CHAPTER 3

DESIGN OF MULTIVARIABLE CONTROLLERS FOR THE IDEAL CSTR USING CONVENTIONAL TECHNIQUES

3.1 INTRODUCTION

PID controllers have been used widely in the industry due to the fact that they have simple structures and they assure acceptable performance for the majority of the industrial processes. Because of their simple structures, PID controllers are easy to design, operate and maintain. Consequently, PID controllers earn their popularity among practitioners in the industry. Beginning with Zeigler and Nichols work, various parameter tuning methods for conventional PID controllers have been proposed. On the other hand, controlling MIMO systems is not straightforward due to the interactions between the channels. The interactive multivariable systems can be controlled by either of the following:

a) a multivariable or centralized MIMO controller or

b) a set of SISO controllers

When the controlled process is nonlinear, a fixed gain PID controller cannot produce satisfactory control performance in all the operating regions. If a fixed gain controller is utilized, the closed loop performance could degrade even to the point of process instability. The proposed control technique uses a linear controller to control a nonlinear system using gain scheduling approach. Gain scheduling means that the tuned parameters of the
controller at each operating point are collected in the form of a table; the global controller observes the state of the process and chooses proper parameters from the table.

The following sections present the multivariable control of an ideal CSTR using decentralized and decoupling control schemes.

3.2 DECENTRALIZED CONTROL

Among the various multivariable controllers for MIMO systems, the decentralized PID controllers have been deployed extensively due to their less complexity, high performance and easy implementation. The design of multiple single loop controllers for multivariable systems proceeds in two stages:

1. Judicious choice of loop pairing
2. Controller tuning for each individual loop.

The main obstacle for proper controller tuning using decentralized control scheme is the interactions that exist between the control loops of the multi-loop system.

3.2.1 Design of Decentralized Controllers

In this section, the decentralized controllers are designed for the local linear models at the three chosen operating points. Figure 3.1 shows the block diagram representation of decentralized control of an ideal CSTR. The manipulated variables are the feed flow rate \( u_1 \) and coolant flow rate \( u_2 \). The outputs are the effluent concentration \( y_1 \) and reactor temperature \( y_2 \). The controller parameters of the two loops such as \( g_{c1} \) and \( g_{c2} \) are obtained using IMC based design procedure.
The open loop input-output relationship can be written in the matrix form as

\[
\begin{bmatrix}
  y_1 \\
  y_2
\end{bmatrix} =
\begin{bmatrix}
  g_{11} & g_{12} \\
  g_{21} & g_{22}
\end{bmatrix}
\begin{bmatrix}
  u_1 \\
  u_2
\end{bmatrix}
\]

(3.1)

The transfer function of the PI controller is given by

\[
G_c(s) = K_c + K_i \frac{1}{s}
\]

(3.2)

where \( K_c \) is the proportional gain and \( K_i \) is the Integral gain.

In order to design the decentralized controller, the appropriate pairings among inputs and outputs are chosen using Relative Gain Array (RGA) analysis to weaken the interactions between them.
3.2.2 Relative Gain Array (RGA)

The first step in designing the decentralized controllers is to select the input-output variable pairing, i.e., which output should be paired with which input (Ogunnaike and Ray 1994). It is determined with the help of relative gain array.

The gain matrix for a two input- two output system is obtained as,

\[
G(0) = \begin{bmatrix}
g_{11}(0) & g_{12}(0) \\
g_{21}(0) & g_{22}(0)
\end{bmatrix} = \begin{bmatrix}
k_{11} & k_{12} \\
k_{21} & k_{22}
\end{bmatrix}
\]  

(3.3)

The relative gain array is the matrix that contains the individual relative gain as elements \( \lambda_{ij} \) such that

\[
\lambda_{ij} = \frac{\text{gain between input } j \text{ and output } i \text{ with all other loops open}}{\text{gain between input } j \text{ and output } i \text{ with all other loops closed}}
\]

