CHAPTER 3

THREE PHASE INDUCTION MOTOR MODEL

3.1 INTRODUCTION

Electrical machines play an important role in many types of industrial process. Among all the types of AC machines, induction machines especially cage type are commonly used in industries, since they are economical, rugged, reliable and are available in the range of fractional horse power to multi mega watt capacity. There are two types of constructions generally used, one is squirrel cage rotor type and another is wound-rotor motor. In wound-rotor machine, rotor has three-phase distributed winding similar to that of stator, but in cage rotor type, instead of windings, thick conductor bars are arranged (axially) in a cage like structure (Bose 2006).

In this work a three-phase squirrel cage induction motor has been considered for carrying out simulation study. Three-phase induction motor is a nonlinear system with very fast dynamics. The Figure 3.1 shows the cross section of a three-phase squirrel cage induction motor, where each phase winding in the stator is represented by a concentrated coil and are displaced by 120°. The windings are distributed sinusoidally and embedded in the slots. The air gap in the machine is practically uniform (non-salient pole). Rotor bars are made up of copper or aluminium which are uniformly distributed and embedded in ferromagnetic material with bars terminating in common ring at each end of rotor. More often these conductors are slightly skewed to reduce magnitudes of harmonic torques (Krause et al 2002).
3.2 EQUIVALENT CIRCUIT

A simple per phase equivalent circuit model is an important tool for analysis and performance prediction at steady state condition. The Figure 3.2 shows the simplified per phase equivalent model referred to stator side in which the core loss resistance is neglected. This assumption is valid for steady state performance prediction as deviation lies within 5% from that of true model (Bose 2006). Here $v_s$ is the stator supply voltage and $I_r'$ is stator equivalent of rotor currents. It should be noted that the equivalent parameter can be obtained using stator to rotor turns ratio. $R_s$ and $L_{ls}$ represent per phase stator resistance and stator leakage reactance respectively. Similarly $R_r'$ and $L_{lr}'$ denote per phase rotor resistance and leakage reactance respectively, referred to stator side. $L_m$ is the magnetizing inductance representing the core. ‘s’ is the slip of induction motor in per unit value. Electromagnetic torque production in induction motor is due to the interaction of air gap flux and
rotor mmf (Magneto motive force) and can be represented by the Equation (3.1).

\[ T_e = \frac{3P}{2} \hat{\psi} \hat{I}_r \sin \delta \]  

(3.1)

where \( P \) is the no of pole pairs, \( \hat{\psi}_r \) is the peak value of air gap flux linkage per pole, \( \hat{I}_r \) is the peak value of rotor current and \( \delta \) represents the angle between the air gap flux linkage and rotor current phasors.

![Simplified equivalent circuit of three-phase induction motor referred to stator](image)

Figure 3.2 Simplified equivalent circuit of three-phase induction motor referred to stator

3.3 DYNAMIC MODELLING OF INDUCTION MOTOR

As mentioned earlier an equivalent circuit model is only useful for studying steady state performance of induction motor, implying that all electrical transients are neglected during load changes and stator frequency changes. Such a variation arises in applications involving variable speed drives (Krishnan 2001). A good model should incorporate all the important effects which should capture both steady state and dynamic behavior of the machine. The dynamic model should be a good approximation of the real machine.
3.3.1 Two phase machine (d-q Model)

The dynamic behavior of an ac machine is somewhat complex because the three-phase rotor windings move with respect to stationary three phase stator windings. The machine model can be described by differential equations with time varying mutual inductances, but such a model tends to be very complex (Bose 2006).

Dynamic model of a three-phase induction motor can be obtained using an equivalent two phase motor in direct and quadrature axis represented by only two sets of winding one for stator and other for rotor as shown in Figure 3.3. \( d^s \) and \( q^s \) represents the stator direct and quadrature axis windings respectively while \( d^r \) and \( q^r \) denotes the direct and quadrature axis rotor winding. The rotor rottes at an angular velocity of \( \omega_r \). This conceptually simplifies an n phase machine into two phase machine by power balance.

