CHAPTER – 3
DESIGN AND DEVELOPMENT OF JOURNAL BEARING

3.0 INTRODUCTION

A bearing is a system of machine elements whose function is to support an applied load by reducing friction between the relatively moving surfaces. In engineering application, bearing acts as supports, providing stability, free and smooth rotation. The importance of bearings may be understood from the supporting requirement of Machine Tool Spindles, Engine Crankshafts, Transmission or Line-shafts in workshops, etc. Therefore, the design of a bearing is very important. To have a proper understanding of the design of a bearing, the theory behind the design is a crucial issue and a designer or researcher has thorough understanding of the subject. As a researcher of the present work, the theory related to bearings and the necessary material for the design and development of Hydrodynamic Journal Bearing are discussed in the following.

3.1 CLASSIFICATION OF BEARINGS

The classification of the bearings is discussed in the following briefly.

3.1.1 Based on the Nature of Contact:

Bearings are broadly classified into two categories based on the nature of contact: sliding contact bearings and rolling contact bearings or anti-friction bearings.

1. Sliding contact Bearings: In sliding contact bearings, as shown in Figure - 3.1(a), the sliding takes place along the surfaces of contact between the moving element and the fixed element. The sliding contact bearings are also known as plain bearings.

2. Rolling Contact Bearings: In rolling contact bearings as shown in Figure - 3.1(b), the steel balls or rollers, are interposed between the moving and fixed element. The balls offer rolling friction at two points for each ball or roller.

Figure – 3.1.(c) and Figure -3.1.(d) represent the simple subdivisions of Fluid Film Bearings and Rolling Contact Bearings respectively, that accounts for the nature of the lubricant, the mode of operation, the direction of motion, the nature of the load and the geometric form of the bearing.
Figure - 3.1(a): Sliding Contact Bearing.  Figure - 3.1(b): Rolling Contact Bearing.

Nature of lubrication

Liquid - Gas

Mode of operation

Hydrodynamic - Squeeze - Hydrostatic - Hybrid

Direction of load

Journal - Combined - Thrust

Nature of load

Steady - Dynamic

Geometric form

(e.g., partial or complete journal bearings)

Figure - 3.1(c): Division of Sliding Contact Bearings
3.1.2 Based on Direction of load: The bearings are classified based on the direction of the load acting on the journal.

1. Radial bearings and 2. Thrust bearings

1. Radial Bearings: In radial bearings, the load acts perpendicular to the direction of motion of the moving element as shown in Figure - 3.1(e).

2. Thrust bearings: In these bearings the load acts parallel to the axis of the shaft, as shown in Figure - 3.1(f).
3.2. CLASSIFICATION OF JOURNAL BEARINGS

The journal bearings are classified into the following.

3.2.1 Based on L/D ratio: Based on the ratio of bearing length (L) to diameter (D) and they are called as,

(i) Short Bearing: If L/D < = 0.5
(ii) Finite Bearing: If 0.5 < L/D < = 1.5
(iii) Long Bearing: If L/D > 1.5
3.2.2 **Based on Nature of Lubrication:** Based on the nature of lubrication, the sliding contact bearings are classified as follows;

(i) **Thick film or full fluid bearings:** The working surfaces are completely separated by the lubricant.

(ii) **Thin film or semi-fluid bearing:** Although lubricant is present in the bearing, the working surfaces may partially contact each other, at least part of the time.

3.2.3 **Based on Type of Lubricant:** In general, lubricants are three types namely; Solid Lubricants, Semi-Solid Lubricants and Fluid Lubricants. Based on these;

(i) **Dry Bearings:** In these bearings, the two surfaces rub each other and the surfaces are given with a solid coating of Graphite, Nylon, etc.

(ii) **Oil bearings:** In these bearings, the two rubbing surfaces are separated by liquid fluid or oil.

(iii) **Gas bearings:** In gas bearings, the two rubbing surfaces are separated by a gas. The gases used are carbon dioxide, air. If air is the media, then it is called Air Bearings. Air bearings are used in dentists' grinding machines.

3.2.4 **Based on Fluid Pressure:** The bearings are classified according to the source of the fluid film pressure and they are,

(i) **Hydrodynamic Bearings:** In this, the pressure in the fluid film is developed by the rotation of the journal; which in-turn resists the load on the bearing. The advantage is that the external equipment does not require create pressure in the fluid film. The disadvantage is that the pressure that resists load is developed only when the journal rotates beyond certain minimum speed. During starting and stopping, metal to metal contact exists (Figure - 3.2 (a)).

(ii) **Hydrostatic Bearings:** In the hydrostatic lubrication lubricant is supplied through holes into the clearance space at high pressure. The pressurized lubricant supports the load on the bearing. Advantage: The contact between the journal and bearing is eliminated even when the journal is at rest. Dis-advantage: Expensive, as external equipment is necessary to supply the lubricant at thigh pressure (Figure - 3.2 (b)).
(iii) **Squeeze Film Bearings:** Squeeze film is a phenomenon of two lubricated surfaces approaching each other with a normal velocity. The thin film of lubricant between the two surfaces acts as a cushion which prevents the surfaces from making instantaneous contact. The relation between load carrying capacity and the rate of approach is generally the focal point of most squeeze film analysis (Figure - 3.3).

(iv) **Porous Bearings:** These are self-contained units, with a bearing of porous bush. In small units, oil soaked felt or pad retains the oil, which flows due to capillary action through the pores of the porous bearing shell (Figure - 3.4).

3.2.5 **Based on Geometric form of the Bearing:** The bearings are classified according to their bush shape and they are,

(i) **Circular Bearings:** If the bearing cross section is in circular form, then it is called circular bearing. They are two types, (a) Full Circular Bearing and (b) Partial Bearing.

(ii) **Non-Circular Bearings:** If the cross of the bearing is non-circular, then it is called Non-Circular Bearing (Figure - 3.5). They are, (a) Elliptical Bearing, (b) Offset-Half Bearing and (c) Orthogonally Displaced Bearing (d) Spiral journal bearing considering two semi circles of different radii.

3.2.6 **Based on Number of Lobes:** The bearings are classified based on the number of lobes that it is having and they are,

(i). **Single Lobe Bearings:** The bearing is a single piece, made from a shell or hollow shaft.

(ii). **Multi Lobe Bearings:** The bearing is a combination of two or more pieces (Figure - 3.6). They are (a) Two Lobe Bearing, (b) Three Lobe Bearing and (c) Four Lobe Bearing.

Figure - 3.2(a): Hydrodynamic Journal

Figure - 3.2(b): Hydrostatic
Figure 3.3: Squeeze film Bearing

Figure 3.4: Porous Bearing [121]

(a) Elliptical Bearing
(b) Off-Set Half Bearing

(c) Orthogonally Displaced Bearing
(d) Spiral bearing

Figure 3.5: Non-circular bearings
3.3 MECHANISM OF PRESSURE DEVELOPMENT

The phenomenon of pressure development in journal bearings was discovered and observed that the developed peak pressure was several times higher than the mean pressure, Tower, 1883 [114]. Later on, the Tower’s results were verified and deduced that the lubrication of bearings was due to the hydrodynamic action of the journal, Reynolds, 1886 [94]. To understand the process, consider a simple slider consisting of two plates separated by a thin lubricant film (Figure - 3.7). Length of plates is perpendicular to the page and thus ensuring the flow will take place in only one direction.
Figure - 3.7: Two parallel surfaces separated by a liquid film

Upper plate moves with velocity U. Assuming no slip condition, the velocity of fluid U will vary from “0” at the stationary surface to U at the moving surface. Volume rate of flow at AA’ and BB’ are equal. Instead of the two plates being kept parallel to each other, one of the plates is inclined as shown in Figure - 3.8, velocity distribution varies at AA’ and BB’. Assuming that the velocity distribution is linear at CC’, the flow continuity will be satisfied with the pressure buildup, since, the flow will have both pressure and velocity induced terms. Thus, a convergent film will generate positive pressure, if there exits relative velocity and the surfaces are separated by viscous fluid film.

3.4 HYDRODYNAMIC ACTION IN SOLID JOURNAL BEARING

The three phases of lubrication which exist in Hydrodynamic Journal Bearing are briefly mentioned in the following.

(i) Starting of the journal from rest: If the two lubricating surfaces are pressed together by a load, a large amount of the lubricant will be squeezed-out and the metal surfaces will come more or less into contact. When the shaft starts to rotate, friction of the journal against the bearing is high and certain amounts of abrasion will always occur – Figure - 3.9(a).

(ii) Operating with imperfectly lubricated surfaces: As the journal begins to rotate as shown in Figure - 3.9(b), it tends to roll up towards the right or left side of the bearing (the contact being at some point ‘P2’ for clockwise rotation). Due to the molecular attraction, the wedge shaped oil film is drawn in between the rubbing surfaces. In thin film lubrication, there exists an unstable condition due to the contact of the metal surfaces time to time under certain conditions of low rubbing speeds or high unit loads.
Figure - 3.8: Flow between two inclined surfaces.
Running with perfectly lubricated surfaces: The rotating journal acts as a pump and the oil pressure increases with an increase in speed. If the surface velocity of the journal increases above that of imperfect lubrication and there is a sufficient oil supply, enough oil pressure will be developed to raise the shaft completely at the bearing. The lubricant oil (carried by the pumping action) pushes the shaft to the left or right side (the point of nearest approach to the bearing surface moves to P3). Since the clearance must exist in order to obtain the eccentric position of the shaft, necessary for the pumping action, part of the oil is discharged at the ends of the bearings. Accordingly, the oil pressure decreases from the middle of the bearing towards each end – Figure - 3.9(c).

![Diagram of journal bearing](image)

(a) – at rest  (b) – at start  (c) – during running

Figure - 3.9: Hydrodynamic Action in Journal Bearing.

3.5 BASIC EQUATIONS FOR FLUID FILM LUBRICATION.

Fluid film lubrication is based on viscous flow. Viscous flow has the following important characteristics:

(i) Viscous resistance of fluid increases with the deformation rate.

