Chapter 4

Four component unmagnetized and magnetized dusty plasma containing nonthermal electrons

4.1 Introduction

Since the discovery of the dust-acoustic wave (DAW) (Rao et al. (1990)), there has been a great deal of interest in investigating numerous collective processes in dusty plasmas. Specifically, attention has been focused on waves and instabilities as well as coherent nonlinear wave structures in weakly coupled dusty plasmas. The history, occurrence and characterization of dusty plasma in space and laboratory environments are well described and documented in different monographs (Verheest (2000); Shukla and Mamun (2002)). In a weakly coupled dusty plasma, the presence of charged dust grains can modify the propagation of existing plasma wave spectra through the quasi-neutrality condition when the dust grains are stationary, whereas the dust dynamics provides the possibility of new wave modes such as dust-acoustic wave (Rao et al. (1990)), dust Alfven wave (Shukla (1992)) and dust-whistler wave (Shukla and Rahman (1996); Shukla (1999)) etc. Dust grains
are usually negatively charged as they collect electrons from the background plasma (Goertz (1989); Verheest (2000); Shukla and Mamun (2002); Vladimir and Ostrikov (2005)). The presence of positively charged dust particles has also been observed in different regions of space such as Cometary tails (Chow et al. (1993); Mendis and Rosenberg (1994); Mendis (2002); Sakanaka and Spassovska (2003); Mamun (2008)), Jupiter’s magnetosphere (Horanyi et al. (1993)), the earth’s polar mesosphere (Havnes et al. (2001)) and in the Martian atmosphere (Shukla and Rosenberg (2006)). There are different mechanisms by which dust grains become positively charged (Fortov et al. (1998a;1998b)). These are (i) photoemission in the presence of a flux of ultra-violet (UV) photons, (ii) thermionic emission induced by the radiative heating, and (iii) secondary emission of electrons from the surface of the dust grains. Wang et al. (2005) have studied the properties of positively and negatively charged dust particles. Rao et al. (1990) were the first to report the existence of dust-acoustic (DA) wave and these were verified experimentally by Barkan et al. (1995). On the other hand, dusty plasmas containing grains of opposite polarity have been investigated theoretically (Sakanaka and Shukla (2000); Angelo (2001); Angelo (2002); Mamun and Shukla (2002b); Sayed and Mamun (2007a); Shukla et al. (2007)) and experimentally (Horanyi et al. (1993); Mendis and Rosenberg (1994); Horanyi (1996); Angelo (2001); Mendis (2002)). Sayed and Mamun (2007a) studied the characteristics of small amplitude solitary wave structures in cold four-component dusty plasma containing both positively and negatively dust particles. On the other hand, Shukla et al. (2007) have reported a purely growing instability in positive-negative dusty plasma.

Observations of space plasmas and particle in cell simulation have confirmed that the particle distributions play a crucial role in characterizing the physics
of wave structures. To explain the observation of solitary wave structures with density depression, the role of nonthermal electron distribution on characterization of solitary wave/solitons have been reported (Cairns et al. (1995a;1995b)). Most of the investigations in dusty plasma containing opposite polarity dust particles have been studied with Maxwellian electron and ion distributions (Sakanaka and Shukla (2000); Shukla and Shorbagy (2005); Sayed and Mamun (2007a); El-Taibany et al. (2008)). The nonthermal distributions associated with the particle flows resulting from the force fields present in the space and astrophysical plasmas, have abundance of super thermal particles. Since the electron and ion distributions play important role for the formation of nonlinear structures, it is therefore, interesting to study the coherent nonlinear wave structures with non-Maxwellian distribution of electrons/ions. In this chapter, we have considered both unmagnetized and magnetized dusty plasma systems consisting of positively and negatively charged adiabatic dust particles with maxwellian ions and nonthermal electrons. It is noticed that the presence of opposite polarity dust components not only significantly modify the basic properties of the solitary potential structures, but also leads to existence of the positive and negative solitary potential structures in both unmagnetized and magnetized dusty plasmas.

In this chapter, the characteristics of dust-acoustic solitary waves (DASWs) in unmagnetized and magnetized four component dusty plasma are studied. The Korteweg-de Vries (KdV) equation which describes the basic features of the electrostatic solitary structures is derived using reductive perturbation method and solved for solitary wave solution for unmagnetized and magnetized four component dusty plasma. The effect of nonthermal electrons in unmagnetized and magnetized plasma and the effect of externally applied magnetic field are found to modify
significantly the properties of dust-acoustic solitary potential.

4.2 Governing equations for unmagnetized dusty plasma

An unmagnetized four component dusty plasma system consisting of positively and negatively charged dust particles, Boltzmann distributed ions and electrons with nonthermal distribution. The dynamics of dust-acoustic wave (DAWs) is governed by the normalized two-dust-fluid equations.

\[
\frac{\partial n_1}{\partial t} + \frac{\partial (n_1 u_1)}{\partial x} = 0 \quad (4.1)
\]

and

\[
\frac{\partial u_1}{\partial t} + u_1 \frac{\partial u_1}{\partial x} = \frac{\partial \phi}{\partial x} \quad (4.2)
\]

for negatively charged dust grain,

\[
\frac{\partial n_2}{\partial t} + \frac{\partial (n_2 u_2)}{\partial x} = 0 \quad (4.3)
\]

and

\[
\frac{\partial u_2}{\partial t} + u_2 \frac{\partial u_2}{\partial x} = -\tilde{\alpha} \beta_1 \frac{\partial \phi}{\partial x} \quad (4.4)
\]

for positively charged dust grain.

