Introduction

Graph theory is directly related to many branches of mathematics, including discrete mathematics, analysis, probability, topology, and combinatorics and its extensive applications in many other such as social, natural, biological and computer sciences. Indeed, graph theory is a theory of domination on graphs and graph colorings.

The formal mathematical definition of the notion of domination was given by Claude Berge in 1960 and the [13] in 1975. In 1975, Cockayne and Hedetniemi [15] studied graph theoretically published a 1976 survey of the results that had been determined. A complete and detailed mathematical foundation, as well as a complete graph theory, is presented in particular of domination theory.

In 1952, the field of graph colorings was developed by the beginning with the origin of the Four Color Problem. In graph theory, several topics between major topics in graph theory and graph colorings, including domination, as well as other emerging topics in the colorings, stable colorings, distance colorings related to the Clasical Assignment Problem, 2-colored distinguishing coloring, and domination coloring [11]. This concept was introduced to graphs was introduced by Cookyne et al [16]. This general notion inspired by a new concept, total-dominator coloring.
Graph Theory is intimately related to many branches of Mathematics, including algebra, numerical analysis, probability, topology and combinatorics and its results have applications in many areas such as social, natural, biological and computer sciences. Two of such sub fields are the theory of Domination on graphs and graph colorings.

The formal mathematical definition of the topic of domination was given by Claude Berge [3] in 1958 and Ore [13] in 1962. In 1972, Cockayne and Hedetniemi began to study it and ultimately published a 1975 survey of the results that had been obtained by that time. Claude Berge has established the mathematical foundations, not only of graph theory in general, but in particular of domination theory.

In 1852, the field of graph colorings has developed by the beginning with the origin of the Four Color Problem. It explores connections between major topics in graph theory and graph colorings, including domination, as well as such emerging topics as list colorings, rainbow colorings, distance colorings related to the Channel Assignment Problem, vertex/edge distinguishing coloring and dominator coloring [8]. The concept total domination in graphs was introduced by Cockayne et al [4]. This general notion inspired by a new concept, total dominator coloring.
In this thesis, an attempt has been made to define certain new domination parameters and graph theoretic parameters and study their bound, characterizations and their properties related to other graph theoretic parameters. Also we find the characterization of structure of graphs with equal domination parameters.

By a graph \( G = (V, E) \), we mean a finite undirected graph without multiple edges and loops.

This thesis consists of six chapters as given below:

Chapter 1 Preliminaries

Chapter 2 Characterization of Structure of Graphs with equal domination parameters and Non-domination on Graphs

Chapter 3 Dominator Colorings and Safe Clique Partitions on Graphs

Chapter 4 Total Dominator Colorings in Graphs

Chapter 5 Total Dominator Colorings in Paths, Cycles and Caterpillars

Chapter 6 H-connected Domination on Graphs

In the last we have included the references and subject index.
### List of Symbols