(ii) The flow is a non-reversible change as the work done in viscous flow appears as heat in the liquid.

(iii) A liquid becomes less viscous as its temperature is raised.

(iv) The viscosity of liquid in non-conformal contact geometries usually increases as the pressure increases.

The basic lubrication theory is based on the solution of a particular form of Navier-Stokes equation.
3.5.1 Navier-Stokes Equation [97,98]

Considering the dynamic equilibrium of a fluid element under surface forces, body forces, and inertia forces, the Navier-Stokes equation can be derived, Bernard Hamrock, 1994 [14].

(i) Surface Forces:

The stresses on the surface of a viscous fluid element are shown in Figure-3.10. Where $\sigma$ is the normal stress and $\tau$ is the shear stress. The five relationships suitable to the surface stresses are presented in the following.

![Figure - 3.10: Stresses on Surfaces of a Fluid Element.](image)

(i) The stresses on the fluid element must be symmetric for equilibrium of moments.

Hence,

$$\tau_{xy} = \tau_{yx} \quad \tau_{yz} = \tau_{zy} \quad \tau_{zx} = \tau_{xz}$$  \hspace{1cm} (3.1)

(ii) The average of the three normal stress components is equal to the hydrostatic pressure in the fluid.

$$\sigma_x + \sigma_y + \sigma_z = -3p$$  \hspace{1cm} (3.2)

(iii) The magnitude of the shear stress depends on the rate at which the fluid is being distorted.

$$\tau_{ij} = \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$$  \hspace{1cm} (3.3)

where $\mu$ = absolute viscosity.

$u_i$ = components of velocity vector, m/s ($u_x = u, u_y = v, u_z = w$).

$x_i$ = components of coordinate vector, m ($x_x = x, x_y = y, x_z = z$).
(iv) The magnitude of the normal stress can be written as

$$\sigma_i = -p + \lambda_a \varepsilon_a + 2\mu \frac{\partial u_i}{\partial x_i}$$  \hspace{1cm} (3.4)$$

where $\lambda_a$ is an anther viscosity coefficient.

$$\varepsilon_a = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$$, is the dilatation and it measures the expansion of the fluid.

(v) Using the above equations, the second viscosity coefficient can be expressed in terms of the absolute viscosity,

$$\lambda_a = -2 \mu / 3$$  \hspace{1cm} (3.5)$$

Therefore normal stress

$$\sigma_i = -p - \frac{2}{3} \mu \varepsilon_a + 2\mu \left( \frac{\partial u_i}{\partial x_i} \right)$$  \hspace{1cm} (3.6)$$

and shear stress

$$\tau_{ij} = \tau_{ji} = \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$$  \hspace{1cm} (3.7)$$

The surface forces resulting from these stresses are expressed as

$$\frac{\partial \sigma_i}{\partial x_i} \text{d}x \text{d}y \text{d}z \quad \text{and} \quad \frac{\partial \tau_{ij}}{\partial x_j} \text{d}x \text{d}y \text{d}z$$  \hspace{1cm} (3.8)$$

(ii) Body Forces:

An external force field, generally gravity, provides the force needed to accelerate the fluid element. Considering the components of the external force field per unit mass as $X$, $Y$ and $Z$, the body forces on an element can be expressed as

$$X \rho \text{d}x \text{d}y \text{d}z \quad , \quad Y \rho \text{d}x \text{d}y \text{d}z \quad , \quad Z \rho \text{d}x \text{d}y \text{d}z$$  \hspace{1cm} (3.9)$$

where $\rho$ is the density.

(iii) Inertia Forces

Considering the $X$ component of velocity: $u = f(x,y,z,t)$. thus,

$$Du = \frac{\partial u}{\partial t} \text{d}t + \frac{\partial u}{\partial x} \text{d}x + \frac{\partial u}{\partial y} \text{d}y + \frac{\partial u}{\partial z} \text{d}z$$
In the limit as \( dt \to 0 \), \( \frac{dx}{dt} = u, \frac{dy}{dt} = v, \frac{dz}{dt} = w \). Thus,

\[
\frac{Du}{dt} = \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z}.
\]  
(3.10)

similarly,

\[
\frac{Dv}{dt} = \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z}.
\]  
(3.11)

\[
\frac{Dw}{dt} = \frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z}.
\]  
(3.12)

The resultant inertia forces on the element are expressed as

\[
\rho \frac{Du}{dt} \, dx \, dy \, dz, \quad \rho \frac{Dv}{dt} \, dx \, dy \, dz, \quad \rho \frac{Dw}{dt} \, dx \, dy \, dz.
\]  
(3.13)

(iv) Equilibrium Condition

For the dynamic equilibrium of the fluid element, the resultant inertia force is set equal to the sum of the body and surface forces. Thus by eliminating the common factor \( dx dy dz \),

\[
\rho \frac{Du}{dt} = \rho X + \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{xz}}{\partial y} + \frac{\partial \tau_{yz}}{\partial z},
\]  
(3.14)

\[
\rho \frac{Dv}{dt} = \rho Y + \frac{\partial \tau_{yx}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z},
\]  
(3.15)

\[
\rho \frac{Dw}{dt} = \rho Z + \frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_z}{\partial z},
\]  
(3.16)

Using equations (3.6) and (3.7), the Navier-Stokes Equation in Cartesian coordinates are obtained as

\[
\rho \frac{Du}{dt} = \rho X \frac{\partial p}{\partial x} + \frac{\partial}{\partial x} \left[ \mu \left( \frac{2}{3} \frac{\partial u}{\partial x} + \frac{2}{3} \frac{\partial u}{\partial x} + \frac{\partial u}{\partial x} \right) \right] + \frac{\partial}{\partial y} \left[ \mu \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial y} \right) \right] + \frac{\partial}{\partial z} \left[ \mu \frac{\partial u}{\partial z} + \frac{\partial w}{\partial z} \right].
\]  
(3.17)
The terms on the left-hand side are inertia terms. On the right-hand side, the first term is body force term, the second is pressure gradient term and the rest three are viscous terms. These equations are for a Newtonian fluid and also valid for viscous compressible flow with varying viscosity.

For cylindrical polar coordinates with \( r, \theta, z \)

\[
x = r \cos \theta, \quad y = r \sin \theta, \quad z = z
\]

For spherical coordinates with \( r, \theta, \phi \)

\[
x = r \sin \theta \cos \phi, \quad y = r \sin \theta \sin \phi, \quad z = r \cos \theta
\]

### 3.5.2 Continuity Equation

There are three equations and four unknowns: \( u, v, w \) and \( p \) in the Navier-Stokes equations. For complete solution fourth equation is needed. It is supplied by the continuity equation. The \( x \)-component of mass flow per unit area at the center of the volume is \( \rho u \). This flux changes from point to point as shown in Figure - 3.11. The net out flow of mass per unit time is thus (considering unit dimension in Y-direction)

\[
\left[ \rho u + \frac{1}{2} \frac{\partial (\rho u)}{\partial x} dx \right] dz + \left[ \rho w + \frac{1}{2} \frac{\partial (\rho w)}{\partial z} dz \right] dx \\
- \left[ \rho u - \frac{1}{2} \frac{\partial (\rho u)}{\partial x} dx \right] dz - \left[ \rho w - \frac{1}{2} \frac{\partial (\rho w)}{\partial z} dz \right] dx \\
= \left[ \frac{\partial}{\partial x} (\rho u) + \frac{\partial}{\partial z} (\rho w) \right] dx dz
\]
This is equal to the rate of mass decrease within the element, i.e., \(-\left(\frac{\partial \rho}{\partial t}\right) dx \, dz\). Thus,

\[
\frac{\partial p}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial z}(\rho w) = 0
\]  
(3.22)

Inclusion of Y-direction results the continuity equation

\[
\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0
\]  
(3.23)

If the density is constant, then the continuity equation becomes

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0
\]  
(3.24)

### 3.6 REYNOLDS EQUATION

The pressure distribution in fluid film lubrication is expressed in differential equation form known as the Reynolds Equation, Reynolds, 1886 [94]. The parameters involved in it are viscosity, force density and the film thickness of lubricant. The Reynolds equation is derived in two different ways from the Navier-Stokes and Continuity Equations and directly from the principle of mass conservation.
Based on Navier-Stokes and Continuity Equations

The generalized Reynolds Equation is derived from the Navier-Stokes and Continuity Equation after making a few assumptions, which are also known as the basic assumptions in the theory of lubrication [21]. The assumptions are as follows:

(a) Inertia and body force terms are negligible compared to pressure and viscous terms.

(b) No variation of pressure across the fluid film, i.e., \( \frac{\partial p}{\partial y} = 0 \).

(c) No slip is assumed at the fluid solid boundary.

(d) No external forces act on the fluid film.

(e) Flow is considered to be viscous and laminar.

(f) The derivatives of \( u \) and \( w \) with respect to \( y \) are much larger than other derivatives of velocity components.

(g) Fluid film thickness \( h \) is very small compared to the bearing length \( L \).

\( \left( \frac{h}{L} \right) \) is of the order \( 10^{-3} \)

The use of these assumptions reduces Equations (3.17) and (3.19) to

\[
\frac{\partial p}{\partial x} + \frac{\partial}{\partial y} \left( \mu \frac{\partial u}{\partial y} \right) = 0 \tag{3.25}
\]

\[
\frac{\partial p}{\partial z} + \frac{\partial}{\partial y} \left( \mu \frac{\partial w}{\partial y} \right) = 0 \tag{3.26}
\]

Taking viscosity to be constant, the velocity components \( u \) and \( w \) can be obtained from Equations (3.25) and (3.26) by integrating twice with respect to \( y \). The four constants of integration are evaluated by using the boundary conditions for \( u \) and \( w \) as shown in Figure - 3.12.
Boundary conditions for the fluid film.