The system of equations is closed with poisson’s equation as follows:

\[
\frac{\partial^2 \phi}{\partial x^2} = n_1 - (1 + \mu_1' - \mu_2')n_2 + n_e - n_i \quad (4.5)
\]

The densities of electrons and ions are \( \mu_1'(1 - \beta \sigma \phi + \beta \sigma^2 \phi^2)\exp[\sigma \phi] \) and \( \mu_2' \exp[-\phi] \) respectively.

Here \( n_1(n_2) \) is the negative (positive) dust number density normalized by its equilibrium value \( n_{10}(n_{20}) \). \( u_1(u_2) \) is the negative (positive) dust fluid speed.
normalized by $c_1 = \sqrt{\frac{z_1 k_B T_i}{m_1}}$. $\phi$ is wave potential normalized by $\frac{k_B T_i}{c_1}$, $\tilde{\alpha} = \frac{z_2}{z_1}$, $\beta' = \frac{m_1}{m_2}$, $\mu'_e = \frac{n_{i0}}{z_1 n_{i10}}$, $\mu'_i = \frac{n_{e0}}{z_1 n_{e10}}$, $\sigma = \frac{T_i}{T_e}$. $z_1(z_2)$ is the number of electrons (protons) residing on a negative (positive) dust particle. $m_1(m_2)$ is the mass of the negative (positive) dust particle. $T_i(T_e)$ is the temperature of ions (electrons). $k_B$ is the Boltzmann constant and $e$ is the electronic charge. The time variable $t$ is normalized by $\omega_p^{-1} = (\frac{m_1}{4 \pi z_1^2 e^2 n_{i10}})^{\frac{1}{2}}$ and space variable $x$ is normalized by $\lambda_D = (\frac{z_1 k_B T_i}{4 \pi z_1^2 e^2 n_{i10}})^{\frac{1}{2}}$.

### 4.3 Derivation of Korteweg-de Vries (KdV) equation for unmagnetized system

Using reductive perturbation method we have derived the KdV equation. Stretching coordinates are $\xi = \epsilon^{1/2}(x - v_0 t)$ and $\tau = \epsilon^{3/2}t$, where $\epsilon$ is a small parameter measuring the weakness of dispersion and $v_0$ is the Mach number (the phase speed of the dust-acoustic (DA) waves normalized by $c_1$). Equations (4.1)-(4.5) can be expressed in terms of $\xi$ and $\tau$ as

\[
\epsilon^{3/2} \frac{\partial n_1}{\partial \tau} - v_0 \epsilon^{1/2} \frac{\partial n_1}{\partial \xi} + \epsilon^{1/2} \frac{\partial (n_1 u_1)}{\partial \xi} = 0 \tag{4.6}
\]

\[
\epsilon^{3/2} \frac{\partial u_1}{\partial \tau} - v_0 \epsilon^{1/2} \frac{\partial u_1}{\partial \xi} + \epsilon^{1/2} u_1 \frac{\partial u_1}{\partial \xi} = \epsilon^{1/2} \frac{\partial \phi}{\partial \xi} \tag{4.7}
\]

\[
\epsilon^{3/2} \frac{\partial n_2}{\partial \tau} - v_0 \epsilon^{1/2} \frac{\partial n_2}{\partial \xi} + \epsilon^{1/2} \frac{\partial (n_2 u_2)}{\partial \xi} = 0 \tag{4.8}
\]

\[
\epsilon^{3/2} \frac{\partial u_2}{\partial \tau} - v_0 \epsilon^{1/2} \frac{\partial u_2}{\partial \xi} + \epsilon^{1/2} u_2 \frac{\partial u_2}{\partial \xi} = -\tilde{\alpha} \epsilon^{1/2} \frac{\partial \phi}{\partial \xi} \tag{4.9}
\]

\[
\epsilon \frac{\partial^2 \phi}{\partial \xi^2} = n_1 - (1 + \mu'_e - \mu'_i) n_2 + \mu'_e(1 - \beta \sigma \phi + \beta \sigma^2 \phi^2) \exp[\sigma \phi] - \mu'_i \exp[-\phi] \tag{4.10}
\]

To strike an appropriate balance between nonlinearity and dispersion terms the dependent variables $n_1$, $n_2$, $u_1$, $u_2$ and $\phi$ are expanded about equilibrium values
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in the power of $\epsilon$ as

$$\begin{pmatrix} n_{1,2} \\ u_{1,2} \\ \phi \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \sum_{r=1}^{\infty} \epsilon^r \begin{pmatrix} n_{1,2}^r \\ u_{1,2}^r \\ \phi^r \end{pmatrix}$$