![Figure 3.3 Two phase machine (d-q axis) representation](image)

In order to eliminate time varying inductance problem arising in differential equation model, it is necessary to refer the machine variables namely voltages, currents and flux linkages to a common reference frames. A
change of variables that formulates a transformation of the 3-phase variables of stationary circuit elements to arbitrary reference frame may be expressed (Krause et al 2002) as follows

\[ f_{qd0s} = K_s f_{abcs} \]  \hspace{1cm} (3.2)

where

\[
(f_{dq0s})^T = \begin{bmatrix} f_{qs} & f_{ds} & f_{0s} \end{bmatrix}^T
\]

\[
(f_{abcs})^T = \begin{bmatrix} f_{as} & f_{bs} & f_{cs} \end{bmatrix}^T
\]

the transformation matrix \( K_s \) is given as

\[
K_s = \frac{2}{3} \begin{bmatrix}
\cos \theta & \cos(\theta - \frac{2\pi}{3}) & \cos(\theta + \frac{2\pi}{3}) \\
\sin \theta & \sin(\theta - \frac{2\pi}{3}) & \sin(\theta + \frac{2\pi}{3}) \\
\frac{1}{2} & \frac{1}{2} & \frac{1}{2}
\end{bmatrix}
\]  \hspace{1cm} (3.3)

\[
\frac{d\omega}{dt} = \theta \]  \hspace{1cm} (3.4)

In the above equation \( \omega \) is the angular velocity and \( \theta \) is the angular displacement of the arbitrary reference frame. Most commonly used reference frames for representing the dynamic model of three-phase induction motor are

- Stationary reference frame \( (\omega = 0) \)
- Synchronously rotating reference frame \( (\omega = \omega_s, \text{synchronous velocity}) \)
- Rotor reference frame \( (\omega = \omega_r) \)
Figure 3.4 shows the dynamic equivalent circuit of three-phase squirrel cage induction motor (rotor short circuited) in equivalent two phase representation referred to an arbitrarily rotating reference frame.

![Dynamic d-q equivalent circuit of induction motor](image)

**Figure 3.4** Dynamic d-q equivalent circuit of induction motor in an arbitrary reference frame (a) q^c-axis circuit (b) d^c-axis circuit.

where, \(v_{ds}\) and \(v_{qs}\) are the direct and quadrature axis voltages; \(i_{ds}\) and \(i_{qs}\) are direct and quadrature axis stator currents; \(i_{dr}\) and \(i_{qr}\) are direct and quadrature axis rotor currents; \(\psi_{ds}\) and \(\psi_{qs}\) are direct and quadrature axis stator
flux linkages; $\psi_{dr}$ and $\psi_{qr}$ are direct and quadrature axis rotor flux linkages; $\omega_c$ is the angular velocity of the arbitrary reference frame.

The differential equation model in terms of voltages and currents can be represented in matrix form (Peter Vas 1998) as given in Equation (3.5).

\[
\begin{bmatrix}
    \mathbf{v}_{qs} \\
    \mathbf{v}_{ds} \\
    \mathbf{v}_{qr} \\
    \mathbf{v}_{dr}
\end{bmatrix} = \begin{bmatrix}
    R_s + pL_s & \omega_c L_s & pL_m & \omega_c L_m \\
    -\omega_c L_s & R_s + pL_s & -\omega_c L_m & pL_m \\
    pL_m & (\omega_c - \omega_r) L_m & R_r + pL_r & (\omega_c - \omega_r) L_r \\
    -(\omega_c - \omega_r) L_m & pL_m & -(\omega_c - \omega_r) L_r & R_r + pL_r
\end{bmatrix} \begin{bmatrix}
    \mathbf{i}_{qs} \\
    \mathbf{i}_{ds} \\
    \mathbf{i}_{qr} \\
    \mathbf{i}_{dr}
\end{bmatrix}
\]

where the operator $p = \frac{d}{dt}$

In this work the induction motor model considered is the stationary reference frame dynamic model. The d-q equivalent circuits and the corresponding voltage equations can be obtained by substituting $\omega_c = 0$ in the Figure 3.4 and Equation (3.5) respectively.

### 3.3.2 Dynamic state space model

The rotor flux oriented extended state space model represented in stationary reference frame (Barut et al 2007) has been used in our study. This is the generally employed model because of its application to rotor flux oriented vector control. This is because in rotor flux orientation, there is a natural decoupling existing between the direct axis component of the measured current and quadrature axis rotor flux and vice versa. This simplifies the model and control strategy. In stator flux or magnetizing flux oriented vector control this coupling exist and the decoupling methodology such as feed forward control have to be employed (Bose 2006).
The state space representation of a dynamic system in general can be represented by the following equation.

\[
\frac{d}{dt} X = F1[X] + BV_s
\]  

(3.6)