(3.4)

The RGA matrix obtained using the gain matrix is

\[
\Lambda = \begin{bmatrix}
k_{11}k_{22} & -k_{12}k_{21} \\
k_{11}k_{22}-k_{12}k_{21} & k_{11}k_{22}-k_{12}k_{21}
\end{bmatrix}
\]

(3.5)

The RGA matrix obtained at each of the three operating points is given in Table 3.1. It is found from Table 3.1 that \( \lambda_{11} \) is negative at all the three operating points. Since loops should not be formed with negative relative gains, \( y_1 \) should be paired with \( u_2 \) and \( y_2 \) with \( u_1 \).
Table 3.1 RGA matrices at the three operating points

<table>
<thead>
<tr>
<th>Operating point</th>
<th>RGA</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q=103, q_c=97, C_A=0.0748, T=445.3$</td>
<td>$\begin{bmatrix} -2.0871 &amp; 3.0871 \ 3.0871 &amp; -2.0871 \end{bmatrix}$</td>
</tr>
<tr>
<td>$q=100, q_c=100, C_A=0.0882, T=441.2$</td>
<td>$\begin{bmatrix} -2.2722 &amp; 3.2722 \ 3.2722 &amp; -2.2722 \end{bmatrix}$</td>
</tr>
<tr>
<td>$q=97, q_c=103, C_A=0.1055, T=436.8$</td>
<td>$\begin{bmatrix} -2.5998 &amp; 3.5998 \ 3.5998 &amp; -2.5998 \end{bmatrix}$</td>
</tr>
</tbody>
</table>

Accordingly, in this work, the effluent concentration is controlled based on the coolant flow rate and the reactor temperature is controlled by feed flow rate as directed by the RGA matrix. The linear models at the three operating points are determined by the linearization procedure explained in section 2.3. It is found that the transfer function of the effluent concentration versus the coolant flow rate is in the form

$$\frac{k_p}{\tau^2 s^2 + 2\zeta \tau s + 1}$$

(3.6)

For tuning the PI controller parameters IMC based tuning approach is taken up and the procedure proposed by Bequette (2003) yields the following controller parameters:

Proportional gain $K_c = \frac{2\zeta \tau}{k_p \lambda}$ and

Integral time constant is $\tau_I = 2\zeta \tau$.

The transfer function for the reactor temperature versus the feed flow rate is in the form

$$\frac{k_p (-\beta s + 1)}{\tau^2 s^2 + 2\zeta \tau s + 1}$$

(3.7)
The IMC based tuning procedure yields the following controller parameters:

\[ K_c = \frac{2\zeta T}{k_p (\beta + \lambda)} \]

and

\[ \tau_i = 2\zeta T \]

The PI controller parameters at the three operating points are presented in Tables 3.2 and 3.3. It should be noted that the controller gain has been found to be a function of the filter parameter \( \lambda \). The filter parameter values are chosen by trial and error method and the selection of the appropriate filter parameter value is decided by the performance index measured in the closed loop.

**Table 3.2  IMC based PI controller settings for effluent concentration control using decentralized control scheme**

<table>
<thead>
<tr>
<th>Operating point</th>
<th>( K_c )</th>
<th>( K_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>q=103, ( q_c = 97 ), ( C_A = 0.0748, T = 445.3 )</td>
<td>44.8769</td>
<td>128.4768</td>
</tr>
<tr>
<td>q=100, ( q_c = 100 ), ( C_A = 0.0882, T = 441.2 )</td>
<td>30.9472</td>
<td>103.7452</td>
</tr>
<tr>
<td>q=97, ( q_c = 103 ), ( C_A = 0.1055, T = 436.8 )</td>
<td>17.7596</td>
<td>80.8726</td>
</tr>
</tbody>
</table>

