CHAPTER - IV

ANALYSIS OF ELECTRONIC AND OBSERVER DESIGN OF TRANSISTOR CIRCUITS

4.1 Introduction

Single-term Walsh series technique has been used to analyse linear time varying and nonlinear networks, and smoothing circuits by Palanisamy and Arunachalam [117,118]. Subbayyan and Zakariah [142] also applied the STWS approach to the computer aided design of electronic circuits. In this chapter, we extend the STWS method to analyse an electronic circuit and the observer design of a transistor circuit which are represented by the singular differential equations.

4.2 Analysis of Electronic Circuits

Consider a physical model of a circuit discussed on p 347 of Chua and Lin [47] as shown in Figure 3.

Figure 3.
The following hybrid equation is obtained [89]:

\[
\begin{bmatrix}
  i_1 & i_4 & v_2 & v_3 \\
 0 & 0 & 0 & 1 \\
 1 & 0 & 0 & 0 \\
-1 & -1 & 0 & 1 \\
\end{bmatrix}
\begin{bmatrix}
  \dot{v}_1 \\
  \dot{v}_2 \\
  \dot{v}_3 \\
\end{bmatrix}
= \begin{bmatrix}
  \begin{bmatrix}
    0 & 0 & -1 & -1 \\
    0 & 0 & 0 & 1 \\
    1 & 0 & 0 & 0 \\
-1 & -1 & 0 & 1 \\
  \end{bmatrix}
  \begin{bmatrix}
    v_1 \\
    v_4 \\
    i_2 \\
    i_3 \\
  \end{bmatrix}
+ \begin{bmatrix}
    0 & 0 \\
    0 & -1 \\
    1 & 0 \\
-1 & 0 \\
  \end{bmatrix}
\end{bmatrix}
\]

(4.1)

Since \( i_c = C \dot{v}_c \) and \( v_L = L_i \) substituting \( i_1 = 2\dot{v}_1 \), \( i_2 = 2\dot{v}_2 \), \( v_3 = 2i_3 \) and \( v_4 = 2i_4 \) into equation (4.1) we obtain

\[
\begin{bmatrix}
  2\dot{v}_1 \\
  i_4 \\
  \dot{v}_2 \\
  2i_3 \\
\end{bmatrix}
= \begin{bmatrix}
  0 & 0 & -1 & -1 \\
  0 & 0 & 0 & 1 \\
  1 & 0 & 0 & 0 \\
-1 & -1 & 0 & 1 \\
\end{bmatrix}
\begin{bmatrix}
  \dot{v}_1 \\
  \dot{v}_2 \\
  \dot{v}_3 \\
\end{bmatrix}
+ \begin{bmatrix}
  \dot{v}_1 \\
  \dot{v}_4 \\
  \dot{i}_2 \\
  \dot{i}_3 \\
\end{bmatrix}
\]

(4.2)

After rearranging terms, we finally obtain the descriptor (generalized state space) systems equation:

\[
\begin{bmatrix}
  2 & 2 & 0 & 0 \\
  0 & 0 & 2 & 2 \\
  0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
  \dot{v}_1 \\
  \dot{v}_2 \\
  i_3 \\
  i_4 \\
\end{bmatrix}
= \begin{bmatrix}
  0 & 0 & -1 & 0 \\
  -1 & 0 & 1 & 0 \\
  1 & -1 & 0 & 0 \\
  0 & 0 & 1 & -1 \\
\end{bmatrix}
\begin{bmatrix}
  v_1 \\
  v_2 \\
  i_3 \\
  i_4 \\
\end{bmatrix}
+ \begin{bmatrix}
  0 & 0 \\
  -1 & 0 \\
  1 & 0 \\
  0 & -1 \\
\end{bmatrix}
\]

(4.2)

This is of the form

\[
K\ddot{x}(t) = Ax(t) + Bu(t)
\]

(4.3)

where the matrix K is singular.

In some cases the variables have some inherent meanings (voltages, currents, positions, velocities, accelerations) or the coefficient matrices have some special structures which may be lost by
manipulating a system of the form (4.3) into an ordinary state-space system.