The following symbols are used in this thesis.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G = (V, E)$</td>
<td>The graph $G$ with vertex set $V$ and edge set $E$</td>
</tr>
<tr>
<td>$V = V(G)$</td>
<td>Vertex set of the graph $G$</td>
</tr>
<tr>
<td>$E = E(G)$</td>
<td>Edge set of the graph $G$</td>
</tr>
<tr>
<td>$p$</td>
<td>Number of vertices in the graph</td>
</tr>
<tr>
<td>$q$</td>
<td>Number of edges in the graph</td>
</tr>
<tr>
<td>$T$</td>
<td>Tree</td>
</tr>
<tr>
<td>$\langle S \rangle$</td>
<td>Sub graph induced by a set $S \subseteq V$</td>
</tr>
<tr>
<td>$\overline{G}$ or $G^c$</td>
<td>The complement of a graph $G$</td>
</tr>
<tr>
<td>$\deg(v)$</td>
<td>Degree of the vertex $v \in V(G)$</td>
</tr>
<tr>
<td>$\deg_H(v)$</td>
<td>$H$-degree of the vertex $v$</td>
</tr>
<tr>
<td>$d_H^*(v)$</td>
<td>The $H$-connected degree of a vertex $v$</td>
</tr>
<tr>
<td>$\delta(G)$</td>
<td>The minimum degree of $G$</td>
</tr>
<tr>
<td>$\delta_H(G)$</td>
<td>The minimum $H$-degree of $G$</td>
</tr>
<tr>
<td>$\delta^*_H(G)$</td>
<td>The minimum $H$-connected degree of $G$</td>
</tr>
<tr>
<td>$\Delta(G)$</td>
<td>The maximum degree of $G$</td>
</tr>
<tr>
<td>$\Delta_H(G)$</td>
<td>The maximum $H$-degree of $G$</td>
</tr>
<tr>
<td>$\Delta^*_H(G)$</td>
<td>The maximum $H$-connected degree of $G$</td>
</tr>
<tr>
<td>$K_p$</td>
<td>The complete graph with $p$ vertices</td>
</tr>
<tr>
<td>$K_{m,n}$</td>
<td>A complete bipartite graph with $m + n$ vertices</td>
</tr>
<tr>
<td>$K_{1,n}$</td>
<td>The star graph on $n+1$ vertices</td>
</tr>
</tbody>
</table>
\begin{itemize}
  \item $B(r, s)$: The bistar graph
  \item $P_p$: The path graph with $p$ vertices
  \item $C_p$: The cycle graph with $p$ vertices
  \item $W_p$: The wheel graph with $p$ vertices
  \item $\deg(v)$: The degree of a vertex $v$
  \item $diam(G)$: Diameter of a graph $G$
  \item $N(v)$: The open neighborhood set of a vertex $v$
  \item $N_H(v)$: The H-open neighborhood set of a vertex $v$
  \item $N_H^*(v)$: The open H-connected neighborhood set of a vertex $v$
  \item $N_H^{**}(v)$: The strong open H-connected neighborhood set of a vertex $v$
  \item $N[v]$: The closed neighborhood set of a vertex $v$
  \item $N_H[v]$: The H-closed neighborhood set of a vertex $v$
  \item $N(S)$: The open neighborhood set of $S \subseteq V$
  \item $N_H(S)$: The open H-neighborhood set of $S \subseteq V$
  \item $N[S]$: The closed neighborhood set of $S \subseteq V$
  \item $N_H[S]$: The closed H-neighborhood set of $S \subseteq V$
  \item $G - v$: Graph obtained by removal of a vertex $v$ from $G$
  \item $G - e$: Graph obtained by removal of an edge $e$ from $G$
  \item $\omega(G)$: The clique number of $G$
  \item $e(u)$: Eccentricity of a vertex $u$ of $G$
  \item $r(G)$: The radius of $G$
  \item $mK_2$: $m$ copies of complete graph $K_2$
  \item $G_1 + G_2$: Join of graphs $G_1$ and $G_2$
  \item $G_1 \cup G_2$: Union of graphs $G_1$ and $G_2$
  \item $G_1 \circ G_2$: Corona of graphs $G_1$ and $G_2$
  \item $\alpha(G)$: Vertex-covering number of $G$
\end{itemize}
\[ \beta_0(G) \]  
Vertex-independence number of G

\[ \beta_H(G) \]  
Vertex H-independence number of G

\[ \beta_{hi}(G) \]  
Upper H-connected independence number of G

\[ \gamma(G) \]  
Domination number of G

\[ \gamma_t(G) \]  
Total domination number of G

\[ \gamma_u(G) \]  
Uniform domination number of G

\[ \gamma_{HA}(G) \]  
H-adjacent domination number of G

\[ \gamma_{HC}(G) \]  
H-connected domination number of G

\[ \gamma_{HS}(G) \]  
Strongly H-connected domination number of G

\[ \Gamma(G) \]  
Upper domination number of G

\[ \Gamma_{hi}(G) \]  
Upper H-connected domination number of G

\[ \Gamma_{hs}(G) \]  
Upper strongly H-connected domination number of G

\[ \varrho(G) \]  
Independent domination number of G

\[ pn[v,S] \]  
Private neighbor of v with respect to \( S \subseteq V \)

\[ ir(G) \]  
Irredundance number of G

\[ IR(G) \]  
Upper irredundance number of G

\[ \chi(G) \]  
Chromatic number of G

\[ \chi_d(G) \]  
Dominator chromatic number of G

\[ \chi_{td}(G) \]  
Total dominator chromatic number of G

\[ \chi_s(G) \]  
Clique partition number of G

\[ \chi_{sa}(G) \]  
Safe clique partition number of G

\[ [x] \]  
Smallest integer less than or equal to x

\[ \lfloor x \rfloor \]  
Largest integer greater than or equal to x
Content Overview

Introduction of the thesis briefs the contents of the thesis by pointing out certain important results in each and every chapter and also describes the structure of the thesis.