(4.11)

Now substituting equation (4.11) into equations (4.6)-(4.10) and considering the coefficient of $\epsilon^{3/2}$ from equations (4.6)-(4.9) and $\epsilon$ from (4.10); we get

$$n_1^1 = \frac{u_1^1}{v_0}$$

(4.12)

$$u_1^1 = -\frac{\phi_1^1}{v_0}$$

(4.13)

$$n_2^1 = \frac{u_2^1}{v_0}$$

(4.14)

$$u_2^1 = \frac{\tilde{\alpha}\beta_1^1\phi_1^1}{v_0}$$

(4.15)

$$n_1^1 - (1 + \mu_e^{'e} - \mu_i^{'i})n_2^1 + \mu_e^{'e}(\sigma - \beta)\phi_1^1 + \mu_i^{'i}\phi_1^1 = 0$$

(4.16)

now using equations (4.12)-(4.16), we have

$$n_1^1 = -\frac{\phi_1^1}{v_0}$$

(4.17)

$$n_2^1 = \frac{\tilde{\alpha}\beta_1^1\phi_1^1}{v_0}$$

(4.18)

Equation (4.18) is the linear dispersion relation for the DA waves propagating in unmagnetized dusty plasma. Equating the coefficient of $\epsilon^{5/2}$ from equations (4.6)-(4.9), $\epsilon^2$ from equation (4.10) and using equations (4.11)-(4.16); one obtains

$$\frac{\partial n_1^1}{\partial \tau} - v_0 \frac{\partial n_1^2}{\partial \xi} + \frac{\partial u_1^2}{\partial \xi} + \frac{\partial (n_1^1 u_1^1)}{\partial \xi} = 0$$

(4.19)

$$\frac{\partial u_1^1}{\partial \tau} - v_0 \frac{\partial u_1^2}{\partial \xi} + u_1^1 \frac{\partial u_1^1}{\partial \xi} = \frac{\partial \phi^2}{\partial \xi}$$

(4.20)
4.3. Derivation of Korteweg-de Vries (KdV) equation for unmagnetized system

\[
\frac{\partial n_1^2}{\partial \tau} - v_0 \frac{\partial n_2^2}{\partial \xi} + \frac{\partial u_2^2}{\partial \xi} + \frac{\partial (n_1^2 u_1^2)}{\partial \xi} = 0 \quad (4.21)
\]

\[
\frac{\partial u_1^1}{\partial \tau} - v_0 \frac{\partial u_2^2}{\partial \xi} + u_2^1 \frac{\partial u_1^1}{\partial \xi} = -\tilde{\alpha} \beta_1 \frac{\partial \phi^2}{\partial \xi} \quad (4.22)
\]

\[
\frac{\partial^2 \phi^1}{\partial \xi^2} = n_1^2 - (1 + \mu' - \mu_i) n_2^2 + \mu_i' (\sigma - \beta) \phi^2 + \mu_e' [\beta (1 - \sigma) + \sigma^2/2] (\phi^1)^2 + \mu_i' \phi^2 - \frac{\mu_i (\phi^1)^2}{2} \quad (4.23)
\]

Now, using equations (4.11),(4.18) and equations (4.19)-(4.23), and eliminating second order quantities i.e., \(n_1^2, n_2^2, u_1^2, u_2^2\) and \(\phi^2\); we finally obtain

\[
\frac{\partial \phi^1}{\partial \tau} + \tilde{A} \phi^1 \frac{\partial \phi^1}{\partial \xi} + B' \frac{\partial^3 \phi^1}{\partial \xi^3} = 0 \quad (4.24)
\]

where \(\tilde{A}'\) and \(B'\) are nonlinear and dispersion coefficients, which are given as

\[
\tilde{A}' = \frac{3 \tilde{\alpha}^2 \beta_1^2 (1 + \mu_e' - \mu_i') - 3 - 2 v_0^3 (\mu_e' \sigma^2 + 2 \beta \mu_e' (1 - \sigma) - \mu_i')}{2 v_0 (1 + \tilde{\alpha} \beta_1 (1 + \mu_e' - \mu_i'))} \quad (4.25)
\]

\[
B' = \frac{v_0^3}{2 [1 + \tilde{\alpha} \beta_1 (1 + \mu_e' - \mu_i')]} \quad (4.26)
\]