**Table 3.3  IMC based PI controller settings for reactor temperature control using decentralized control scheme**

<table>
<thead>
<tr>
<th>Operating point</th>
<th>( K_c )</th>
<th>( K_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>q=103, ( q_c = 97 ), ( C_A = 0.0748, T = 445.3 )</td>
<td>0.1908</td>
<td>0.5461</td>
</tr>
<tr>
<td>q=100, ( q_c = 100 ), ( C_A = 0.0882, T = 441.2 )</td>
<td>0.1136</td>
<td>0.3807</td>
</tr>
<tr>
<td>q=97, ( q_c = 103 ), ( C_A = 0.1055, T = 436.8 )</td>
<td>0.0304</td>
<td>0.1385</td>
</tr>
</tbody>
</table>
3.2.3 Gain Scheduled Control

Gain scheduling is a common practice used to control nonlinear plants in a variety of engineering applications. Bequette (2003) has outlined the traditional gain scheduled process control. A typical gain scheduled design procedure for nonlinear plants is as follows:

1. The designer selects several operating points which span the range of operation of the process.
2. At each of these operating points, the designer constructs a linear time-invariant approximation of the plant and designs a linear controller for the linearized plant model.

In between the operating points, the parameters or gains of the controllers are then interpolated, or scheduled, thus resulting in a global controller applicable to the whole range of operation.

3.2.3.1 Gain scheduler design (decentralized control)

The gain scheduler output is based on multiple local linear controllers. To combine multiple local linear PI controller outputs, a method for partitioning the operating space is to be devised. The choice of variables to be used to characterize the operating regimes are highly problem dependent. It is observed that the dynamic behaviour of the CSTR system changes significantly depending upon the operating point. The controllers based on the different operating points are listed in Tables 3.2 and 3.3 and the controllers are switched between the three operating regions to provide control over the entire operating region.
The closed loop simulation studies are carried out on the ideal CSTR using gain scheduled decentralized control using IMC tuned PI control settings. In order to assess the tracking capability of the designed PI controllers, set point variations in effluent concentration as given in Figure 3.2 (a) and the set point variations in the reactor temperature as given in the Figure 3.3 (a) have been introduced. The set point for effluent concentration is changed from 0.0882 mol/l to 0.0748 mol/l, 0.0748 mol/l to 0.0882 mol/l, and 0.0882 mol/l to 0.1105 mol/l. The set point for reactor temperature is changed from 441.2 K to 445.3 K, 445.3 K to 441.2 K and 441.2 K to 436.8 K. The variation in the controller outputs are presented in Figures 3.2(b) and 3.3(b). From the responses it can be inferred that the gain scheduled decentralized control scheme using IMC tuned PI controllers designed for the ideal CSTR are able to maintain the variables concentration and the reactor temperature at the desired set points.

The disturbance rejection capabilities of the gain scheduling of IMC tuned decentralized controllers for the ideal CSTR have been analyzed in the presence of a step change in the feed temperature. A positive and a negative step change in the feed temperature of magnitude 1 K has been introduced at time 50 min and 100 min respectively. The variations in effluent concentration are given in Figure 3.4(a) and the variations in the reactor temperature are given in the Figure 3.5(a). The variation in the controller outputs are presented in Figures 3.4(b) and 3.5(b). From the responses it can be inferred that the designed gain scheduled decentralized control scheme using IMC tuned PI controllers is capable of load disturbance rejection for the ideal CSTR.
Figure 3.2 Servo response for effluent concentration control using decentralized control scheme

Figure 3.3 Servo response for reactor temperature control using decentralized control scheme
Figure 3.4  Regulatory response for effluent concentration control using decentralized control scheme

Figure 3.5  Regulatory response for reactor temperature control using decentralized control scheme
In the following section the performance of the controllers are still further improved by considering the interactions among the loops.