In chapter 1, we collect the necessary basic definitions and graph theoretical results which would be required for the subsequent chapters.

In chapter 2, we characterize the structure of graphs for which the upper domination number is equal to the uniform domination number. Also we introduce a new parameter called non-domination number of a graph. A set $S \subseteq V$ is said to be a maximal non-dominating set (mn-d set) if every super set of $S$ is a dominating set of $G$. The non-domination number $\lambda$ of $G$ is the minimum cardinality taken over all mn-d-sets of $G$. The upper non-domination number $\Lambda$ of $G$ is the maximum cardinality of a non-dominating set of $G$. Some results have been proved on this new parameter.

In chapter 3, an attempt has been made to study a bound for $\chi_d(G)$ for bipartite graphs with minimum degree one in terms of domination number. Also we study a sufficient condition for the existence of a graph $G$ with $\Delta(G) \leq p - 2$ to have no safe clique partition and the relation between the safe clique partition concerning mostly metric invariants such as diameter and radius of $G$. Gera et. al proved the following theorem.
Theorem 1. Let $G$ be a tree of order $p \geq 5$. A maximum matching $M^*$ of $G$ is unsafe if and only if $G$ is the wounded spider $W_{a,b}$ ($a \geq 1$, $b \geq 0$).

We attempt to study the characterization of trees which have no safe clique partition and an alternative proof for the theorem 1 mentioned above.

Also we find the bounds for dominator chromatic number for a caterpillar and obtain some results concerning the dominator chromatic number.

In chapter 4, we introduce the parameter called total dominator chromatic number of a graph. A total dominator coloring of a graph $G$ is a proper coloring with the additional property that every vertex in $G$ properly dominates a color class. The total dominator chromatic number is denoted by $\chi_{td}(G)$ and is defined by the minimum number of colors needed in a total dominator coloring of $G$.

We attempt to find a general bound in terms of total domination number and chromatic number. Also we obtain a characterization for lower and upper bound and some results concerning $\chi_{td}(G)$ connecting other graph theoretic parameters.

In chapter 5, we determine the bounds for total dominator chromatic number in paths, cycles and caterpillars.

In chapter 6, we introduce the parameters called $H$-connected domination number, strongly $H$-connected domination number on graphs and some observations have been made among these two parameters.
Let $G = (V, E)$ be a graph and $H$ be a fixed non-trivial proper sub graph of $G$. Two vertices $u$ and $v$ in $V(G)$ are said to be $H$-connected if there exist a sub graph $H_i$ of $G$ which is isomorphic to $H$ contains a $(u,v)$-path. A path joining two vertices $u$ and $v$ in $G$ is said to be a $H$-path if each edge in $P$ is incident on two $H$-adjacent vertices. i.e., any two consecutive vertices in $P$ are $H$-adjacent. A graph $G$ is $H$-connected if every pair of vertices in $G$ are joined by a $H$-path. A set $D \subseteq V(G)$ is said to be $H$-connected dominating set if every vertex in $V - D$ is $H$-connected to some vertex in $D$.

Also we introduce a new concept strongly $H$-connected domination number of $G$. Let $G$ be a graph and $H$ be a non-trivial proper sub graph of $G$. Two vertices $u$ and $v$ in $G$ are said to be strongly $H$-adjacent if $\exists$ an induced sub graph $H_i$ of $G$, which is isomorphic to $H$, contains a $(u,v)$-path. A set $D \subseteq V(G)$ is said to be strongly $H$-connected dominating set if every vertex in $V - D$ is strongly $H$-connected to some vertex in $D$.

We study on these two new concepts and some observations have been made of these two parameters.