Equation (4.24) is the KdV equation describing the nonlinear propagation of the DA waves in unmagnetized four component dusty plasma with nonthermal electron distribution. The steady state solution of this KdV equation is obtained by transforming the independent variables \(\xi\) and \(\tau\) to \(\zeta = \xi - u_0 \tau\) and \(\tau = \tau\), where \(u_0\) is a constant velocity normalized by \(c_1\), and imposing the appropriate boundary conditions. Thus, one can express the steady state solution of the KdV equation as

\[
\phi^1 = \phi_m'' \text{sech}^2[\zeta/w], \quad (4.27)
\]

where the peak amplitude \(\phi_m''\) and the width \(w\) are given by:

\[
\phi_m'' = 3 u_0 / \tilde{A}' \quad (4.28)
\]

and

\[
w = 2 \sqrt{B'/u_0} \quad (4.29)
\]
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It is obvious from equations (4.28) and (4.29) that as $u_0$ increases; the amplitude (width) of the solitary waves increases (decreases). It is clear from equations (4.25), (4.27) and (4.28) that the solitary potential profile is positive (negative) if $\tilde{A} > (\leq) 0$. To find the parametric regimes for which positive and negative solitary potential profiles exist, we have numerically analyzed $\tilde{A}'$ and obtain $\tilde{A}' = 0$ ($\beta'_1$ versus $\mu'_i$) curves for $\sigma = 0.5$, $\beta = 0.3$, $\tilde{\alpha} = 0.01$, $\mu'_e = 0.1$ (solid curve), $\mu'_e = 0.15$ (dotted curve) and $\mu'_e = 0.2$ (dashed curve). The curves for $\tilde{A}' = 0$ are displayed in figure 4.1. It shows that we can have positive (negative) solitary potential profile for the parameters whose values lie above (below) $\tilde{A}' = 0$ curves. We have also shown graphically that how the amplitude, width and Mach number $v_0$ of the positive and negative solitary profiles vary with $\mu'_i$ and $\beta$. These are displayed in figure 4.2(a) to figure 4.4(b).
4.4. Discussion of unmagnetized dusty plasma

Figure 4.2: (a) Variation of peak amplitude $\phi_m''$ of positive solitary potential against $\mu_i'$ for different values of $\beta$ with $\sigma = 0.5$ and $\tilde{\alpha}_1 = 1.5$. (b) Variation of amplitude $\phi_m''$ of negative solitary potential against $\mu_i'$ for different values of $\beta$ with $\sigma = 0.5$ and $\tilde{\alpha}_1 = 0.1$. 
From figure 4.2(a), it depicted that the peak amplitude ($\phi''_m$) of positive solitary profiles increases with nonthermal parameter $\beta$ and $\mu'_i$ for the chosen set of parameters. We have also observed from figure 4.2(b), that the amplitude of negative solitary potential profile ($|\phi'_m|$) decreases with $\beta$ but increases with $\mu'_i$.

Figures 4.3(a) and 4.3(b), shows that the width ($w$) of both positive and negative solitary potential profiles decreases with $\mu'_i$ and increases with $\beta$. From numerical analysis, it is observed that $\beta$ has a significant effect on the variation of amplitude as well as width.

It is clear from figures 4.4(a) and 4.4(b) that the Mach number ($v_0$) of both positive and negative solitary potential profiles increases with $\beta$ but decreases with $\mu'_i$. The parameters that we have considered here are relevant to different regions of space environments (cometary tails, mesosphere, Jupiter’s magnetosphere, etc.). Finally, for $\beta = 0$, the results of the present investigation agree with Sayed and Mamun (2007a).

### 4.5 Governing equations for magnetized dusty plasma

We consider fully ionized collisionless dusty plasma consisting of warm adiabatic positively and negatively charged dust particles, Maxwellian ions, and nonthermally distributed electrons in an external static magnetic field $B = B_0\hat{z}$. The behavior of nonlinear wave structure in this plasma system is described by the following set of fluid equations:

\[
\frac{\partial n_j}{\partial t} + \nabla . (n_ju_j) = 0 \tag{4.30}
\]

\[
\frac{\partial u_1}{\partial t} + (u_1, \nabla)u_1 - \nabla \phi + \frac{\sigma_1}{n_1} \nabla p_1 + \omega_{c1}(u_1 \times B) = 0 \tag{4.31}
\]
Figure 4.3: (a) Variation of width $w$ of positive solitary potential profile against $\mu_i'$ for different values of $\beta$ with $\sigma = 0.5$ and $\tilde{\alpha}\beta'_1 = 1.5$. (b) Variation of width $w$ of negative solitary potential profile against $\mu_i'$ for different values of $\beta$ with $\sigma = 0.5$ and $\tilde{\alpha}\beta'_1 = 0.1$. 
Figure 4.4: (a) Variation of Mach number $v_0$ of positive solitary potential profile against $\mu'_i$ for different values of $\beta$ with $\sigma = 0.5$ and $\tilde{\alpha}\beta'_1 = 1.5$. (b) Variation of Mach number $v_0$ of negative solitary potential profile against $\mu'_i$ for different values of $\beta$ with $\sigma = 0.5$ and $\tilde{\alpha}\beta'_1 = 0.1$. 
\[
\frac{\partial u_2}{\partial t} + (u_2, \nabla) u_2 + \hat{\alpha} \beta_1' \nabla \phi + \frac{\sigma_2}{n_2} \nabla p_2 + \omega_c (u_2 \times B) = 0 \tag{4.32}
\]

The subscript \( j = 1(2) \) corresponds to negatively (positively) charged dust particles. The scaling parameters are same as that of unmagnetized plasma model. \( \sigma_1 = 1 \) and \( \sigma_2 = \frac{\delta_1 T_2}{T_1} \). The time variable \( t \) is normalized by inverse of plasma frequency, \( \omega_{pi}^{-1} = \sqrt{(m_1/4\pi n_{10} z_1^2 e^2)} \) and space variable \( \nabla \) is normalized by Debye length \( \lambda_D = \sqrt{(Z_1 k_B T_i/4\pi Z_1^2 n_{10} e^2)} \) respectively. \( \omega_{c1} \) and \( \omega_{c2} \) are the negative and positive charged dust cyclotron frequencies normalized to plasma frequency.

