3.12}
$$

where $w_p$ and $\Omega_e$ are the ambient plasma frequency and gyrofrequency (in $\text{rad s}^{-1}$) of an electron respectively.

The Cerenkov condition in a magneto plasma for emission of a wave at an angle $\Theta$ to the magnetic field direction is

$$
V_{\parallel} \cos \Theta = \frac{c}{n}
\tag{3.13}
$$

For conditions of $w_p, \Omega_e \gg \omega$ and $\Omega_e \cos \Theta >> w$, the whistler-mode refractive index $n$ can be given by

$$
n^2 = \frac{w_p^2}{w \Omega_e \cos \Theta}
\tag{3.14}
$$

From equations (3.13) and (3.14), the frequency of whistler-mode Cerenkov wave can be given as (Ondoh, 1990)

$$
f(\text{Hz}) = \frac{w_p^2 W \cos \Theta}{500 \pi \Omega_e}
\tag{3.15}
$$

in which $W(\text{keV}) = 250 V_{\parallel}^2 / c^2$ is the energy of an electron with
parallel velocity \( V_{ii} \). The value of \( \theta \) determined from equation (3.13) is substituted in the expressions of \( T_{11}, T_{13} \) and \( T_{33} \) which appear in the power expression (3.7).

The volume \( V_T \) of a flux tube is calculated from the following expression (Park, 1972):

\[
V_T = \frac{R_e^4 L^4}{R_1^3} (1+3 \sin^2 \theta_1)^{1/2} \left[ \frac{35}{64} \sin \theta_1 + \frac{7}{64} \sin 3 \theta_1 
+ \frac{7}{320} \sin 5 \theta_1 + \frac{1}{448} \sin 7 \theta_1 \right] \quad (3.16)
\]

To calculate the magnetic field intensity of plasmaspheric ELF hiss emissions in terms of cyclotron instability mechanism, we use expressions (3.2) and (3.3) of the first part of the chapter. The gain in wave instability due to electron cyclotron instability in the equatorial region is given by (Thorne and Barfield, 1976)

\[
G = 2 \int_{UR} \gamma^2 dt = \frac{2 \gamma L R_e}{V_g} \quad (3.17)
\]

The unstable region (UR) is within \( \pm 20^\circ \) of the geomagnetic equator (Inan et al., 1978; Roederer, 1970), and beyond these points the wave growth vanishes. In equation (3.17), \( V_g \) and \( \gamma \) are the group velocity and wave growth rate respectively which are defined as (Kennel and Petschek, 1966; Inan, 1977)

\[
V_g = 2c \frac{\Omega_e}{\nu_p} \left( \frac{\nu}{\nu_p} \right)^{1/2} \left( 1 - \frac{\nu}{\nu_p} \right)^{3/2} \quad (3.18)
\]

and
\[ \gamma = \pi A \gamma \Omega_e \]

in which

\[ A = \frac{1}{2} \log_e \left( \frac{1}{\sin \alpha} \right) \]

is the pitch angle anisotropy, \( \gamma \), the ratio of the density of energetic electrons to that of thermal electrons, and \( \alpha_e \) is the equatorial loss cone angle. The gain in decibels can be written as (Church and Thorne, 1983)

\[ \text{Gain(dB)} = 10 \log \left( e^G \right) \]

\[ 3.8 \text{ RESULTS AND DISCUSSION} \]