At $y = 0$, \[ u = u_b, \quad w = w_b, \] (3.27)

at $y = h$, \[ u = u_a, \quad w = w_a. \] (3.28)

Thus the velocity components $u$ and $w$ are obtained as

\[
\begin{align*}
u &= \frac{1}{2\mu} \frac{\partial p}{\partial x} (y - h) + \frac{h - y}{h} u_b + \frac{y}{h} u_a \\
w &= \frac{1}{2\mu} \frac{\partial p}{\partial z} (y - h) + \frac{h - y}{h} w_b + \frac{y}{h} w_a
\end{align*}
\] (3.29)

The velocity gradients are

\[
\begin{align*}
\frac{\partial u}{\partial y} &= \frac{1}{2\mu} \frac{\partial p}{\partial x} (2y - h) + \frac{u_a - u_b}{h} \\
\frac{\partial w}{\partial y} &= \frac{1}{2\mu} \frac{\partial p}{\partial x} (2y - h) + \frac{w_a - w_b}{h}
\end{align*}
\] (3.31)

The Reynolds Equation is formed by using the velocity components $u$ and $w$ in the continuity equation (3.23). Before doing so, it is convenient to express the continuity equation in an integral form.

\[
\int_0^h \left[ \frac{\partial p}{\partial t} + \frac{\partial p}{\partial x} (\rho u) + \frac{\partial p}{\partial y} (\rho v) + \frac{\partial p}{\partial y} (\rho w) \right] dy = 0
\] (3.33)
A general rule of integration states that

\[ \int_0^h \frac{\partial}{\partial x} [f(x,y,z)] dy = -f(x,h,z) \frac{\partial h}{\partial x} + \frac{\partial}{\partial x} \left[ \int_0^h f(x,y,z) dy \right] \]  

(3.34)

If \( \rho \) is assumed to be the mean density across the film, then \( u \) component term in the integrated continuity equation is

\[
\int_0^h \frac{\partial}{\partial x} (\rho u) dy = - (\rho u)_{y=h} \frac{\partial h}{\partial x} + \frac{\partial}{\partial x} \left[ \int_0^h (\rho u) dy \right]
\]

\[
= - \rho u_a \frac{\partial h}{\partial x} + \frac{\partial}{\partial x} \left[ \rho \int_0^h u dy \right]
\]

(3.35)

For \( u \) component, direct integration gives

\[
\int_0^h \frac{\partial}{\partial y} (\rho u) dy = \rho (u_a - u_b)
\]

(3.36)

and for \( w \) component

\[
\int_0^h \frac{\partial}{\partial y} (\rho w) dy = - \rho w_a \frac{\partial h}{\partial z} + \frac{\partial}{\partial z} \left[ \rho \int_0^h w dy \right]
\]

(3.37)

Thus from equation (3.6.9), the integrated continuity equation becomes

\[
h \frac{\partial p}{\partial t} + \rho u_a \frac{\partial h}{\partial x} + \left( \rho \int_0^h u dy \right) + \rho (u_a - u_b)
\]

\[
- \rho w_a \frac{\partial h}{\partial z} + \frac{\partial}{\partial z} \left( \rho \int_0^h w dy \right) = 0
\]

(3.38)

The integrals in this equation represent the volume flow rates per unit width in the \( x^- \) and \( z^- \) directions defined as

\[
q_x = \int_0^h u dy
\]

(3.39)

\[
q_z = \int_0^h w dy
\]

(3.40)
Using \( w \) from equation (3.39) and (3.40), results

\[
q_x = \frac{h^3}{12\mu} \frac{\partial \rho}{\partial x} + \frac{u_a + u_b}{2} h
\]

\[
q_z = \frac{h^3}{12\mu} \frac{\partial \rho}{\partial z} + \frac{w_e + w_b}{2} h
\]

In these two equations, the first term on the right-hand side represents the well-known Poiseuille (or pressure) flow and the second term represents the Couette (or velocity) flow [5,6,7,8]. Using these two equations for flow rates in equation (3.38), yields the general Reynolds equation

\[
\frac{\partial}{\partial x} \left( \frac{\rho h^3}{12\mu} \frac{\partial \rho}{\partial x} \right) + \frac{\partial}{\partial z} \left( \frac{\rho h^3}{12\mu} \frac{\partial \rho}{\partial z} \right) = \frac{\partial}{\partial x} \left( \frac{\rho h (u_a + u_b)}{2} \right) + \frac{\partial}{\partial z} \left( \frac{\rho h (w_e + w_b)}{2} \right)
\]

\[
+ \rho (u_a - u_b) - \rho \frac{u_e}{\rho} \frac{\partial h}{\partial x} - \rho \frac{w_e}{\rho} \frac{\partial h}{\partial z} + h \frac{\partial \rho}{\partial t}
\]

\[
(3.43)
\]

(ii) Significance of Different Terms

The first-two terms on the left-hand side are the Poiseuille terms. They describe the net flow rates due to pressure gradients. The first-two terms on the right-hand side are the Couette terms. Which describe the net entraining flow rates due to velocity of the surfaces. The third to fifth terms on the right-hand side describe the net flow rates due to a squeezing action and the last term denotes the flow due to local expression as a result of local time rating of density. The squeeze flow them (\( = \rho \frac{\partial h}{\partial t} \)) includes the normal squeeze term, \( \rho (v_a - v_b) \) and translational squeeze terms \( -\rho u_a \frac{\partial h}{\partial x} \) \( -\rho u_a \frac{\partial h}{\partial z} \).

The normal squeeze term results from the difference in the normal velocities and the translational squeeze term results from the translation of inclined surfaces. Of the two Couette terms, each can be further expanded to form three separate terms. For example,

\[
\frac{\partial}{\partial x} \left( \frac{\rho h (u_a + u_b)}{2} \right) = \frac{h (u_a + u_b)}{2} \frac{\partial \rho}{\partial x} + \frac{\rho h}{2} \frac{\partial}{\partial x} (u_a + u_b) + \frac{\rho (u_a + u_b)}{2} \frac{\partial h}{\partial x}
\]

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The first term on the right-hand side is termed as density wedge term, since it is concerned with the rate at which lubricant density changes in the sliding direction. The second term is termed as stretch term. It considers the rate at which surface velocity changes in the sliding direction. The last term is the physical wedge term.

(ii) Special Cases [72,83,88]

Considering the case of pure tangential motion under steady-state conditions,

\[ \frac{\partial h}{\partial t} = 0 \text{ and } V_b = 0 \text{ and } V_a = u_a \frac{dh}{dx} + w_a \frac{dh}{dz}, \]

the Reynolds equation given in Equation (3.43) becomes

\[ \frac{\partial}{\partial x} \left[ \frac{\rho h^3}{\mu} \frac{\partial p}{\partial x} \right] + \frac{\partial}{\partial z} \left[ \frac{\rho h^3}{\mu} \frac{\partial p}{\partial z} \right] = 12 \bar{u} \frac{\partial}{\partial x} (\rho h) + 12 \bar{w} \frac{\partial}{\partial z} (\rho h) \]  

(3.44)

Where, \( u = \frac{(u_a + u_b)}{2}, \quad w = \frac{(w_a + w_b)}{2} \).

Considering hydrodynamic lubrication, fluid properties may be taken as constant. Also, the motion is pure sliding, i.e., \( \bar{W} = 0 \), thus, the Reynolds equation reduces to

\[ \frac{\partial}{\partial x} \left[ \frac{\rho h^3}{\mu} \frac{\partial p}{\partial x} \right] = 12 \bar{u} \mu \frac{\partial (\rho h)}{\partial z} \]

(3.45)

If side leakage is neglected, then the second term can be omitted thus,

\[ \frac{d}{dx} \left[ \rho h^3 \frac{\partial p}{\partial x} \right] = 12 \bar{u} \mu \frac{\partial (\rho h)}{\partial z} \]

(3.46)

Integrating equation (3.6.22) with respect to \( x \),

\[ \frac{d p}{d x} = 12 \bar{u} \mu \left( \rho h - (\rho h)_m \right) \]

(3.47)

Where the subscript \( m \) refer to the condition at a point where \( \frac{dp}{dx} = 0 \), such as the point of maximum pressure. Equation (3.47) is known as the integrated form of the Reynolds equation. Up to this no assumptions have been made regarding the density and viscosity. For a perfect gas (\( \rho \propto \rho \))
for an incompressible fluid (constant $\rho$),

$$\frac{d p}{dx} = 12 \bar{u} \mu \frac{(p_h - p_m h_m)}{p h^3}$$  \hspace{1cm} (3.48)

(iv) Dimensionless numbers:

Fluid film lubrication problems belong to a class of flow condition known as slow viscous motion where pressure and viscous terms predominate. The Reynolds equation derived earlier is based on this assumption that inertia and body force terms are negligible compared to pressure and viscous terms. The significance of the terms within the Reynolds equation is better described by the introduction of various dimensionless numbers. By defining the following characteristic parameters, the dimensionless numbers are defined.

Let $l, b$ and $h$ are the characteristic length in $x, y$ and $z$ directions respectively; $u, v$ and $w$ are the characteristic velocity in $x, y$ and $z$ directions respectively; $t$ the characteristic time, $\rho$ is the characteristic force density and $\mu$ is the characteristic absolute viscosity.

**Reynolds Number:**

In fluid mechanics, the relative importance of inertia to viscous forces is judged from the value of the Reynolds number Re,

$$\text{Re} = \frac{\text{Inertia force}}{\text{Viscous force}} = \frac{\rho ul}{\mu}$$

But in fluid film lubrication, because of the dominance of the viscous term $\frac{\partial^2 u}{\partial z^2}$,

the modified Reynolds numbers in $x, y$ and $z$ - directions are defined as
Also, the squeeze number is defined as
\[ \sigma_s = \frac{\rho h^2}{\mu t} \]

All these numbers are of the order \( \left( \frac{h}{l} \right) \), i.e., typically \( 10^{-3} \). Thus it is clear that in typical hydrodynamic lubrication, the viscous forces are much greater than the inertia forces. Still there are Taylor Number, Froude Number and Euler Number, but they are not discussed here since the research is on Reynolds Equation. Anyhow one can refer, Prasanta Sahoo, 2005 [88].