The un-normalized Poisson’s equation is given by

\[
\nabla^2 \phi = 4\pi e z_1 n_1 - 4\pi e z_2 n_2 + 4\pi e n_e - 4\pi e n_i \tag{4.33}
\]

Where the first term on the right hand side of the above equation is the total contribution due to negatively charged dust particles, second term is the contribution of positively charged dust particles, third term describes the nonthermally distributed electrons and last term represents the ion contribution due to Maxwellian distribution. We disturb the densities of various terms on the right hand side of the above equation into equilibrium values plus perturbed quantities and used the charge neutrality of the background plasma to obtain the poisson’s equation in normalized form as follow:

\[
\nabla^2 \phi = n_1 - (1 + \mu_e' - \mu_i') n_2 + \mu_e'(1 - \beta \sigma \phi + \beta \sigma^2 \phi^2) \exp[\sigma \phi] - \mu_i' \exp[-\phi] \tag{4.34}
\]

\[
p_j = C' n_j' \tag{4.35}
\]

\( C' \) in the above equation is a constant, \( \gamma' \) is the ratio of the specific heats at constant pressure \( c_p \) and constant volume \( c_v \).

It is worth mentioning that

\[
\gamma' = 1 + 2/F \tag{4.36}
\]
Here $F$ represents the degrees of freedom and for our model $\gamma' = 5/3$.

Gradient of $p_j$ is obtained from equation (4.35) and is given by

$$\nabla p_j = \frac{5}{3} \nabla n_j n_j^{2/3}$$

(4.37)

### 4.6 Derivation of Korteweg-de Vries (KdV) equation for magnetized system

In the small amplitude approximation we study ion acoustic solitary waves or dust-acoustic solitary waves by using reductive perturbation method. This method is basically a weakly nonlinear theory with small but finite amplitude, which leads to scaling of the independent variables through the stretching coordinates (Washimi and Taniuti (1966)).

$$\xi = \epsilon^{1/2}(l_x x + l_y y + l_z z - V_0 t) \quad \text{and} \quad \tau = \epsilon^{3/2} t$$

, where $\epsilon$ is a small parameter measuring the weakness of dispersion and $V_0$ is the solitary wave velocity. $l_x$, $l_y$ and $l_z$ are the direction cosines of the wave vector $k$ along the $x$, $y$ and $z$ axis respectively. In reductive perturbation method, we further expand the perturbed quantities about their equilibrium values in the powers of $\epsilon$ as follows:

$$n_j = 1 + \epsilon n_j^1 + \epsilon^2 n_j^2 + \epsilon^3 n_j^3 + \ldots,$$

$$u_{jx} = 0 + \epsilon^{3/2} u_{jx}^1 + \epsilon^2 u_{jx}^2 + \epsilon^{5/2} u_{jx}^3 + \ldots,$$

$$u_{jy} = 0 + \epsilon^{3/2} u_{jy}^1 + \epsilon^2 u_{jy}^2 + \epsilon^{5/2} u_{jy}^3 + \ldots,$$

$$u_{jz} = 0 + \epsilon u_{jz}^1 + \epsilon^2 u_{jz}^2 + \epsilon^3 u_{jz}^3 + \ldots,$$

$$\phi = 0 + \epsilon \phi^1 + \epsilon^2 \phi^2 + \epsilon^3 \phi^3 + \ldots,$$

(4.38)
Using stretched coordinates along with equation (4.38) in equations (4.30)-(4.32) and (4.34) yield the following equations to the lowest order in $\epsilon$ as:

\[ n_1^1 - (1 + \mu'_e - \mu'_i)n_1^2 + \mu'_e(\sigma - \beta)\phi^1 + \mu'_i\phi^1 = 0 \quad (4.39) \]

\[ n_1^1 = \frac{(\mu'_e(\sigma - \beta) + \mu'_i)\phi^1}{(1 + \tilde{\alpha}\beta'_1(1 + \mu'_e - \mu'_i))} \quad (4.40) \]

\[ n_2^1 = \frac{(\mu'_e(\sigma - \beta) + \mu'_i)\tilde{\alpha}\beta'_1\phi^1}{(1 + \tilde{\alpha}\beta'_1(1 + \mu'_e - \mu'_i))} \quad (4.41) \]

\[ u_{1z}^1 = \frac{l_z\phi^1}{(3l_z^3 - V_0)} \quad (4.42) \]

\[ u_{2z}^1 = -\frac{\tilde{\alpha}\beta'_1l_z\phi^1}{(3l_z^3 - V_0)} \quad (4.43) \]

\[ V_0 = M'l_z \quad (4.44) \]

\[ M' = \left(\frac{5}{3} + \frac{(1 + \tilde{\alpha}\beta'_1(1 + \mu'_e - \mu'_i))}{(\mu'_e(\sigma - \beta) + \mu'_i)}\right)^{1/2} \quad (4.45) \]

Equation (4.44) represents the dispersion relation for a given dusty plasma system.