Fig. 3.3 shows the variation of the experimentally observed magnetic field intensity of plasmaspheric ELF hiss emissions with the wave frequency for three equatorial observations of August, 5, September 4 and September 28, 1977 on board GEOS-1 satellite. Fig. 3.3 is reproduced from Fig. 2 of Hayakawa et al. (1987). Hereafter, we shall refer the emissions of 5th August as type I emissions and those of September 4 and 28 as type II emissions. Type I emissions exhibit a rather flat distribution in a frequency range of 300 - 450 Hz, with the peak intensity of 1.44 \( \times 10^{-4} \frac{\Omega}{\text{Hz}} \) at 350 Hz. Direction finding results show that these emissions are observed at the inner edge of the plasmapause and have quasi-longitudinal wave normals (\( \theta^\circ 5-15^\circ \)) (Hayakawa et al., 1987). Quasi-linear electron cyclotron instability (Sazhin), 1984, Hayakawa et al. 1986) can be held responsible for the
FIG. 3.3 - MAGNETIC FIELD INTENSITY VERSUS FREQUENCY CURVES OF PLASMASPHERIC ELF HISS EMISSIONS AS OBSERVED BY GEOS-1 SATELLITE (AFTER HAYAKAWA ET AL., 1987).
generation of such type of emissions. Type II emissions have the peak intensity of \( \approx 6 \times 10^{-6} \frac{\gamma^2}{\text{Hz}^{-1}} \). Type II emissions of 4th September 1977 have a secondary peak also with an intensity of \( 1.33 \times 10^{-6} \frac{\gamma^2}{\text{Hz}^{-1}} \) at \( \approx 550 \) Hz. Such double peaks observed in the intensity spectra of DP hiss emissions have been satisfactorily explained in part I of the chapter by quasi-linear electron cyclotron instability mechanism proposed by Sazhin (1984). The similarity of primary peaks in the intensity spectra of type I and type II emissions in Fig. 3.3 indicates that both types of emissions should be generated by the same mechanism. However, type II emissions have large wave normal angles ranging from 20° to 80° which suggest that these emissions might be generated in the Cerenkov or Landau resonance mechanism (Hayakawa et al., 1987).

In order to know the generation mechanism of plasmaspheric ELF hiss emissions with large wave normal angles (oblique wave normals) observed aboard GEOS-1 satellite, we first consider incoherent Cerenkov radiation mechanism and calculate the frequencies of the whistler-mode Cerenkov waves at the different values of the wave normal angle \( \theta \) by using equation (3.15) and assuming their generation at \( L = 4.8 \) and 5.5, just near the inner edge of the plasma-pause in the equatorial plane. The plasmapause is located at \( L = 5.8 \) on 4th September and at \( L = 5 \) on 28th September, 1977 (Hayakawa et al., 1987). The electron densities at the \( L \) - values of 4.8 and 5.5 are taken to be 200
and 100 el. cm\(^{-3}\) respectively which roughly correspond to the diffusive equilibrium model (Angerami and Thomas, 1963). The plasma frequency is calculated using the relation

\[ \frac{\nu_p}{2\pi} (\text{kHz}) = 8.98 n_0^{1/2} \]

where \(n_0\) is in el. cm\(^{-3}\) and the electron gyrofrequency \(\Omega_e\) is calculated from the relation \(\frac{\Omega_e}{2\pi} (\text{kHz}) = 873.6/L_2^2\). The frequencies of whistler-mode Cerenkov waves calculated from equation (3.15) at the \(L\)-values of 4.8 and 5.5 are plotted against wave normal angle \(\theta\) in Fig. 3.4 which shows that larger frequency components are radiated out at smaller values of wave normal angle \(\theta\). The energies of the resonant electrons used in the calculations are also indicated in Fig. 3.4. The calculated frequency spectrum of whistler-mode Cerenkov waves is in good agreement with the observed frequency spectrum of Fig 3.3. At this stage, the Cerenkov resonance mechanism seems to be a possible generation mechanism for oblique wave normals.