3.7 FLOW RATE AND SHEAR FORCE

While deriving the Navier-Stokes and Reynolds Equations, the flow rates and shear stresses are found. Equation (3.41) and (3.42) provide the flow rates in \( X \) and \( Z \) directions.

\[
q_x = \frac{h^3}{12 \mu} \frac{\partial p}{\partial x} + \frac{(u_a + u_b)}{2} h 
\]

(3.50)

\[
q_z = \frac{h^3}{12 \mu} \frac{\partial p}{\partial z} + \frac{(w_a + w_b)}{2} h 
\]

(3.51)

For a Newtonian fluid, shear stresses are given in Equation (3.37). Thus the shear force \( F \) over the bearing surface can be obtain as

\[ F = \iint \tau \ dA \]  

(3.52)

3.8 INFINITELY LONG-JOURNAL BEARING [92]

It is assumed that the bearing is infinitely long in the axial direction, i.e., the pressure in that direction is constant. This approach is valid for length to diameter ratio \( \frac{L}{D} \) more than 1.5. Thus, omitting \( \left( \frac{\partial p}{\partial z} \right) \) term the governing equation may be written as
Using polar coordinates, \( x = r \theta \) and \( dx = R \, d \theta \), Equation (3.53) may be written as

\[
\frac{d}{dx} \left( \frac{h^3 \, dp}{dx} \right) = 6 \mu \omega R \, \frac{dh}{dx}
\]  
(3.53)

In order to find a solution the above equation, the film thickness, \( h \) must be expressed in terms of \( \theta \) (Refer Appendix - A).

\[ h = 1 + \epsilon \cos \theta \]  
(3.55)

Integrating Equation (3.54) with the condition that at

\[ h = h_m, \quad \frac{dp}{d \theta} = 0 \]

\[ \frac{dp}{d \theta} = 6 \mu \omega R^2 \left( \frac{h - h_m}{h^3} \right) \]  
(3.56)

Integrating again with respect to \( \theta \),

\[ p = \frac{6 \mu \omega R^2}{c^2} \left[ \frac{d \theta}{(1 + \epsilon \cos \theta)^2} - \frac{h_m}{c} \frac{d \theta}{(1 + \epsilon \cos \theta)^3} \right] + C_1 \]  
(3.57)

where \( C_1 \) is the constant of integration.

To evaluate the integrals, Sommerfeld in 1904 used the substitutions, known as Sommerfeld substitutions, as

\[
\cos \gamma = \frac{\epsilon + \cos \theta}{1 + \epsilon \cos \theta}
\]

\[
\sin \theta = \frac{(1 - \epsilon^2)^{1/2} \sin \gamma}{1 + \epsilon \cos \gamma}
\]

\[
\cos \gamma = \frac{\cos \gamma - \epsilon}{1 - \epsilon \cos \gamma}
\]
\[ d\theta = \frac{(1 - \varepsilon^2)^{1/2} d\gamma}{1 - \varepsilon \cos \gamma} \quad (3.58) \]

Here, \( \gamma \) is known as the Sommerfeld variable.

Making use of equation (3.54), equation (3.55) can be evaluated as

\[
p = \frac{6 \mu \omega R^2}{c^2} \left[ \frac{\gamma - \varepsilon \sin \gamma}{(1 - \varepsilon^2)^{3/2}} - \frac{h_m}{c(1 - \varepsilon^2)^{3/2}} \left( \gamma - 2 \varepsilon \sin \gamma + \frac{\varepsilon^2 \gamma}{2} + \frac{\varepsilon^2 \sin (2\gamma)}{4} \right) \right] + C_i
\]

\[
(3.59)
\]

The constants \( C_i \) and \( h_m \) are still to be evaluated [91,54]. For this, however, boundary conditions are required.

3.9 BOUNDARY CONDITIONS

Three types of boundary conditions are available for bearings.

(i) Full-Sommerfeld Boundary Condition

In the full-Sommerfeld boundary condition, it is assumed that

\[(i) \quad p = 0 \quad \text{at} \quad \theta = 0 \quad (\gamma = 0) \quad \text{and} \quad \theta = 2\pi .\]

This indicates the film is full, i.e., up to 2\(\pi\). The boundaries \(\theta=0\) and \(\theta=2\pi\) will transform into the same boundaries in the \(\gamma\) variable. The first boundary condition gives \(C_i = 0\) and the second provides

\[ h_m = \frac{2 c (1 - \varepsilon^2)}{2 + \varepsilon^2} \quad (3.60) \]

Substituting \(h_m\) form Equation (3.60) in Equation (3.59) and using \(C_i = 0\), and reverting to \(\theta\) coordinate from \(\gamma\), the pressure distribution comes to

\[
p = \frac{6 \mu \omega \left( \frac{R}{c} \right)^2 \varepsilon \sin \theta \left( 2 + \varepsilon \cos \theta \right)}{(2 + \varepsilon^2)(1 + \varepsilon \cos \theta)^2} \quad (3.61)
\]

This can be written in non-dimensional form as
Since the film thickness is small compared to the shaft radius, the curvature of the film can be neglected. Therefore, as shown in Figure-3.9.1(a), the film can be unwrapped from around the shaft and it is a periodic stationary profile with wavelength \(2 \pi r\). The value of \(\Theta\) measures the angular position from the position of maximum film thickness and the minimum film thickness occurs at \(\Theta = \pi\). The shape of the pressure distribution is also shown in Figure-3.9.1(b). The positive pressure is developed in the convergent film \((0 \leq \Theta \leq \pi)\) and the negative pressure in the divergent film \((\pi < \Theta < 2\pi)\). The pressure distribution is skewed symmetrically.

From equation (3.62), \(\Theta_m\) corresponds to the maximum pressure location is

\[
\Theta_m = \cos^{-1}\left(\frac{3\varepsilon}{2 + \varepsilon^2}\right) \quad (3.63)
\]

The maximum pressure is then given as

\[
\bar{P}_m = \frac{3\varepsilon \left(4 - 5\varepsilon^2 + \varepsilon^4\right)^{1/2} \left(4 - \varepsilon^2\right)}{2 \left(2 + \varepsilon^2\right) \left(1 - \varepsilon^2\right)^2} \quad (3.64)
\]

The maximum pressure occurs in the second quadrant and the minimum pressure occurs in the third quadrant. If \(\varepsilon \to 0\), \(\Theta_m \to \pm \frac{\pi}{2}\) and \(\bar{P}_m = 0\). If \(\varepsilon \to 1\), \(\Theta_m \to \pm \frac{\pi}{2}\) and \(\bar{P}_m \to \infty\).

(a) Load Carrying Capacity

The components of the resultant load along and perpendicular to the line of centers are determined using the pressure (Refer Figure - 3.14).

\[
W_x = W \cos \phi = -L \int_0^{2\pi} p \cos \theta \ r \ d\theta \quad (3.65)
\]

\[
W_z = W \sin \phi = -L \int_0^{2\pi} p \sin \theta \ r \ d\theta \quad (3.66)
\]
Figure 3.13: (a) Unwrapped film shape and (b) Shape of Pressure distribution for Full Sommerfeld solution in a journal bearing.

Figure 3.14: Coordinate Systems and Force Components in a Journal Bearing.

$L$ is the axial length of the journal and $\phi$ is the attitude angle which is defined as the angle through which load vector has to be rotated in the direction of journal rotation to bring it the line of centers.
Integrating equation (3.65) with substitution of $\frac{dp}{d\theta}$ taking from equation (3.8.4) along with the Sommerfeld Substitutions

$$W_z = W \cos \phi = 0$$

Since $W \neq 0$, $\cos \phi = 0$ and hence $\phi = \pi / 2$. It means the displacement of the shaft is always at the right angle to load vector. The experimental evidences do not support this trend. $W_z$ is zero because the contributions from the convergent and divergent films cancel each other.

The other load component $W_z$ is evaluated as

$$W_z = W \sin \phi = \frac{12 \mu \pi \omega RL \varepsilon \left(\frac{r}{c}\right)^2}{\left(2 + \varepsilon^2\right)\left(1 - \varepsilon^2\right)^{3/2}}$$

Thus, the total load is $W = W_z$.

The equation (3.9.9), may be rewritten as

$$S = \frac{\mu N \left(\frac{R}{c}\right)^2}{P} = \frac{2 + \varepsilon^2}{12 \pi^2 \varepsilon} \left(1 - \varepsilon^2\right)^{1/2}$$

Where $S$ is the Sommerfeld number, $N$ is the speed of the journal in rps and $P$ is unit load $\left(= \frac{W}{2RL}\right)$.

The shear stress at the journal surface is given as

$$\tau_j = \frac{\eta \omega R}{h} + h \frac{dp}{2R \ d\theta}$$

(b) Friction Force

The friction force at the journal surface may be obtained as

$$F_j = \int_0^{2\pi} \tau_j LR \ d\theta = \frac{\mu \omega R^2 L}{c} \frac{4\pi \left(1 + 2\varepsilon^2\right)}{\left(2 + \varepsilon^2\right)\left(1 - \varepsilon^2\right)^{3/2}}$$

Thus, the coefficient of friction on the journal surface is given as
The friction force on the bearing surface may be obtained in a similar way and is given as

\[ F_b = \frac{\mu \omega R^2 L}{c} 4\pi \left( 1 - \varepsilon^2 \right)^{3/2} \left( 2 + \varepsilon^2 \right) \]  

(3.73)

It may be noted that the Sommerfeld number \((S)\) and the friction variable \(\left( \frac{R}{c} \right) \mu_j\) are dependent on the eccentricity ratio \((\varepsilon)\) only. Considering the equilibrium of journal under applied load \(W\) and friction forces in journal and bearing surfaces, the condition may be written as

\[ RF_j = RF_b + We \]  

(3.74)

Which means friction at the journal surface is more than that at the bearing surface. So it is adequate to know the coefficient of friction on the journal surface in order to find the power loss due to fluid friction.