### 3.3 DESIGN OF DECOUPLING CONTROLLERS

Figure 3.6 Block diagram representation of decoupled control scheme for the multivariable control of CSTR. The manipulated variables are the feed flow rate ($u_1$) and coolant flow rate ($u_2$). The outputs are the effluent concentration ($y_1$) and reactor temperature ($y_2$). In this work, the decouplers are designed using static decoupling method, to reduce the interaction brought in by cross coupling. It consists of two steps: first, to design the decouplers and second, to design the controllers for decoupled systems.

The static decouplers are designed as follows:

\[
g_{11} = \frac{-g_{12}(0)}{g_{11}(0)} \tag{3.8}
\]

\[
g_{12} = \frac{-g_{21}(0)}{g_{22}(0)} \tag{3.9}
\]
In the presence of decouplers, the multivariable system behaves like two independent loops, for which the controllers can be designed independently. The static decouplers at the three operating points are given in Table 3.4.

### Table 3.4 Static decouplers at the three operating points

<table>
<thead>
<tr>
<th>Operating point</th>
<th>$g_{11}$</th>
<th>$g_{12}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q=103,q_c=97,C_A=0.0748, T=445.3$</td>
<td>1.8577</td>
<td>0.7962</td>
</tr>
<tr>
<td>$q=100,q_c=100,C_A=0.0882, T=441.2$</td>
<td>1.7499</td>
<td>0.8229</td>
</tr>
<tr>
<td>$q=97,q_c=103,C_A=0.1055, T=436.8$</td>
<td>1.6416</td>
<td>0.8434</td>
</tr>
</tbody>
</table>

The equations for the two loops of a 2 x 2 multivariable system including the decouplers are given by

\[
y_1 = \left( g_{11} - \frac{g_{12} g_{21}}{g_{22}} \right) v_1 \tag{3.10}
\]

\[
y_2 = \left( g_{22} - \frac{g_{12} g_{21}}{g_{11}} \right) v_2 \tag{3.11}
\]

Substituting the values of $g_{ij}$ in equations (3.10) and (3.11), the controllers are designed using IMC based tuning method. The PI controller parameters obtained at different operating points are given in Tables 3.5 and 3.6.
Table 3.5  IMC based PI controller settings for effluent concentration control using decoupled control scheme

<table>
<thead>
<tr>
<th>Operating point</th>
<th>$K_C$</th>
<th>$K_I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>q=103, $q_c=97$, $C_A=0.0748$, $T=445.3$</td>
<td>123.4212</td>
<td>354.339</td>
</tr>
<tr>
<td>q=100, $q_c=100$, $C_A=0.0882$, $T=441.2$</td>
<td>137.1404</td>
<td>459.5857</td>
</tr>
<tr>
<td>q=97, $q_c=103$, $C_A=0.1055$, $T=436.8$</td>
<td>154.2477</td>
<td>702.083</td>
</tr>
</tbody>
</table>

Table 3.6  IMC based PI controller settings for reactor temperature control using decoupled control scheme

<table>
<thead>
<tr>
<th>Operating point</th>
<th>$K_C$</th>
<th>$K_I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>q=103, $q_c=97$, $C_A=0.0748$, $T=445.3$</td>
<td>1.8359</td>
<td>5.2559</td>
</tr>
<tr>
<td>q=100, $q_c=100$, $C_A=0.0882$, $T=441.2$</td>
<td>1.359</td>
<td>4.487</td>
</tr>
<tr>
<td>q=97, $q_c=103$, $C_A=0.1055$, $T=436.8$</td>
<td>0.808</td>
<td>3.678</td>
</tr>
</tbody>
</table>