Now, we calculate the magnetic field intensity in W x Hz\(^{-1}\) per electron at the \(L\)-values of 4.8 and 5.5 in the equatorial plane by considering incoherent Cerenkov radiation mechanism and using equations 3.7-3.12 of the previous section. The wave frequency is taken to be 350 Hz at which the maximum intensity of type II emissions has been observed (Hayakawa et al., 1987). The pitch angle of radiating electrons is assumed to be zero because such electrons give rise to emissions of larger intensity (Prakash et al., 1979). The energies of the electrons are taken to be
FIG. 3.4 - PLOTS SHOWING VARIATION OF WHISTLER-MODE CERENKOV WAVE FREQUENCY WITH WAVE NORMAL ANGLE.
0.1 and 0.14 keV because at these energies the calculated frequencies of the whistler-mode Cerenkov waves are consistent with the observed frequencies and higher intensities are radiated out by low energy electrons. The calculated equatorial intensities at the $L$-values of 4.8 and 5.5 for 350 Hz waves are found to be $2.5732 \times 10^{-33}$ and $2.3674 \times 10^{-32}$ W Hz$^{-1}$ per electron respectively. We calculate the volumes of the flux tubes by employing equation (3.16) along the field lines corresponding to $L = 4.8$ and 5.5, each of which has across section of 1 cm$^2$ at 500 km in the ionosphere and extends to the equatorial plane. The volumes of these flux tubes are found to be $2.2520 \times 10^{11}$ cm$^3$ and $3.9217 \times 10^{11}$ cm$^3$ respectively. Now, we have to consider a suitable energy spectra for the particles used in the present calculations so as to determine their densities in the flux tubes. The energy spectra of $\sim 10^5$ el. cm$^{-2}$ s$^{-1}$ sterad$^{-1}$ observed aboard Hitch-hiker I (1963-25 B) satellite for outer belt electrons (Katz, 1966) is unsuitable for present calculations because these electrons have energy $\gtrsim 1$ MeV. O'Brien(1966) has reported a maximum flux of $\sim 5 \times 10^6$ el cm$^{-2}$ s$^{-1}$ sterad$^{-1}$ for energetic electrons of energy $\gtrsim 40$ keV in the region of $L$-values of 4.8 - 5.5. Since the observed flux at lower energies is always higher than that at higher energies (Katz, 1966), we consider a flux of $5 \times 10^8$ el. cm$^{-2}$ s$^{-1}$ sterad$^{-1}$ at the $L$-values under consideration for 0.1 and 0.14 keV electrons. This is two orders of magnitude higher than that for electrons of energy $\gtrsim 40$
keV. From this value of flux, we find the densities of 0.1 and 0.14 keV electrons in the flux tubes along the field lines corresponding to the L-values of 4.8 and 5.5 to be 10.17 and 8.85 cm⁻³, respectively. We further assume that the flux remains constant along the field lines under consideration and multiply these densities with the tube volumes and the intensities calculated in the equatorial plane to obtain intensities in $W m^{-2} Hz^{-1}$ there. The calculated equatorial intensities at the L-values of 4.8 and 5.5 are found to be $6.05 \times 10^{-17}$ and $8.22 \times 10^{-16} W m^{-2} Hz^{-1}$ respectively which are equivalent to $1.98 \times 10^{-11}$ and $2.36 \times 10^{-10} \gamma Hz^{-1}$ respectively (see conversion formula 2.15). These values are 4 to 5 orders of magnitude lower than the observed intensities of type II emissions. Although incoherent Cerenkov instability mechanism radiates out waves at large wave normal angles near resonance cone, the radiated intensities, being 4 to 5 orders of magnitude lower than the observed intensities, rule out the possibility of generation of oblique wave normals in this mechanism. Further, the condition $\Omega_c \cos \Theta \gg \omega$ assumed in the derivation of equation (3.5) which has been used in the calculation of the frequency spectrum of the radiated whistler-mode Cerenkov waves, is not rigorously true for all values of $\Theta$ at $L = 4.8$ and 5.5. Thus, type II emissions cannot be generated by Cerenkov instability mechanism.