(ii) Half – Sommerfeld Boundary Condition

It has been noticed that the idealized full-journal bearing solution using full-Sommerfeld boundary conditions leads to the skewed symmetrical pressure distribution shown in Figure - 3.15. The pressure in the divergent film region is negative (lower than the ambient pressure). Such conditions are rarely encountered in real bearings. If the negative pressure in the divergent zone is ignored, it might lead to some reasonable result. An approach that limits the analysis to the convergent film \(0 < \theta < \pi\) is known as the Half-Sommerfeld solution. The Half-Sommerfeld boundary conditions may be written as

\[ p = 0 \quad \text{at} \quad \theta = 0 \]
\[ p = 0 \quad \text{at} \quad \pi \leq \theta \leq 2\pi \]  

(3.75)

The pressure distribution as obtained with Full - Sommerfeld condition, can be applied here for the region \(\theta = 0\) to \(\pi\). Thus

\[ p = \frac{6 \mu \omega \left( \frac{R}{c} \right)^2 \varepsilon \sin \theta \left( 2 + \varepsilon \cos \theta \right)}{(2 + \varepsilon^2)\left( 1 + \varepsilon \cos \theta \right)^2} \quad \text{for} \quad 0 \leq \theta \leq \pi \]

and \(p = 0\) at \(\pi \leq \theta \leq 2\pi\)  

(3.76)
This pressure distribution is shown in Figure 3.15.

Figure 3.15: Shape of Pressure Distribution for Half-Sommerfeld Boundary Condition in a Journal Bearing.

(a) Load Carrying Capacity

The load components in the direction of centers and in the direction perpendicular to the line of centers are,

\[
W_x = W \cos \phi = -LR \int_0^\pi p \cos \theta \, d\theta \tag{3.77}
\]

\[
W_z = W \sin \phi = -LR \int_0^\pi p \sin \theta \, d\theta \tag{3.78}
\]

Performing the integration,

\[
W_x = 12 \mu \omega RL \left( \frac{R}{c} \right)^2 \frac{\varepsilon^2}{(2 + \varepsilon^2)(1 - \varepsilon^2)} \tag{3.79}
\]

\[
W_z = 6 \mu \omega RL \left( \frac{R}{c} \right)^2 \frac{\pi \varepsilon}{(2 + \varepsilon^2)(1 - \varepsilon^2)^{3/2}} \tag{3.80}
\]

The total load is given by

\[
W = \sqrt{W_x^2 + W_z^2} = \frac{6\mu \omega RL \left( \frac{R}{c} \right)^2 \varepsilon \left[ \pi^2 - \varepsilon^2 \left( \pi^2 - 4 \right) \right]^{1/2}}{2 + \varepsilon^2 (1 - \varepsilon^2)} \tag{3.81}
\]
(b) Attitude Angle

The attitude angle $\phi$ is given as

$$\phi = \tan^{-1}\left( \frac{W_{c}}{W_{x}} \right) = \tan^{-1}\left( \frac{\pi \sqrt{1 - e^2}}{2 e} \right)$$

(3.82)

It may be noted here that $W_i$ for Half-Sommerfeld solution is one half of the corresponding full-Sommerfeld solution. It is so because the resultant contribution of negative pressure perpendicular to the line of centers is neglected in the present case. Moreover, $W_i$ is not zero as it was observed in the Full - Sommerfeld case, since the contributions from the convergent and divergent films do not cancel each other here.

(iii) Reynolds Boundary Conditions / Gumbel’s Boundary condition

Though the Half-Sommerfeld solution leads to more realistic prediction, it fails to satisfy the continuity of mass-flow condition at the end of the pressure curve. The pressure suddenly becomes zero at $\theta = \pi$ and then stays at zero from $\pi$ to $2\pi$. This shortcoming can be overcome by a better boundary condition, known as Reynolds boundary condition, which may be stated as given below and the corresponding diagram is shown in Figure - 3.16.

\[
\begin{align*}
(i) & \ p = 0 \ \text{at} \ \theta = 0 \\
(ii) & \ p = 0 \ \text{at} \ \theta_2 \leq \theta \leq 2\pi \ (\theta_2 > \pi) \\
(iii) & \frac{dp}{d\theta} = 0 \ \text{at} \ \theta = \theta_2
\end{align*}
\]

(3.83)

![Figure - 3.16: Shape of Pressure Distribution for a Journal Bearing using Reynolds Boundary Condition.](image)
Thus boundary condition takes care of the film rupture at $\theta = \Theta_2$, using the conditions (i) and (iii), and employing Sommerfeld substitutions.

$$p = \frac{6\mu \omega R^2}{c^2 (1-\varepsilon^2)^{3/2}} \left[ \gamma - \varepsilon \sin \gamma - \frac{(2+\varepsilon^2)\gamma - 4\varepsilon \sin \gamma + \varepsilon^2 \sin \gamma \cos \gamma}{2(1+\varepsilon \cos (\gamma_2 - \pi))} \right]$$

(3.84)

Where $\cos \gamma = \frac{\varepsilon + \cos \theta}{1 + \varepsilon \cos \theta}$ and $\gamma_2$ corresponds to $\Theta_2$

The boundary condition (ii) when used in equation (3.84), leads to

$$\varepsilon \left[ \sin \gamma_2 \cos \gamma_2 - \gamma_2 \right] + 2 \left[ -\gamma_2 \cos \gamma_2 + \sin \gamma_2 \right] = 0$$

(3.85)

For a particular $\varepsilon$, $\gamma_2$ can be found from equation (3.85) and hence $\Theta_2$ can be found.

(a) Load Carrying Capacity [35]

The load components are given as

$$W_x = -3\eta \omega r L \left( \frac{R}{c} \right)^2 \frac{(1-\cos \gamma_2)^2}{(1-\varepsilon^2)(1-\varepsilon \cos \gamma_2)}$$

(3.86)

$$W_z = 6\mu \omega RL \left( \frac{R}{c} \right)^2 \frac{(\sin \gamma_2 - \gamma_2 \cos \gamma_2)}{(1-\varepsilon^2)^{3/2}(1-\varepsilon \cos \gamma_2)}$$

(3.87)

The total load is given by

$$W = \frac{3\mu \omega RL \left( \frac{R}{c} \right)^2}{(1-\varepsilon^2)^{3/2}(1-\varepsilon \cos \gamma_2)} \left[ \varepsilon^2 \left(1-\cos \gamma_2\right)^4 \left(1-\varepsilon^2\right)^{3/2} + 4(\sin \gamma_2 - \gamma_2 \cos \gamma_2)^2 \right]^{1/2}$$

(3.88)

(b) Attitude Angle

The attitude angle $\phi$ is given as

$$\phi = \tan^{-1} \left[ \frac{2(1-\varepsilon^2)^{1/2}(\sin \gamma_2 - \gamma_2 \cos \gamma_2)}{\varepsilon \left(1-\cos \gamma_2\right)^2} \right]$$

(3.89)
(c) Friction Variable

The friction variable is

$$\left( \frac{R}{c} \right)_f = \frac{\varepsilon \sin \phi}{2} + \frac{2 \pi^2 S}{(1 - \varepsilon^2)^{1/2}}$$  \hspace{1cm} (3.90)

3.10 INFINITELY SHORT JOURNAL BEARING:

If the bearing is infinitely short or narrow, the flow due to pressure gradient in the x-direction can be neglected. The governing equation in this case takes the form

$$\frac{\partial}{\partial z} \left[ h^3 \frac{\partial p}{\partial z} \right] = 6 \mu \omega R \frac{dh}{dx}$$  \hspace{1cm} (3.91)

In polar coordinates with \( x = R \theta \) and \( dx = R \, d\theta \)

$$\frac{\partial}{\partial z} \left[ h^3 \frac{\partial p}{\partial z} \right] = 6 \mu \omega \frac{dh}{d\theta}$$  \hspace{1cm} (3.92)

Assuming no misalignment, the film thickness is a function of \( \theta \) only and the right-hand side is independent of \( z \). Thus integrating twice with respect to \( z \),

$$p = \frac{6 \mu \omega}{h^3} \left( \frac{dh}{d\theta} \right) \left( \frac{z^2}{2} \right) + C_1 z + C_2$$  \hspace{1cm} (3.93)

where \( C_1 \) and \( C_2 \) are constants of integration to be evaluated from boundary conditions: \( p = 0 \) \hspace{1cm} at \hspace{1cm} \( z = \pm L/2 \). Using these boundary conditions, the pressure distribution is given as

$$p = \frac{3\mu \omega}{c^2} \left( \frac{L^2}{4} - z^2 \right) \frac{\varepsilon \sin \theta}{(1 + \varepsilon \cos \theta)^3}$$  \hspace{1cm} (3.94)

This pressure distribution is parabolic in z-direction whereas the trigonometric function dictates the circumferential variation of pressure. It may also be noted that Equation (3.94) is applicable to \( \theta = 0 \) \hspace{1cm} to \hspace{1cm} \( \theta = \pi \), as the pressure becomes negative for \( \theta > \pi \). Thus it is assumed that the positive pressure region from \( \theta = 0 \) \hspace{1cm} to \hspace{1cm} \( \theta = \pi \)
carries the total load of the bearing (Half-Sommerfeld assumption). The location of the maximum pressure may be obtained by setting $\frac{\partial p}{\partial \theta} = 0$. Therefore,

$$-3h^{-4}\left(\frac{dh}{d\theta}\right)^2 + h^{-3}\frac{d^2 h}{d\theta^2} = 0$$  \hspace{1cm} (3.95)$$

Since,

$$h = c(1 + \epsilon \cos \theta), \quad \frac{dh}{d\theta} = -c \epsilon \sin \theta \quad \text{and} \quad \frac{d^2 h}{d\theta^2} = -c \epsilon \cos \theta,$$

Substituting these in equation (3.95)

$$\theta_m = \cos^{-1}\left[\frac{1 - (1 - 24\epsilon^2)^{1/2}}{4\epsilon}\right]$$ \hspace{1cm} (3.96)$$

if $\epsilon \to 0$ then $\theta \to \pm \pi/2$ and if $\epsilon \to 1$ then $\theta_m \to \pm \pi$