Similarly the $x$ and $y$ components of the perturbed velocity from the momentum balance equation are given as follows:

\[ u_{1x}^1 = \left[ -\frac{l_y}{\omega_{c1}} + \frac{5l_y}{3\omega_{c1}(\frac{2}{3} - M'^2)} \right] \frac{\partial \phi^1}{\partial \xi} \quad (4.46) \]

\[ u_{2x}^1 = \left[ \frac{\tilde{\alpha}\beta'_1l_y}{\omega_{c2}} - \frac{5\tilde{\alpha}\beta'_1\sigma_2l_y}{3\omega_{c2}(\frac{2}{3} - M'^2)} \right] \frac{\partial \phi^1}{\partial \xi} \quad (4.47) \]

\[ u_{1y}^1 = \left[ \frac{l_x}{\omega_{c1}} - \frac{5l_x}{3\omega_{c1}(\frac{2}{3} - M'^2)} \right] \frac{\partial \phi^1}{\partial \xi} \quad (4.48) \]

\[ u_{2y}^1 = \left[ -\frac{\tilde{\alpha}\beta'_1l_x}{\omega_{c2}} + \frac{5\tilde{\alpha}\beta'_1\sigma_2l_x}{3\omega_{c2}(\frac{2}{3} - M'^2)} \right] \frac{\partial \phi^1}{\partial \xi} \quad (4.49) \]
Following the KdV equation:

Using the same procedure we obtain next higher order quantities given as follows:

\[
\begin{align*}
\frac{u_{1x}^2}{\omega_c^2} &= -\frac{V_0 l_x}{\omega_c^2} \left[ 1 - \frac{1}{(1 - \frac{3M^2}{5})} \right] \frac{\partial^2 \phi^1}{\partial \xi^2} \\
\frac{u_{2x}^2}{\omega_c^2} &= \frac{V_0 l_x \tilde{\alpha}_{1} \beta_1}{\omega_c^2} \left[ 1 - \frac{\sigma_2}{(1 - \frac{3M^2}{5})} \right] \frac{\partial^2 \phi^1}{\partial \xi^2} \\
\frac{u_{1y}^2}{\omega_c^2} &= \frac{V_0 l_y \tilde{\alpha}_{1} \beta_1}{\omega_c^2} \left[ -1 + \frac{1}{(1 - \frac{3M^2}{5})} \right] \frac{\partial^2 \phi^1}{\partial \xi^2} \\
\frac{u_{2y}^2}{\omega_c^2} &= \frac{V_0 l_y \tilde{\alpha}_{1} \beta_1}{\omega_c^2} \left[ 1 - \frac{\sigma_2}{(1 - \frac{3M^2}{5})} \right] \frac{\partial^2 \phi^1}{\partial \xi^2}
\end{align*}
\]

\[
(l_x^2 + l_y^2 + l_z^2) \frac{\partial^2 \phi^1}{\partial \xi^2} = n_1^2 - (1 + \mu_e - \mu_i) n_2^2 + (\mu_e (\sigma - \beta) + \mu_i) \phi^2 + [\mu_e ((1 - \sigma) \beta + \sigma^2/2) - \frac{\mu_i}{2}] (\phi^1)^2
\]

Using the same procedure we obtain next higher order quantities given as follows:

\[
\begin{align*}
-V_0 \frac{\partial n_{1x}^2}{\partial \xi} + \frac{\partial n_{1y}^2}{\partial \tau} + l_x \frac{\partial u_{1x}^2}{\partial \xi} + l_y \frac{\partial u_{1y}^2}{\partial \xi} + l_z \frac{\partial (u_{1x}^2 + n_{1y}^2)}{\partial \xi} &= 0 \\
-V_0 \frac{\partial n_{2x}^2}{\partial \xi} + \frac{\partial n_{1y}^2}{\partial \tau} + l_x \frac{\partial u_{2x}^2}{\partial \xi} + l_y \frac{\partial u_{2y}^2}{\partial \xi} + l_z \frac{\partial (u_{2x}^2 + n_{2y}^2)}{\partial \xi} &= 0 \\
-V_0 \frac{\partial u_{1z}^2}{\partial \xi} + \frac{\partial u_{1z}^2}{\partial \tau} + l_x u_{1z} \frac{\partial n_{1z}^2}{\partial \xi} = l_z \frac{\partial \phi^2}{\partial \xi} - \frac{5l_z}{3} \frac{\partial n_{1}^2}{\partial \xi} + \frac{5l_z}{9} \frac{n_{1}^4}{\partial \xi} \\
-V_0 \frac{\partial u_{2z}^2}{\partial \xi} + \frac{\partial u_{2z}^2}{\partial \tau} + l_x u_{2z} \frac{\partial n_{2z}^2}{\partial \xi} = -l_z \tilde{\alpha}_{1} \beta_1 \frac{\partial \phi^2}{\partial \xi} - \frac{5l_z}{3} \frac{n_{2}^2}{\partial \xi} + \frac{5l_z}{9} \frac{n_{2}^4}{\partial \xi}
\end{align*}
\]