We now assume that type I emissions of August 5, 1977 are
generated in the equatorial region at L= 5.9 near the inner edge of the plasmapause (relatively sharp plasmapause located at Lpp = 5.50 - 6.20) and type II emissions of September 4 and September 28, 1977 are generated at the equatorial locations of L = 5.5 (very sharp plasmapause at Lpp = 5.8) and 4.8 (very gradual gradient, plasmapause location Lpp ≈ 5.0) respectively under electron cyclotron resonance instability mechanism (Sazhin, 1984). To calculate the intensity of these emissions, we first determine the characteristic energy of the energetic electrons likely to be responsible for generation of these emissions. From equation (3.2), the characteristic energies of the energetic electrons for \( f_{\text{max}} = 350 \text{ Hz} \) are estimated to be within \( \sim 10-20 \text{ keV} \) range. The electron density at \( L = 5.9 \) is taken to be \( 50 \text{ cm}^{-3} \). This value of electron density is taken from Fig 1 of Parrot and Lefevre (1986) which is based on the electron density measurements performed by the S-304 mutual impedance technique on board GEOS-1 satellite. In the calculations, the values of \( \frac{dn_j}{dt} \) is assumed to be \( 3 \times 10^{-4} \text{ cm}^{-3} \text{ s}^{-1} \) at all the three L-values under consideration which is the same as adopted by Sazhin (1984) at \( L = 3.6 \) and Nishino and Tanaka (1994) at \( L = 4.8 \) in the study of plasmaspheric ELF/VLF hiss emissions. The calculated values of the characteristic energy are used to estimate the peak power spectral densities from equation (3.3) for \( f_{\text{max}} = 350 \text{ Hz} \) at the three equatorial locations of \( L = 4.8, 5.5 \) and 5.9. The
calculated and observed intensities along with other parameters are presented in table 3.2.

**Table 3.2 - Observed and Calculated Equatorial Intensities with Other Parameters Used**

<table>
<thead>
<tr>
<th>L=L₀</th>
<th>W(keV)</th>
<th>n_{eq} (cm⁻³)</th>
<th>/L⁻³</th>
<th>Intensity in γ²Hz⁻¹ Calculated</th>
<th>Observed</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.8</td>
<td>16.90</td>
<td>200</td>
<td>1.1165x10⁶</td>
<td>4.2787x10⁻⁵</td>
<td>9.00x10⁻⁶</td>
</tr>
<tr>
<td>5.5</td>
<td>9.93</td>
<td>100</td>
<td>1.1803x10⁶</td>
<td>3.7575x10⁻⁵</td>
<td>6.25x10⁻⁶</td>
</tr>
<tr>
<td>5.9</td>
<td>10.55</td>
<td>50</td>
<td>8.6827x10⁵</td>
<td>4.1563x10⁻⁵</td>
<td>1.44x10⁻⁴</td>
</tr>
</tbody>
</table>

Table 3.2 shows that the calculated values of intensity at the L-values of 4.8 and 5.5 are greater than the observed intensities by about 7 and 8 dB respectively. However, the calculated intensity at \( L = 5.9 \) is lower than the observed intensity by about 5 dB. We are of the opinion that the difference in the calculated and observed intensities is due to the propagation effect. Type I emissions which have quasi-longitudinal wave normals (\( 0°-5°-15° \)) at the observing site just at the inner edge of the plasmapause (Hayakawa et al., 1987) might have crossed the equatorial source region at least once nearly aligned with the earth's magnetic field between the times of their generation and observation similar to the "cyclic waves" discussed by Thorne et al. (1979) and so they are likely to be amplified further. Therefore, we calculate the power gain for 350
Hz waves at L = 5.9 in the equatorial region. At this location, the equatorial loss cone angle $\chi_0$ is estimated to be 0.049 rad. which gives anisotropy $A = 0.1658$. Now, assuming a flux of $4 \times 10^5$ el. cm$^{-2}$ s$^{-1}$ sterad$^{-1}$ at L = 5.9 (O'Brien 1966) for 10.55 keV electrons we get $\gamma = 2.0342 \times 10^{-5}$. Putting these values in equation (3.19), we get the wave growth rate $\gamma = 0.2832$ rad s$^{-1}$ for 350 Hz waves. Substituting the appropriate values of different parameters in equation (3.18), we find that $V_g = 1.0136 \times 10^7$ m s$^{-1}$.