3.3.1  Gain Scheduler Design (Decoupled Control)

In this sub-section, the design of gain scheduled controller settings at different operating points to provide a complete control over the entire operating region of the process is discussed. The scheduler considered is the set point. Similar to the decentralized control, the responses are obtained by switching over the three operating points. The following section deals with the simulation results obtained by using decoupled control scheme.
The closed loop simulation studies are carried out using gain scheduled decoupled controller using IMC tuned PI controllers. In order to assess the tracking capability of the designed PI controllers, set point variations in effluent concentration as given in Figure 3.7(a) and the set point variations in the reactor temperature as given in the Figure 3.8 (a) have been introduced. The set point for effluent concentration is changed from 0.0882 mol/l to 0.0748mol/l, 0.0748mol/l to 0.0882mol/l and 0.0882 mol/l to 0.1105 mol/l. The set point for reactor temperature is changed from 441.2 K to 445.3 K, 445.3 K to 441.2 K and 441.2 K to 436.8 K. The variation in the controller outputs are presented in Figures 3.7(b) and 3.8(b). From the responses it can be inferred that the gain scheduled decoupled controllers using IMC tuned PI controllers designed for the ideal CSTR are able to maintain the variables concentration and the reactor temperature at the desired set points. The performance of the decoupled control scheme is better than the decentralized control scheme for the ideal CSTR.

The disturbance rejection capabilities of the gain scheduled decoupled control scheme using IMC tuned PI controllers for the ideal CSTR have been analyzed in the presence of a step change in the feed temperature. A positive and a negative step change in the feed temperature of magnitude 1K has been introduced at time 50 min and 100 min respectively. The variations in effluent concentration are given in Figure 3.9(a) and the variations in the reactor temperature are given in the Figure 3.10(a). The variation in the controller outputs are presented in Figures 3.9(b) and 3.10(b). From the responses it can be inferred that the gain scheduled decoupled control scheme using IMC tuned PI controllers provides better results when compared to the gain scheduled decentralized control scheme using IMC tuned PI controllers for the ideal CSTR.
(a) Variations in effluent concentration

(b) Variations in coolant flow rate

Figure 3.7 Servo response for effluent concentration control using decoupled control scheme

(a) Variations in reactor temperature

(b) Variations in feed flow rate

Figure 3.8 Servo response for reactor temperature control using decoupled control scheme
Figure 3.9 Regulatory response for effluent concentration control using decoupled control scheme

Figure 3.10 Regulatory response for reactor temperature control using decoupled control scheme
In all the simulation runs, the CSTR is simulated using the nonlinear first principles model given in equations (2.4) and (2.5) and the state variables (effluent concentration and reactor temperature) are computed by solving the nonlinear differential equations using the differential equation solver in Matlab 7.0. Table 3.7 provides the performance measures for servo problem and Table 3.8 provides the performance measures for regulatory problem using decentralized control scheme. Table 3.9 provides the performance measures for servo problem and Table 3.10 provides the performance measures for regulatory problem using decoupled controllers. The comparison of the performance measures such as IAE and ISE values between the decentralized and decoupled control schemes show that the performance is better with the decoupled controllers than with the decentralized controllers.

**Table 3.7 Performance measures for servo problem using decentralized control scheme**

<table>
<thead>
<tr>
<th>Sl. No.</th>
<th>Time interval (min)</th>
<th>IAE Effluent Conc.</th>
<th>ISE Effluent Conc.</th>
<th>IAE Reactor Temp</th>
<th>ISE Reactor Temp</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>0-5</td>
<td>0.0011</td>
<td>5.583e-7</td>
<td>0.2161</td>
<td>0.0298</td>
</tr>
<tr>
<td>2.</td>
<td>5-50</td>
<td>0.0239</td>
<td>0.0001</td>
<td>6.497</td>
<td>11.97</td>
</tr>
<tr>
<td>3.</td>
<td>50-100</td>
<td>0.0475</td>
<td>0.0003</td>
<td>12.56</td>
<td>22.91</td>
</tr>
<tr>
<td>4.</td>
<td>100-150</td>
<td>0.1039</td>
<td>0.0007</td>
<td>34.64</td>
<td>52.82</td>
</tr>
</tbody>
</table>