Equations (4.39)–(4.58) can be used to eliminate \( \frac{\partial n_{1}^2}{\partial \xi}, \frac{\partial n_{2}^2}{\partial \xi}, \frac{\partial u_{1z}^2}{\partial \xi}, \frac{\partial u_{2z}^2}{\partial \xi}, \frac{\partial \phi^2}{\partial \xi} \) and following straightforward but long algebraic manipulations, we finally obtain the following KdV equation:

\[
\frac{\partial \phi^1}{\partial \tau} + a \phi^1 \frac{\partial \phi^1}{\partial \xi} + b \frac{\partial^2 \phi^1}{\partial \xi^2} = 0
\]
Coefficients $a$ and $b$ of nonlinear and dispersive terms appearing in equation (4.59) are given by following expressions

\[
\begin{align*}
a &= \frac{1}{C} \left[ \frac{1}{M'^2}(3 - \frac{5}{9}M'^2) \right] \left( \frac{1}{1 - \frac{5}{9}M'^2} \right) \left( 1 - \frac{\alpha^2\beta'^2}{1 + \alpha\beta'(1 + \mu_e - \mu_i)} \right) \\
&\quad + \frac{2\mu'_e((1 - \sigma)\beta + \frac{\sigma^2}{2}) - \mu_i'}{1 + \tilde{\alpha}\beta'} \left( 1 + \frac{\mu e^2(1 + \alpha e)}{\mu_e(1 + \mu_e - \mu_i)} \right) \\
\end{align*}
\]

\[ (4.60) \]

Further the expression for '$C$' is given by

\[
C = \frac{-2(\mu'_e(\sigma - \beta) + \mu'_i)^2 M'^2}{V_0(1 + \tilde{\alpha}\beta'(1 + \mu_e - \mu_i))} 
\]

On introducing new variable $\zeta = \xi - u_0\tau$, $u_0$ is a constant velocity, the soliton solution of equation (4.59) in the stationary frame is given as

\[
\phi^1 = \phi''_m \text{sech}^2[(\xi - u_0\tau) / w] 
\]

The quantities $\phi''_m$ and $w$ representing peak amplitude and width of the soliton are given as:

\[
\begin{align*}
\phi''_m &= 3u_0 / a \\
w &= \sqrt{4b/u_0} 
\end{align*}
\]

4.7 Discussion of magnetized dusty plasma

From equations (4.63)-(4.65) we find that as $u_0$ increases, peak amplitude (width) of the solitary waves for a given plasma system increases (decreases).
potential profile is positive (negative) if \( a > ( < 0 ) \) as obvious from the equations (4.59), (4.60) and (4.64). Numerical computation for analyzing the nature of nonlinear coefficient ‘\( a' \) is performed to obtain contour maps for ‘\( a' = 0 \). The parametric regimes for positive and negative solitary potential profiles have been traced out. Apparently the parameter regime is divided into two regions one for compressive solitons and other for rarefactive solitons. It may be noted that from equations (4.60) and (4.64) that the coefficient ‘\( a' \) of the nonlinearity remains positive for the positive solitary potential profile and correspondingly compressive solitons exist in the given plasma system. However, introduction of magnetic field leads to the appearance of compressive and rarefactive solitons. In figure 4.5(a) plots of peak value of amplitude of solitary waves as a function of \( \mu'_i \) for parameters such as \( \sigma = 0.5, \omega_{c1} = 0.02, \omega_{c2} = 0.025 \), are shown for three different values of nonthermal parameter \( \beta \). As obvious from figure 4.5(a), the amplitude of positive solitary potential profile is observed to decrease with the increase of nonthermal parameter but increases with \( \mu'_i \). On the contrary the magnitude of the amplitude (\( | \phi_m '' | \)) of negative solitary potential profile representing rarefactive solitons increases with \( \beta \) (nonthermal parameter) but decreases with \( \mu'_i \) as displayed in figure 4.5(b).

For the sake of completeness, we have also plotted corresponding width \( w \) as a function of \( \mu'_i \) as shown in figure 4.6(a) and figure 4.6(b). Width of both positive and negative potential profiles increases with increase in \( \beta \) but decreases with increase in \( \mu'_i \).