With these values of $\gamma$ and $V_g$ we get from equation (3.17) $G = 2.0999$ and equation (3.20) gives a gain of 9.12 dB in a single transit through the source region. Thus the intensity of $4.1563 \times 10^{-5} \gamma^2$ Hz$^{-1}$ initially radiated out at L = 5.9 in electron cyclotron instability mechanism now becomes $3.3940 \times 10^{-4} \gamma^2$ Hz$^{-1}$ after a single transit through the source region. The value is larger by 3.72 dB than the observed value of $1.44 \times 10^{-4} \gamma^2$ Hz$^{-1}$. The excess intensity by 3.72 dB may very well account for the propagation loss suffered by these waves during the period of their generation and observation aboard GEOS - 1 satellite just inside the plasmapause. Thus we find that type I emissions could very well be generated by the electron cyclotron instability mechanism just near the inner edge of the plasmapause (Hayakawa et al., 1987).

Type II emissions which have oblique wave normals ($\theta \approx 20-80^\circ$) at the observing site that is 0.3 - 0.5 Re inside from the
plasmapause (Hayakawa et al., 1987) do not pass through the equatorial amplification region before their observation due to their large wave normal angles and hence are not amplified further. Instead, they suffer comparatively greater propagation losses including the losses due to Landau damping and collisional as well as spreading losses (Taylor and Shawhan, 1974). The difference of 7-8 dB between calculated and observed intensities of type II emissions seems to be sufficient to account for such losses (Helliwell, 1965).

Earlier ray-tracing studies have indicated that non-ducted waves launched longitudinally at the geomagnetic equator tend to become oblique as they propagate away from it (Aikyo and Ondoh, 1971; Huang and Goertz, 1983). Their wave normals are tilted towards the earth by the general decrease of plasma density with increasing L-values and outward from the earth by the decrease of magnetic field strength with increasing L-value, jointly with the effect of field line curvature. For a wave of arbitrary frequency launched from an arbitrary equatorial point, these competing effects are unlikely to cancel each other exactly; one or another dominating, resulting in that as the wave propagates, its normal becomes more and more inclined to the field, ultimately approaching the resonance cone. With a suitable distribution of the plasma in the magnetosphere, it might be possible that type II emissions propagating as above are magnetospherically
reflected back and reach the GEOS -1 satellite with large wave normal angles. Thus it seems to us that like type I emissions type II emissions are also generated by the electron cyclotron instability mechanism.

In the light of the above results we finally conclude that both type I and type II plasmaspheric ELF hiss emissions observed by GEOS-1 satellite are originated by the same electron cyclotron resonance instability mechanism and that the difference in their intensities is due to propagation effect.

3.9 CONCLUSION

In this part of the chapter, we have carried out detailed intensity calculations for the plasmaspheric ELF hiss emissions observed aboard GEOS -1 satellite both at small (type I emissions) and large (type II emissions) wave normal angles by adopting suitable plasma parameters and considering both the Cerenkov and cyclotron resonance instability mechanisms. The intensities calculated by the Cerenkov instability mechanism are 4 to 5 orders of magnitude lower than the observed intensities and thus rule out the generation of oblique wave normals by this mechanism. After allowing appropriate propagation losses, the intensities calculated by the electron cyclotron instability mechanism are found to be quite consistent with the observed intensities. It is, therefore, concluded that the plasmaspheric ELF hiss emissions observed aboard GEOS -1 satellite both at
small and large wave normal angles are generated by the same electron cyclotron instability mechanism just near the inner edge of the plasmapause; the difference in observed intensities of the two types of the emissions being attributed to the propagation effect rather than the generation effect.