The dynamic state space model of the induction motor considered for simulation studies is given below with usual notations

\[
\begin{bmatrix}
i_{ds} \\
i_{qs} \\
\psi_{dr} \\
\psi_{qr} \\
\dot{\omega}_r
\end{bmatrix} = \begin{bmatrix}
\frac{R_s}{L_\sigma} + \frac{R_i^2L_m}{L_\sigma^2L_\sigma} & 0 & \frac{R_iL_m}{L_\sigma}\omega_r & \frac{L_mP\omega_r}{L_\sigma^2L_\sigma} & 0 \\
0 & -\left(\frac{R_s}{L_\sigma} + \frac{R_i^2L_m}{L_\sigma^2L_\sigma}\right) & -\frac{L_m\omega_r}{L_\sigma L_t} & -\frac{R_i^2L_m}{L_\sigma^2L_\sigma} & 0 \\
\frac{R_i}{L_r} & 0 & -\frac{R_i}{L_r} & -P\omega_r & 0 \\
0 & \frac{R_i}{L_r} & 0 & -\frac{R_i}{L_r} & 0 \\
\frac{-3P}{2J_L} L_m \psi_{qr} & \frac{-3P}{2J_L} L_m \psi_{dr} & 0 & 0 & 0
\end{bmatrix} \begin{bmatrix}
i_{ds} \\
i_{qs} \\
\psi_{dr} \\
\psi_{qr} \\
\omega_r
\end{bmatrix}
\]  

(3.7)
where $\sigma$ is the leakage factor, and $J_L$ represents the total inertia of the induction motor and

$$L_\sigma = \sigma L_s : \sigma = 1 - \left(\frac{L_m}{L_s L_r}\right)^2$$

The state vector is

$$X = \begin{bmatrix} i_{ds} & i_{qs} & \psi_{dr} & \psi_{qr} & \omega_r \end{bmatrix}^T$$

And the input vector

$$V = \begin{bmatrix} v_{ds} & v_{qs} & T_L \end{bmatrix}^T$$

The output equation is represented as follows

$$\begin{bmatrix} i_{ds} \\ i_{qs} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_{ds} \\ i_{qs} \\ \psi_{dr} \\ \psi_{qr} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \omega_m \end{bmatrix}$$

(Equation 3.8)

Equations (3.7) and (3.8) are used to generate true state and measured variables for simulation studies.

The Equation (3.9) represents the augmented model of a three-phase induction motor and is used to carry out one step ahead prediction of the state variables in the nonlinear Kalman filters (Barut et al 2007). It should be noted that the load torque has been considered as an additional state variable in order to generate unbiased state estimates. The state vector for this model,

$$X = \begin{bmatrix} i_{ds} & i_{qs} & \psi_{dr} & \psi_{qr} & \omega_r & T_L \end{bmatrix}^T$$

and input vector,

$$V = \begin{bmatrix} v_{ds} & v_{qs} \end{bmatrix}^T$$
The model equation is given as

\[
\begin{bmatrix}
\dot{i}_{ds} \\
i_{qs} \\
\dot{\psi}_{dr} \\
\dot{\psi}_{qr} \\
\dot{\omega}_r \\
\dot{T}_L
\end{bmatrix} =
\begin{bmatrix}
\left(-\frac{R_s + R_m^2}{L_\alpha} - \frac{R_m^2}{L_t^2 L_\gamma}\right) & 0 & \frac{R_m^2}{L_t^2 L_\gamma} & L_m P_\Omega & L_m P_\Omega & 0 \\
0 & -\left(\frac{R_s + R_m^2}{L_\alpha} - \frac{R_m^2}{L_t^2 L_\gamma}\right) & \frac{L_m P_\Omega}{L_t L_t} & \frac{L_m P_\Omega}{L_t L_t} & \frac{R_m^2}{L_t^2 L_\gamma} & 0 \\
R \frac{L_m}{L_t} & 0 & -R & -P_\Omega & 0 & 0 \\
0 & R \frac{L_m}{L_t} & -P_\Omega & -R & 0 & 0 \\
-\frac{3P L_m}{2L_t \psi_{qr}} & -\frac{3P L_m}{2L_t \psi_{dr}} & 0 & 0 & 0 & \frac{1}{J_L} \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
i_{ds} \\
i_{qs} \\
\psi_{dr} \\
\psi_{qr} \\
\omega_r \\
T_L
\end{bmatrix}
+ \begin{bmatrix}
\frac{1}{L_\gamma} & 0 \\
0 & \frac{1}{L_\gamma} \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
v_{ds} \\
v_{qs}
\end{bmatrix}
\]

(3.9)

A further extended model is used for combined state and parameter estimation using Joint Extended Kalman filter and Joint Unscented Kalman filter. In the specific case of broken rotor bar detection rotor resistance is treated as the parameter to be estimated and the extended model is given by Equation (3.10).

Here the state vector, \( X = \begin{bmatrix} i_{ds} & i_{qs} & \psi_{dr} & \psi_{qr} & \omega_r & T_L & R_r \end{bmatrix}^T \) and input vector, \( V = \begin{bmatrix} v_{ds} & v_{qs} \end{bmatrix}^T \)
3.4 CONCLUSION

The basic aspects of modeling a three-phase induction motor and its dynamic state space models used in this work for various filters have been highlighted.