Figures 4.7(a) and 4.7(b) display the variation of Mach number \( V_0 \) of positive and negative solitary potential regimes. Obviously \( V_0 \) of both positive and negative potential profiles increases with increase in \( \beta \) but decreases with increase in \( \mu'_i \).
4.7. Discussion of magnetized dusty plasma

Figure 4.5: (a) Variation of peak amplitude $\phi_m''$ of positive solitary potential profile against $\mu_i'$ for different values of $\beta$ with $\sigma = 0.5$, $\sigma_1 = 1$, $\sigma_2 = 1.5$, $\omega_{c1} = 0.02$, $\omega_{c2} = 0.025$ and $\tilde{\alpha}\beta_1' = 1.5$. (b) Variation of peak amplitude $\phi_m''$ of negative solitary potential profile against $\mu_i'$ for different values of $\beta$ with $\sigma = 0.5$, $\sigma_1 = 1$, $\sigma_2 = 1.5$, $\omega_{c1} = 0.02$, $\omega_{c2} = 0.025$ and $\tilde{\alpha}\beta_1' = 0.1$. 
Chapter 4. Four component unmagnetized and magnetized dusty plasma containing nonthermal electrons

Figure 4.6: (a) Variation of width $w$ of positive solitary potential profile against $\mu'_i$ for different values of $\beta$ with $\sigma = 0.5$, $\sigma_1 = 1$, $\sigma_2 = 1.5$, $\omega_{c1} = 0.02$, $\omega_{c2} = 0.025$ and $\tilde{\alpha}_\beta' = 1.5$. (b) Variation of width $w$ of negative solitary potential profile against $\mu'_i$ for different values of $\beta$ with $\sigma = 0.5$, $\sigma_1 = 1$, $\sigma_2 = 1.5$, $\omega_{c1} = 0.02$, $\omega_{c2} = 0.025$ and $\tilde{\alpha}_\beta' = 0.1$. 
4.7. Discussion of magnetized dusty plasma

Figure 4.7: (a) Variation of Mach number $V_0$ of positive solitary potential profile against $\mu'_i$ for different values of $\beta$ with $\sigma = 0.5$, $\sigma_1 = 1$, $\sigma_2 = 1.5$, $\omega_{c1} = 0.02$, $\omega_{c2} = 0.025$ and $\tilde{\alpha}_1 = 1.5$. (b) Variation of Mach number $V_0$ of negative solitary potential profile against $\mu'_i$ for different values of $\beta$ with $\sigma = 0.5$, $\sigma_1 = 1$, $\sigma_2 = 1.5$, $\omega_{c1} = 0.02$, $\omega_{c2} = 0.025$ and $\tilde{\alpha}_1 = 0.1$. 
Chapter 4. Four component unmagnetized and magnetized dusty plasma containing nonthermal electrons

Sakanaka and Shukla (2000) observed that the compressive dust-acoustic potential exist only when there is a significant fraction of positively charged dust grains in unmagnetized plasma. Malik and Bharuthram (1998) have studied the effect of magnetic field strength, wave propagation angle, particle densities and temperature in a magnetized dusty plasma with two ion species. Malik et al. (1998) have also observed that only rarefactive solitons can propagate in a magnetized dusty plasma having finite temperature ions, electrons with Maxwellian distribution and negatively charged dust grains. However introduction of nonthermally distributed electrons in the present investigation adds some new salient features. Particularly in this four component magnetized dusty plasma, both compressive and rarefactive dust-acoustic solitons are observed depending on the sign of nonlinear coefficient ‘a’ in the KdV equation (4.59). It is worth mentioning that in the absence of magnetic field and positive dust component, only rarefactive solitons are obtained since ‘a’ is always negative (Malik et al. (1998); Mamun (1999)).

4.8 Conclusions

In this chapter, we have studied the role of the nonthermally distributed electrons i.e. parameter β on fully ionized collisionless unmagnetized and magnetized dusty plasma system consisting of warm adiabatic positively and negatively charged dust particles having maxwellian ions. Using reductive perturbation method, KdV equations are derived for unmagnetized as well as for magnetized system. The results obtained have several interesting features as follows:

In the unmagnetized case we have peak amplitude (φ′′ m) of positive solitary profiles increases with nonthermal parameter β and μ_0. However, in magnetized case, the amplitude of positive solitary potential profile is observed to decrease with
the increase of nonthermal parameter but increases with $\mu_i'$. For unmagnetized case, amplitude of negative solitary potential profile ($|\phi_m''|$) decreases with $\beta$ but increases with $\mu_i'$. On the contrary for the magnetized case the magnitude of the amplitude ($|\phi_m''|$) of negative solitary potential profile representing rarefactive solitons increases with $\beta$(nonthermal parameter) but decreases with $\mu_i'$. Particularly in this four component magnetized dusty plasma, both compressive and rarefactive dust-acoustic solitons are observed depending on the sign of nonlinear coefficient $'a'$ in the KdV equation (4.59). It is worth mentioning that in the absence of magnetic field and positive dust component, only rarefactive solitons are obtained since $'a'$ is always negative (Malik et al. (1998); Mamun (1999)).

Finally, since dusty plasmas are ubiquitous component of space and astrophysical environments and magnetic field is usually present in space environment, the results of the present investigation may be of useful help to identify charge coagulation, charge separation and investigating new wave modes in unmagnetized and magnetized four component dusty plasma system.