The maximum pressure occurs when $\theta = \theta_m$ and $z = 0$. Thus the expression for maximum pressure is

$$p_m = \frac{3 \mu \omega L^2 \sin \theta_m}{4c^2(1 + \epsilon \cos \theta_m)^3}$$ \hspace{1cm} (3.97)$$

The load components from the pressure development under the Half-Sommerfeld assumption are

$$W_x = -2 \int_0^{\pi L/2} \int p \cos \theta \, r \, d\theta \, dz$$

$$= \frac{\mu \omega R \epsilon L^2 \sin \theta_m}{2c^2} \int_0^{\pi} \frac{\sin \theta \cos \theta}{(1 + \epsilon \cos \theta)^3} \, d\theta$$ \hspace{1cm} (3.98)$$

$$W_z = 2 \int_0^{\pi L/2} \int p \sin \theta \, r \, d\theta \, dz$$

$$= \frac{\mu \omega R \epsilon L^2 \sin \theta_m}{2c^2} \int_0^{\pi} \frac{\sin^2 \theta}{(1 + \epsilon \cos \theta)^3} \, d\theta$$ \hspace{1cm} (3.99)$$

using the Sommerfeld substitutions, the $W_x$ and $W_z$ can be written as
\[ W_x = \frac{\mu \omega RL^3}{c^2} \frac{\varepsilon^2}{(1-\varepsilon^2)^2} \]

\[ W_z = \frac{\mu \omega RL^3}{4c^2} \frac{\pi \varepsilon}{(1-\varepsilon^2)^{3/2}} \]  

(a) Load Carrying Capacity

The total load capacity \( W \) is given as

\[ W = \sqrt{W_x^2 + W_z^2} \]

\[ = \frac{\mu \omega RL^3 \varepsilon}{4c^2} \left[ \pi^2 \left(1-\varepsilon^2\right) + 16\varepsilon^2 \right]^{1/2} \]  

(3.101)

(b) Attitude Angle

The attitude angle \( \phi \) is given by

\[ \phi = \tan^{-1} \left( \frac{W_z}{W_x} \right) \]

\[ = \tan^{-1} \left( \frac{\pi}{4} \frac{1-\varepsilon^2}{\varepsilon} \right) \]  

(3.102)

Since there is no pressure induced shear, the shear force (friction) experienced by the journal (or the bearing) is simply given as

\[ F_j = -F_b = \int_0^{2\pi} \mu \frac{\omega LR^2}{h} d\theta \]

\[ = \mu \frac{\omega LR^2}{c} \frac{2\pi}{(1-\varepsilon^2)^{1/2}} \]  

(3.103)

(c) Friction Variable

The friction variable is given by

\[ \left( \frac{R}{c} \right)_f = \frac{2\pi^2 S}{(1-\varepsilon^2)^{1/2}} \]  

(3.104)

The volume flow of lubricant supplied to the bearing through a central hole is equal to the net rate of outflow along the axis of bearing (z-direction). The total leakage from the sides of the bearing in the convergent film region can be expressed as
\[ Q_{z} = -2 \int_{\theta}^{\pi/2} \left( \frac{R h^3}{12 \mu} \frac{\partial p}{\partial z} \right) d\theta = \varepsilon \omega R L c \] (3.105)

As \( \varepsilon \to 0 \), \( Q_{z} \to 0 \) (no slide leakage) and \( \varepsilon \to 1 \), \( Q_{z} = \omega RLc \) (complete side leakage).

The load bearing capacity as obtained from the infinitely long bearing solution can be compared with the infinitely short-bearing solution by comparing equation (3.81) and (3.101) to obtain.

\[ \frac{W(\text{Short bearing})}{W(\text{Long bearing})} = \left( \frac{L}{R} \right)^2 \frac{\left(2 + \varepsilon^2\right) 16 \varepsilon^2 + \pi^2 \left(1 - \varepsilon^2\right)}{24 \left(1 - \varepsilon^2\right) \left[\pi^2 - \varepsilon^2 \left(\pi^2 - 4\right)\right]^{1/2}} \] (3.106)

Thus the load ratio is functions of \( (L/R) \) and \( \varepsilon \). The infinitely long bearing theory overestimates the load carrying capacity for all \( \varepsilon \) and should be used for length to diameter ratio greater than 1.5. The infinitely short-bearing theory provides a much better estimate for finite bearings with length to diameter ratio less than 0.5.

3.11 DYNAMIC CHARACTERISTICS

Dynamically loaded bearing and stiffness and Damping coefficient

When the journal is oscillating the velocity of the journal introduces time dependent terms on the right hand side of the Reynolds equation as shown in equation \(- (3.107)\) below.

\[ \frac{\partial}{\partial x} \left[ \frac{h^3}{\mu} \frac{\partial p}{\partial x} \right] + \frac{\partial}{\partial z} \left[ \frac{h^3}{\mu} \frac{\partial p}{\partial z} \right] = 6 \bar{u} \frac{\partial}{\partial x} (h) + 12 \frac{\partial}{\partial t} (h) \] (3.107)

Consider the journal under a steady state condition given by eccentricity \( \varepsilon_0 \) and attitude angle \( \omega_0 \) as shown in the Figure - 3.17. Let the journal be displaced by \( \Delta z \) and \( \Delta y \) from equilibrium position, then the vertical and horizontal components of load can be written down using Taylor’s series from the equilibrium position as
Figure – 3.17 : Journal Displacement

\[ W_x = W_{x,0} + W_{x,x} \Delta x + W_{x,y} \Delta y + W_{x,\dot{x}} \Delta \dot{x} + W_{x,\dot{y}} \Delta \dot{y} \]  
(3.108)

\[ W_y = W_{y,0} + W_{y,x} \Delta x + W_{y,y} \Delta y + W_{y,\dot{x}} \Delta \dot{x} + W_{y,\dot{y}} \Delta \dot{y} \]  
(3.109)

The above two equations can be written as

\[ W_x = W_{x,0} + K_{x,x} \Delta x + K_{x,y} \Delta y + C_{x,\dot{x}} \Delta \dot{x} + C_{x,\dot{y}} \Delta \dot{y} \]  
(3.110)

\[ W_y = W_{y,0} + K_{y,x} \Delta x + K_{y,y} \Delta y + C_{y,\dot{x}} \Delta \dot{x} + C_{y,\dot{y}} \Delta \dot{y} \]  
(3.111)

Where

\[ K_{xx} = \left( \frac{\partial W_x}{\partial x} \right)_{x=x_0, y=y_0} = \left( W_{x,x} \right)_{x=x_0, y=y_0} \]

\[ K_{xy} = \left( \frac{\partial W_x}{\partial y} \right)_{x=x_0, y=y_0} = \left( W_{x,y} \right)_{x=x_0, y=y_0} \]

\[ K_{yy} = \left( \frac{\partial W_y}{\partial y} \right)_{x=x_0, y=y_0} = \left( W_{y,y} \right)_{x=x_0, y=y_0} \]

\[ K_{yx} = \left( \frac{\partial W_y}{\partial x} \right)_{x=x_0, y=y_0} = \left( W_{y,x} \right)_{x=x_0, y=y_0} \]

85
Similarly,

\[
C_{xx} = \left( \frac{\partial W_x}{\partial x} \right)_{x=x_0, y=y_0} = \left( W_{x,x} \right)_{x=x_0, y=y_0}
\]

\[
C_{xy} = \left( \frac{\partial W_x}{\partial y} \right)_{x=x_0, y=y_0} = \left( W_{x,y} \right)_{x=x_0, y=y_0}
\]

\[
C_{yx} = \left( \frac{\partial W_y}{\partial x} \right)_{x=x_0, y=y_0} = \left( W_{y,x} \right)_{x=x_0, y=y_0}
\]

\[
C_{yy} = \left( \frac{\partial W_y}{\partial y} \right)_{x=x_0, y=y_0} = \left( W_{y,y} \right)_{x=x_0, y=y_0}
\]

To determine these coefficients, let the pressure distribution in the film be linearized as

\[
P = P_0 + P_x \Delta x + P_y \Delta y + P_{x,x} \Delta x + P_{y,y} \Delta y
\]

(3.112)

Considering the equilibrium of the journal, the above components can be written as

\[
\begin{bmatrix}
W_x \\
W_y
\end{bmatrix} = 2 \int_0^{\phi_0 + \pi / 2} \int (P_0 + P_x \Delta x + P_y \Delta y + P_{x,x} \Delta x + P_{y,y} \Delta y) r \begin{bmatrix}
\cos \theta \\
\sin \theta
\end{bmatrix} dz d\theta
\]

(3.113)

Comparing Equation (3.113) with Equation (3.110), we obtain the following

\[
\begin{bmatrix}
W_{x0} \\
0
\end{bmatrix} = 2 \int_0^{\phi_0 + \pi / 2} \int (P_0) r \begin{bmatrix}
\cos \theta \\
\sin \theta
\end{bmatrix} dz d\theta
\]

(3.114)

\[
\begin{bmatrix}
K_{yy} & K_{yx} \\
K_{xy} & K_{xx}
\end{bmatrix} = \begin{bmatrix}
\alpha_0 + \pi / 2 & \alpha_0 + \pi / 2 \\
\alpha_0 & \alpha_0 \\
2 & 2 \\
0 & 0
\end{bmatrix}
\]

(3.115)

\[
\begin{bmatrix}
C_{yy} & C_{yx} \\
C_{xy} & C_{xx}
\end{bmatrix} = \begin{bmatrix}
\alpha_0 + \pi / 2 & \alpha_0 + \pi / 2 \\
\alpha_0 & \alpha_0 \\
2 & 2 \\
0 & 0
\end{bmatrix}
\]

(3.116)
The above stiffness and damping coefficients are evaluated using MatLab programming and coding is incorporated in Annexure – A. Similar equations are represented and referred by J. S. Rao [95].