By taking \( E_a = 1 + t + \frac{t^2}{2} + \frac{t^3}{3} \) and \( J_b = 1 + t + t^2 \)

we obtain the exact solution of (4.2) as

\[
\begin{align*}
v_1(t) &= -\frac{93}{2}(1 - \sqrt{5})\exp\left(\frac{1 + \sqrt{5}}{8}\right)t - \frac{93}{2}(1 + \sqrt{5})\exp\left(-\frac{1 - \sqrt{5}}{8}\right)t \\
&\quad -27t + \frac{3}{2}t^2 - \frac{t^3}{3} + 163 \\
v_2(t) &= -\frac{93}{2}(1 - \sqrt{5})\exp\left(-\frac{1 + \sqrt{5}}{8}\right)t - \frac{93}{2}(1 + \sqrt{5})\exp\left(-\frac{1 - \sqrt{5}}{8}\right)t \\
&\quad -26t + 2t^2 + 164 \\
i_3(t) &= -93\exp\left(\frac{1 + \sqrt{5}}{8}\right)t - 93\exp\left(\frac{1 - \sqrt{5}}{8}\right)t - 14t + 2t^2 + 106 \\
i_4(t) &= -93\exp\left(\frac{1 + \sqrt{5}}{8}\right)t - 93\exp\left(\frac{1 - \sqrt{5}}{8}\right)t - 15t + t^2 + 105
\end{align*}
\]

with \([v_1(0) \ v_2(0) \ i_3(0) \ i_4(0)]^T = [70 \ 71 \ -80 \ -81]^T\). Using (3.17) and (4.4) the discrete time solution \(x^*(t)\) and the exact solution \(x(t) = [v_1(t) \ v_2(t) \ i_3(t) \ i_4(t)]^T\) are evaluated with \(m=100\) and \(m=200\). The results are shown in Tables 4.1 - 4.4. The related computer source code is presented as Appendix-6.
<table>
<thead>
<tr>
<th>S. No.</th>
<th>t</th>
<th>$v_1(t)$</th>
<th>$v_1^*(t)$</th>
<th>Error</th>
<th>$v_1(t)$</th>
<th>$v_1^*(t)$</th>
<th>Error</th>
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<td>70.00000</td>
<td>0.0000</td>
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Table 4.1 Solutions of (3.17) and (4.4) for $v_1(t)$

<table>
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<th>$v_2(t)$</th>
<th>$v_2^*(t)$</th>
<th>Error</th>
<th>$v_2(t)$</th>
<th>$v_2^*(t)$</th>
<th>Error</th>
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<td>71.00000</td>
<td>0.0000</td>
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<td>82.57135</td>
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<td>89.46178</td>
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Table 4.2 Solutions of (3.17) and (4.4) for $v_2(t)$
### Table 4.3 Solutions of (3.17) and (4.4) for $i_3(t)$

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<th>$i_3(t)$</th>
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<th>Error $i_3(t)$</th>
<th>$i_3^*(t)$</th>
<th>Error $i_3(t)$</th>
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<td>6.0x10^{-5}</td>
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### Table 4.4 Solutions of (3.17) and (4.4) for $i_4(t)$

<table>
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<tr>
<th>S. No</th>
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<th>$i_4(t)$</th>
<th>$i_4^*(t)$</th>
<th>Error $i_4(t)$</th>
<th>$i_4^*(t)$</th>
<th>Error $i_4(t)$</th>
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</thead>
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<td>-102.18270</td>
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<td>-102.18268</td>
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<td>-114.47363</td>
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<td>6.0x10^{-5}</td>
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</tbody>
</table>

From Tables 4.1 - 4.4 we observe that the discrete solution obtained by the STWS method compares well with the exact solution.
4.3 Observer Design of Singular Systems

Little effort has been made to develop a theory of observers for singular systems. In particular, based on a singular value decomposition, El-Tohami et al [61,62] have proposed the reduced order observer for a class of singular systems satisfying a simple rank condition. Also, using the concept of a matrix generalized inverse, Shafai and Carroll [139] have proposed a new method for constructing a minimal (reduced) order observer under a certain observability condition on the constructed observer. Design of observers for singular systems using a descriptor standard form was given by Minamide et al [106]. This implies that various kinds of synthesis problem for controlling singular systems, such as pole shifting problems and observer design problems, might also be solved by analogy to corresponding conventional techniques.

Full order observer, a reduced order observer and a function observer of the state in the descriptor standard form are derived in a way which parallels the observer theory. Geometric design techniques for observers in singular systems have been studied by Lewis [77]. Analysis of linear optimal control systems incorporating observers has been studied using the single-term Walsh series method by Palanisamy and Raghunathan [121]. In this chapter, we extend the STWS method to observer design in generalized state-space systems of transistor circuits.
4.4 Observer Design of Singular Systems Using STWS