**Table 3.8 Performance measures for regulatory problem using decentralized control scheme**

<table>
<thead>
<tr>
<th>Sl. No.</th>
<th>Time interval (min)</th>
<th>IAE Effluent Conc.</th>
<th>ISE Effluent Conc.</th>
<th>IAE Reactor Temp</th>
<th>ISE Reactor Temp</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>50-100</td>
<td>0.0121</td>
<td>1.362e-5</td>
<td>2.909</td>
<td>0.6551</td>
</tr>
<tr>
<td>2.</td>
<td>100-150</td>
<td>0.0228</td>
<td>2.756e-5</td>
<td>5.421</td>
<td>1.295</td>
</tr>
</tbody>
</table>
Table 3.9 Performance measures for servo problem using decoupled control scheme

<table>
<thead>
<tr>
<th>Sl. No.</th>
<th>Time interval (min)</th>
<th>IAE Effluent Conc.</th>
<th>ISE Effluent Conc.</th>
<th>IAE Reactor Temp</th>
<th>ISE Reactor Temp</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>0-5</td>
<td>0.0006</td>
<td>2.874e-7</td>
<td>0.132</td>
<td>0.0164</td>
</tr>
<tr>
<td>2.</td>
<td>5-50</td>
<td>0.0131</td>
<td>3.714e-5</td>
<td>1.585</td>
<td>2.186</td>
</tr>
<tr>
<td>3.</td>
<td>50-100</td>
<td>0.0286</td>
<td>9.312e-5</td>
<td>3.806</td>
<td>5.439</td>
</tr>
<tr>
<td>4.</td>
<td>100-150</td>
<td>0.0487</td>
<td>2.075e-4</td>
<td>7.958</td>
<td>11.8</td>
</tr>
</tbody>
</table>

Table 3.10 Performance measures for regulatory problem using decoupled control scheme

<table>
<thead>
<tr>
<th>Sl. No.</th>
<th>Time interval (min)</th>
<th>IAE Effluent Conc.</th>
<th>ISE Effluent Conc.</th>
<th>IAE Reactor Temp</th>
<th>ISE Reactor Temp</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>50-100</td>
<td>0.0024</td>
<td>1.285e-6</td>
<td>0.5045</td>
<td>0.1011</td>
</tr>
<tr>
<td>2.</td>
<td>100-150</td>
<td>0.0042</td>
<td>2.312e-6</td>
<td>0.8785</td>
<td>0.1867</td>
</tr>
</tbody>
</table>

3.4 CONCLUSION

The multivariable controllers for the ideal CSTR are obtained using decentralized control scheme and decoupled control scheme. For designing the controllers, pairing among the input-output variables is done using RGA analysis. From the RGA matrices obtained at the three chosen operating points, it is found that effluent concentration should be paired with coolant flow and reactor temperature should be paired with feed flow for the satisfactory performance of the controllers. The diagonal elements of the decentralized controllers are tuned using IMC based PID tuning procedure. The draw back in the IMC based tuning method of PID control is that the
effectiveness of the control action depends upon the selection of the tuning parameter $\lambda$, which is done by trial and error method. The controller parameters at the three operating points are switched by using a gain scheduler, forcing new controller parameters as it detects the modifications in the set point to provide control over the entire operating region. In order to improve the performance of the control loops the interactions in the process are considered and the design the multivariable control of CSTR using decoupling control scheme is carried out. The decouplers for the decoupling control are obtained using static decoupling control method. The PI controllers are obtained by IMC based tuning technique for the decoupled process at the chosen operating points. The control parameters at the three operating points are switched by using a gain scheduler to provide control over the entire operating region. The closed loop simulation studies are carried out to obtain the output responses and the corresponding IAE and ISE values. The qualitative and quantitative comparison of the closed loop simulation studies conducted on the ideal CSTR using decentralized and decoupled control schemes reveal that the decoupling control scheme provides better set point tracking and load disturbance rejection than the decentralized control scheme. In the following chapter an attempt is made to improve the performance of the controllers using soft computing techniques.