3.12 STABILITY OF THE JOURNAL BEARING [23,72,107]

Considering the journal to be a single mass with possible translation in x and y coordinates, equations of motion can be written as:

\[ M\ddot{x} + K_{xx}\dot{x} + K_{xy}\dot{y} + C_{xx}x + C_{xy}y = 0 \]  \hspace{1cm} (3.117)

\[ M\ddot{y} + K_{yx}\dot{x} + K_{yy}\dot{y} + C_{yx}x + C_{yy}y = 0 \]  \hspace{1cm} (3.118)

Assuming the harmonic solution, the above two equations leads to a characteristic equation of the form

\[ A_1\overline{s}^4 + A_2\overline{s}^3 + A_3\overline{s}^2 + A_4\overline{s} + A_5 = 0 \]  \hspace{1cm} (3.119)

Where \( \overline{s} = s / \omega \), “s” is a complex Characteristics root and \( \omega \) is the rotating speed of the journal. The \( A_\text{s} \) in equation – (3.119) is functions of the journal mass, the stiffness coefficients and the damping coefficients are given below.

\[
\begin{align*}
A_1 &= \overline{m}^2 \\
A_2 &= \overline{m}(C_{xx} + C_{yy}) \\
A_3 &= \overline{m}(K_{xx} + K_{yy}) + C_{xx}C_{yy} - (C_{xy} + C_{yx}) \\
A_4 &= -K_{xx}C_{yy} + K_{yy}C_{xx} - K_{xy}C_{yx} - K_{yx}C_{xy} \\
A_5 &= K_{xx}K_{yy} - K_{xy}K_{yx}
\end{align*}
\]

and \( \overline{m} = \frac{M\omega^2}{W_0} \)

Where \( M \) is the mass of rotor per bearing

\( W_0 \) is the steady state load

For a bearing operating at a certain Sommerfeld number, its stability is dependent on the mass parameter "\( \overline{m} \)" and the coefficients of equation (3.119). It is required to solve the roots of equation – (3.119) to determine whether the bearing is stable or unstable. At the
threshold of instability, the real part of the unstable root is zero. This leads to rotor stability threshold $\omega_s$ is given by

$$\omega_s = \sqrt{\frac{C_1}{C_2}}$$

Where

$$C_1 = \frac{\bar{K}_{xx} \bar{C}_{yy} + \bar{K}_{yy} \bar{C}_{xx} - \bar{K}_{xy} \bar{C}_{yx} - \bar{C}_{xy} \bar{K}_{yx}}{\bar{C}_{xx} + \bar{C}_{yy}}$$

$$C_2 = \frac{(C_1 - \bar{K}_{xx})(C_1 - \bar{K}_{yy}) - \bar{K}_{xy} \bar{K}_{yx}}{\bar{C}_{xx} \bar{C}_{yy} - \bar{C}_{xy} \bar{C}_{yx}}$$

The stability prevails for any $\sqrt{\frac{C_1}{C_2}}$ when $(C_1 / C_2)$ is negative.

3.13 THEORETICAL PREDICTION OF WHIRL INSTABILITY

To begin the evaluation of whirl instability, the equations of motion must be considered. For this purpose, the rotor system was considered to be a flexible shaft with a lumped mass at the midpoint of the rotor. The rotor was supported on two equally spaced identical journal bearings. In finding the equations of motion three reference points were taken into consideration. They were the bearings’ center, the journal’s center and the rotor’s center (which is located at the point of lumped mass). Observing the sum of the forces on the lumped mass, the first two equations of motion are found.

Considering a Simple rotor in fluid bearing with undamped analysis

$$M \frac{d^2}{dt^2} (x + a(\cos(\omega t))) + K(x - x_0) = 0$$

$$M \frac{d^2}{dt^2} (y + a(\cos(\omega t))) + K(y - y_0) = 0$$

(3.122)

Where $x$, $y$ are the disc deflections and $x_0$, $y_0$ Journal deflection, $K$ is the shaft stiffness. The second two equations of motion are found when summing the forces on the fluid for a stable running condition is given by
The above equation (3.123) is represented in matrix form

\[
\begin{bmatrix}
2K_{xx} + K & 2K_{xy} \\
2K_{yx} & 2K_{yy} + K
\end{bmatrix}
\begin{bmatrix}
x_0 \\
y_0
\end{bmatrix} = \begin{bmatrix}
Kx \\
Ky
\end{bmatrix}
\] (3.124)

The journal deflection from the above equation is given by

\[
x_0 = K \frac{(2K_{xx} + K)x - 2K_{yx}y}{(2K_{xx} + K)(2K_{yy} + K) - 4K_{xy}K_{yx}}
\] (3.125)

\[
y_0 = K \frac{(2K_{yy} + K)y - 2K_{xy}x}{(2K_{xx} + K)(2K_{yy} + K) - 4K_{xy}K_{yx}}
\]

Substituting the above equation (3.124) in equation (3.125)

\[
M\ddot{x} + K_{11}x + K_{12}y = M(\omega_0^2 \cos(\omega t))
\] (3.126)

\[
M\ddot{y} + K_{22}y + K_{21}x = M(\omega_0^2 \sin(\omega t))
\]

Where

\[
K_{11} = \frac{K[K_{yy}(2K_{xx} + K) - 4K_{yx}K_{xy}]}{(2K_{xx} + K)(2K_{yy} + K) - 4K_{xy}K_{yx}}
\]

\[
K_{12} = \frac{K[K_{xx}(2K_{yy} + K) - 4K_{yx}K_{xy}]}{(2K_{xx} + K)(2K_{yy} + K) - 4K_{xy}K_{yx}}
\]

\[
K_{22} = \frac{2K_{yx}K^2}{(2K_{xx} + K)(2K_{yy} + K) - 4K_{xy}K_{yx}}
\]

\[
K_{21} = \frac{2K_{xy}K^2}{(2K_{xx} + K)(2K_{yy} + K) - 4K_{xy}K_{yx}}
\]
The solution for equation (3.126) is

\[ x = u = u_c \cos \omega t + u_s \sin \omega t \]
\[ y = v = v_c \cos \omega t + v_s \sin \omega t \]  

(3.127)

Substituting equations (3.13.6) in (3.13.5) and separating out cosine and sine terms, we obtain

\[ -M_0^2 u_c + K_1 u_c + K_{12} v_c = M_0^2 \]  

(3.128)

\[ -M_0^2 u_s + K_1 u_s + K_{12} v_s = 0 \]  

(3.129)

\[ -M_0^2 v_c + K_2 v_c + K_{21} u_c = M_0^2 \]  

(3.130)

\[ -M_0^2 v_s + K_2 v_s + K_{21} u_s = 0 \]  

(3.131)

From equations (3.129) and (3.131), we get

\[ u_s = \frac{-K_{12}}{K_1 - M_0^2} v_s \]  

(3.132)

\[ v_s = \frac{-K_{21}}{K_2 - M_0^2} u_s \]  

(3.133)

Using equations (3.132) & (3.133) in (3.128) & (3.130), the following solutions can be obtained

\[ u_c = \frac{M_0^2 (K_2 - M_0^2)}{(K_1 - M_0^2)(K_2 - M_0^2) - K_{12} K_{21}} \]  

\[ u_s = \frac{-K_{12} M_0^2}{(K_1 - M_0^2)(K_2 - M_0^2) - K_{12} K_{21}} \]  

\[ v_s = \frac{M_0^2 (K_1 - M_0^2)}{(K_1 - M_0^2)(K_2 - M_0^2) - K_{12} K_{21}} \]  

\[ v_c = \frac{-K_{21} M_0^2}{(K_1 - M_0^2)(K_2 - M_0^2) - K_{12} K_{21}} \]  

(3.134)
We introduce the following parameters

\[ \omega_1^2 = \frac{K_1}{M}; \quad \omega_2^2 = \frac{K_2}{M}; \quad \mu_1 = \frac{K_{12}}{K_1}; \quad \mu_2 = \frac{K_{21}}{K_2} \]

\[ \bar{u}_c = \frac{u_c}{a}; \quad \bar{u}_k = \frac{u_k}{a}; \quad \bar{v}_c = \frac{v_c}{a}; \quad \bar{v}_s = \frac{v_s}{a} \]  \hspace{1cm} (3.135)

Substituting equations (3.135) in (3.134), we get

\[ \bar{u}_c = \frac{\omega_2}{\omega_1} \frac{2}{1 - \frac{\omega_2^2}{\omega_1^2}} \left( 1 - \omega_2^2 \right) \left( 1 - \omega_1^2 \right) - \mu_1 \mu_2 \]  \hspace{1cm} (3.136)

\[ \bar{u}_k = \frac{-\mu_1}{\omega_2^2} \left( \frac{\omega_2^2}{\omega_1^2} \right) \left( 1 - \omega_2^2 \right) \left( 1 - \omega_1^2 \right) - \mu_1 \mu_2 \]  \hspace{1cm} (3.137)

\[ \bar{v}_s = \frac{\omega_2}{\omega_1} \frac{2}{1 - \frac{\omega_2^2}{\omega_1^2}} \left( 1 - \omega_2^2 \right) \left( 1 - \omega_1^2 \right) - \mu_1 \mu_2 \]  \hspace{1cm} (3.138)

\[ \bar{v}_c = \frac{-\mu_2}{\omega_2^2} \left( \frac{\omega_2^2}{\omega_1^2} \right) \left( 1 - \omega_2^2 \right) \left( 1 - \omega_1^2 \right) - \mu_1 \mu_2 \]  \hspace{1cm} (3.139)

The frequency equation can be written as

\[ \left( 1 - \frac{\omega_2^2}{\omega_1^2} \right) \left( 1 - \frac{\omega_2^2}{\omega_2^2} \right) - \mu_1 \mu_2 = 0 \]  \hspace{1cm} (3.140)

Which gives

\[ p^2_{1,2} = \frac{\omega_1^2 + \omega_2^2}{2} \pm \sqrt{\left( \frac{\omega_1^2 - \omega_2^2}{2} \right)^2 + \mu_1 \mu_2 \omega_1^2 \omega_2^2} \]  \hspace{1cm} (3.141)
These two frequencies can be taken as critical speeds of the rotor. It is not necessary that rotors in all cases will behave with two clear critical speeds corresponding to the fundamental rigid bearing critical speed, with whirls between two critical speeds. This is because the cross couple stiffness coefficients \((K_{xy} ; K_{yx})\) can be negative for a hydrodynamic bearing in a certain range of Sommerfeld number. From the above Equation \((3.141)\), if either \(\mu_1, \mu_2\) is negative then there will be two distinct critical speeds only when

\[
\sqrt{1 + (|\mu_1| + |\mu_2|)^2} < \frac{\omega_1^2 + \omega_2^2}{2\omega_1 \omega_2} \quad (3.142)
\]

### 3.14 PHENOMENON OF WHIP AND WHIRL

Large machines mounted on fluid film bearings are exposed to many vibration problems including those generated due to resonances. Fluid film bearings contribute in the dynamic characteristics of these systems by influencing the natural frequencies of the rotor system as a whole. Unlike the well-known mechanical natural frequency, the fluid natural frequency is dependent upon operating factors such as shaft speed and shaft eccentricity in the bearing.