Consider the linear singular system

\[ K\dot{x}(t) = Ax(t) + Bu(t) \quad (4.5a) \]
\[ y = Cx(t) \quad (4.5b) \]

with \( x \in \mathbb{R}^n, u \in \mathbb{R}^m, y \in \mathbb{R}^p \) and \( K \) an \( nxn \) matrix, generally singular. \( y(t) \) is the \( p \)-output vector, and \( A, B \) and \( C \) are constant matrices of appropriate dimensions. \( x(t) \) is an \( n \)-state vector, \( u(t) \) is an \( m \)-input vector. With the STWS approach the given function is expanded as single-term Walsh series in the normalized interval \( s \in [0,1) \) which corresponds to \( t \in [0, 1/m) \) by defining \( t = s/m \), \( m \) being any integer. Equation (4.5) becomes in the normalized interval as

\[ K\dot{x}(s) = \frac{A}{m} x(s) + \frac{B}{m} u(s) \quad (4.6a) \]
\[ y(s) = C x(s), \quad (4.6b) \]

Now expanding \( \dot{x}(s), x(s) \) and \( u(s) \) in STWS as

\[ \dot{x}(s) = G_i \psi_0(s) \]
\[ x(s) = B_i \psi_0(s) \quad (4.7) \]
\[ u(s) = H_i \psi_0(s) \]

the following recursive relationship is obtained with \( E = 1/2 \):

\[ G_i = [K - A/2m]^{-1} [A x(i-1) + BH_i]/m \]
\[ B_i = \frac{G_i}{2} + x(i-1) \]
\[ x(i) = G_i + x(i-1) \quad (4.8) \]
\[ y(i) = C x(i) \]
where \( i = 1, 2, \ldots \) the interval number. The \( x(i) \) and \( B_i \) give the discrete values and BPF values of the state and \( y(i) \) give the output measurement values of the state for any length of time. This is the main advantage of this present method. Even though the matrix \( K \) is singular the difference \([K - A/2m]\) turns out to be nonsingular.

4.5 Observer Design of Transistor Circuits

Consider the simplest model for a transistor in the circuit of Figure-4 discussed by Kang [70] and Lewis [77].

\[ U_1 + X_1 + r_1 x_2 = 0 \]
\[ u_2 + x_3 + r_2(x_4 - a_1x_2) = 0 \]
\[ x_2 = c_1 x_1 \] \hspace{1cm} (4.9)
\[ x_4 = c_2 x_3 \]
\[ y_1 = r_2(a_1 x_2 - x_4) \]
\[ y_2 = r_L a_2 x_4 \]

Figure 4.

The circuit equations are
Here $x_1$, $x_2$, $x_3$ and $x_4$ are the capacitor voltages and assume $r_1 = r_2 = r_L = a_1 = a_2 = c_1 = c_2 = 1$. This yields the singular system

\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 \\
\end{bmatrix}
\dot{x} =
\begin{bmatrix}
0 & 1 & 0 & 0 \\
1 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & -1 & 1 & 1 \\
\end{bmatrix} x
+ 
\begin{bmatrix}
0 & 0 \\
1 & 0 \\
0 & 0 \\
0 & 1 \\
\end{bmatrix} u
\]

(4.10a)

\[
y =
\begin{bmatrix}
0 & 1 & 0 & -1 \\
0 & 0 & 0 & 1 \\
\end{bmatrix} x
\]

(4.10b)

which we symbolize as $\dot{x} = Ax + Bu$, $y = Cx$. The need to destroy the sparsity of the system matrices can be avoided by eliminating algebraic variables to convert it into a state-space form. Therefore we shall study the problem of observing the semi-state $x$ using the output $y$ in the singular system setting.

Let us take $u = [1 \ 0]^T$, then the exact solutions of (4.10) are

\[
y_1(t) = (1-t)\exp(-t)
\]

\[
y_2(t) = (t-2)\exp(-t)
\]

(4.11)

with $x(0) = [0 \ -1 \ 1 \ -2]^T$. Using (4.8) and (4.11) the discrete time solution $y^*(t)$ and the exact solution $y(t)$ are evaluated with $m=100$ and $m=200$. The results are shown in Table 4.5 and the relevant computer program is presented in Appendix-7.
4.6 Conclusions

A simple method is introduced for the analysis of electronic circuits and observer design of generalized state space system of transistor circuits using the STWS. The solutions obtained for the electronic circuit problem compare well with the exact solution. The method can easily be implemented on a digital computer. In the observer design of transistor circuit problem, the output measurement obtained by the STWS approach compares well with the exact solution and it is determined for any length of time. This method is better than the previous BPF and WF techniques [115-121]. By employing large values of m and extreme precision, very accurate results may be obtained. It is also highly stable because the method is based on the trapezoidal rule which is an A-stable method.