Whirl and whip represent the major examples of fluid film resonances. They are characterized by forward precession in a circular orbit with subsynchronous frequency. Journal bearings (which involve radial forces) are the normal location where these instabilities develop. Orbit and frequency spectrum plots can be used to investigate the existence of whirl or whip vibrations.

From the theory of **Complex Dynamic Stiffness**, the interaction among shaft, oil film and bearing plays a major role in the stability. Controlling oil whirl/whip characteristics is achieved through adapting this interaction. The methods used to overcome these instabilities are many and they include: rotor-related solutions, bearing-related solutions and lube oil-related solutions.

### 3.15 FLUID INDUCED VIBRATIONS

Fluid Induced Instabilities (FII) are damaging problems faced in many rotating machinery, e.g., large turbines and compressors. The vibration resulting from these problems limits safe and efficient equipment operation because these problems are
directly related to the machine speed and unfortunately may exist over a range of speeds.

Fluid Induced Vibrations (FIV) are described as a special type of self-excited resonance vibration (refer to Figure - 3.18). They are induced by an internal mechanism (oil film bearing, this particular case) that transfers part of shaft rotational energy back to the shaft as a lateral vibration. This mechanism is very much related to fluids, hence, sometimes called Fluid Generated or Fluid Related Instability.

![Vibration Class Diagram](Image)

Figure 3.18 Vibration classes based on source of excitation

Examples of the FIV are: *Oil Whirl*, *Oil Whip*, *Subsynchronous Resonance* and *Stall*. FIV could be generated in different fluids. Figure - 3.19 below categorizes the instabilities based on fluid at which the instability is generated.

![Fluid Instability Classification](Image)

Figure 3.19 Instability classification based on fluid
Moreover, FIV could be encountered in many locations in the equipment itself like the tips of the blades, bearings and seals. Figure 3.20 classifies these instabilities based on the location.

### 3.16 ROTOR-BEARING: THE INTERACTION AND EFFECTS

Oil whirl and oil whip arise when both Direct and Quadrature Stiffness reduce to zero (for more information, refer to complex dynamic stiffness articles). For an actual rotor, the formulas describing a rotor model, with whirl or whip vibration, are very complicated. Machines mounted on radial fluid film bearings will have two types of resonance: mechanical and fluid. The natural frequency of the whole system (not only the rotor) will be affected by the interaction of the three elements: shaft, oil film and bearing. These elements are partially mechanical (solid materials) and partially fluid (hydraulic oil).

Complex Dynamic Stiffness

\[
\text{Direct Stiffness} \approx \text{Mechanical Based Stiffness} + \text{Quadrature Stiffness} \approx \text{Fluid Based Stiffness}
\]

In the literature, if resonance is not explicitly specified, usually the mechanical resonance is meant.
Notice that the natural frequency of a machine found by the impact test will not be accurate for such machines because it gives you only the natural frequency at zero rpm with minimum fluid film thickness. This explains why natural frequency value collected by an impact test varies sometimes from that observed while the machine runs during start up. To compensate this deficiency, *Frequency Interface Charts* are developed. These charts provide the machine natural frequency as a function of the rotor speed and with normal fluid film thickness. The simplified figure below is an example.

![Frequency interference chart](image)

3.17 **OIL WHIRL / WHIP SYMPTOMS**

The typical symptoms of the whirl/whip vibrations can be summarized as follows:

1) Subsynchronous frequency (noticed in the frequency spectrum)

2) High amplitudes (reaching to machine's alarm limits)

A general oil whip trend is shown in the Figure – 3.23 below.
3.18 STARVED LUBRICATION AND FLOODED LUBRICATION [45]

Non-circular journal bearings is generally used for high speed machinery because of their advantages of stabilization. However, there are some problems using compact size rotating machineries for their high manufacturing cost and complicated structure. Therefore, it is necessary to enhance the stability of more simple bearing than the non-circular bearings in order to use the small size – journal bearings. Hiromu Hashimoto et al [39,44], described the stabilization method of the small-bore rotating machinery of non-circular bearing under two mechanisms, namely starved and flooded lubrication. In the present study, the prediction of whirl and whips in non-circular bearings under flooded lubrication were carried.

3.19 RECORDING AND INSTRUMENTATION [36,71]

Vibration instruments are generally small, hand held, inexpensive, simple to use and self condition devices that gives overall vibration level reading. The instruments used in the vibration analysis are vibration meters, Data collectors, Frequency Domain Analyzers, Time domain instruments, Tracking analyzers. Out-off these instruments Time Domain instruments will provide the time domain display of the vibration waveform. Whereas frequency domain analyzers are specialized instruments that emphasize the analysis of vibration signal. But these instruments are too expensive. In the present work, Time Domain instrument called vibrometer AVD – 80 is used. The time
domain refers to display or analysis of the vibration data as a function of time. The principal advantage of this format is that little or no data are lost prior to inspection. This allows for a great deal of detailed analysis.

3.19.1 Time-Waveform Analysis
Time-waveform analysis involves the visual inspection of the time-history of the vibration signal. The general nature of the vibration signal can be clearly seen and distinctions made between sinusoidal, random, repetitive, and transient events. Non-steady-state conditions, such as run-up and coast-down, are most easily captured and analyzed using time waveforms. High-speed sampling can reveal such defects as broken gear teeth and cracked bearing races, but can also result in extremely large amounts of data being collected — much of which is likely to be redundant and of little use.

3.19.2 Time-Waveform Indices
A time-waveform index is a single number calculated in some way based on the raw vibration signal and used for trending and comparisons. These indices significantly reduce the amount of data that is presented for inspection, but highlight differences between samples. Examples of time-waveform-based indices include the peak level (maximum vibration amplitude within a given time signal), mean level (average vibration amplitude), root-mean-square (RMS) level (peak level= pk-pk); reduces the effect of spurious peaks caused by noise or transient events), and peak-to-peak amplitude (maximum positive to maximum negative signal amplitudes). All of these measures are affected adversely when more than one machinery component contributes to the measured signal. The crest factor is the ratio of the peak level to the RMS level δ peak level=RMS level b; and indicates the early stages of rolling-element-bearing failure. However, the crest factor decreases with progressive failure because the RMS level generally increases with progressive failure.

3.19.3 Time-Synchronous Averaging
Averaging of the vibration signal synchronous with the running speed of the machinery being monitored is called time-synchronous averaging. When taken over many machine cycles, this technique removes background noise and non-synchronous events (random transients) from the vibration signal. This technique is extremely useful where multiple shafts those are operating at only slightly different speeds and in close proximity to one another are being monitored. A reference signal (usually from a tachometer) is always needed.
3.19.4 Negative Averaging

Negative averaging works in the opposite way to time-synchronous averaging. Rather than averaging all the collected data, a baseline signal is recorded and then subtracted from all subsequent signals to reveal changes and transients only. This type of signal processing is useful on equipment or components that are isolated from other sources of vibrations.

SUMMARY

Definition and types of bearings, classification of journal bearings are presented. Mechanism of pressure development and hydrodynamic action in Hydrodynamic Journal Bearing is explained. The Reynolds Equation, which is the basic equation for the analysis of bearing characteristics, Short Bearing, Long Bearing and Finite Bearing analysis are explained. To study the Spiral Journal bearing performance, Reynolds Equation is derived for journal bearing operating in various boundary conditions. The theory and the procedure for the development of the design equations for spiral journal bearings operating under various conditions and the solution methodology for each condition are explained in detail in the next Chapter.
CHAPTER 4

STATIC PERFORMANCE CHARACTERISTICS OF HYDRODYNAMIC SPIRAL JOURNAL BEARING

4.0 INTRODUCTION

Developmental activities associated with Bearings have been widely presented in the earlier chapters. In this chapter, a methodology for obtaining the Reynolds equation for spiral contour is explained in detail. The methodology consists of the analysis of static characteristics of a Spiral bearing in different conditions, which are explained in detail in the following. The derivation of Short Journal bearing and long journal bearing from the Reynolds equation with Full Sommerfeld and Half Sommerfeld boundary conditions has been discussed in Chapter-3. For Circular bearings the boundary conditions are suffice, but for non-circular bearings the other type of boundary conditions which has been already discussed in Chapter -2 are utilized in this Chapter.

4.1 SOLUTION METHODOLOGY

The review of literature reveals that for a finite bearing, the two dimensional Reynolds equation is a partial differential equation. Generally, that equation is solved using (i) Finite Difference Method, (ii) Finite Element Method and (iii) Fourier series technique. Such techniques are cumbersome and require a large computer coding. Therefore, a MATLAB code is written for evaluation of the differential equation.

In the process of design of machine elements such as bearings etc., the designer always requires a simple equation rather than a complex one for the primary design. Therefore, the rapid methods available in literature are commonly used for the design of journal bearings. Such rapid methods are broadly divided into two categories: (1) analytical methods, using certain simplifications in Reynolds Equation, such as infinitely long- and short-bearing approximations or modified forms of these approximations and, (2) methods employing algebraic equations obtained by curve fitting data based on a large number of solutions of Reynolds equation under different operating conditions. The curve fitting equations are simple to implement using computer codes, but are statistical in nature and do not provide a feel of the real problem. Keeping this in view, in the present research work, an