CHAPTER 4

DATA HIDING TECHNIQUES IN FREQUENCY DOMAIN

4.1 INTRODUCTION

Most of the data hiding techniques be it in natural images or medical images are done in either of the two domains i.e., spatial and frequency domain or a combination of both in hybrid method. While spatial techniques involve manipulation of pixels of the cover image, frequency domain techniques involve manipulation of coefficients of the cover image. The coefficients are obtained by transforming the cover image in time domain to a frequency domain through a specific transformation function. Since the manipulation of medical images is involved, spatial domain techniques in spite of their good fidelity criteria exhibit poor tolerance towards a wide range of external attacks which is not desirable. Further, since spatial techniques involve manipulation of pixels, pixel level modification may not be suited for medical images which may cost severely on the content of medical image which is not a comprimisible event to a very small extent. The transform of a signal is just another form of representing the signal. It does not change the information content present in the signal. Hence, the frequency domain transforms is utilized and in specific the multi resolution properties of certain transforms like DWT, contourlet transform and the robustness properties of certain transforms like DCT, SVD. This chapter is organized as follows.

- Section 4.2 outlines the importance of DCT, embedding and extraction in DCT domain.
➢ The multi resolution properties of the DWT and the procedure to embed and extract using the Haar wavelet function is outlined in section 4.3.

➢ Utilization of directional properties of the contourlet transform (CT) and its embedding and extraction is explained in section 4.4.

➢ Section 4.5 highlights the rotation, scaling and translation invariance properties of the SVD Transform.

➢ A comparative analysis of three different embedding techniques in spatial and frequency domain in terms of its peak signal to noise ratio (PSNR) is presented in section 4.6.

➢ Section 4.7 outlines the summary and the significance of data embedding and extraction in the frequency domain.

4.2 DISCRETE COSINE TRANSFORM

A DCT expresses a sequence of finitely many data points in terms of a sum of cosine functions oscillating at different frequencies. DCT’s are important to numerous applications in science and engineering, from lossy compression of audio (e.g. MP3) and images (e.g. JPEG), to spectral methods for the numerical solution of partial differential equations. The use of cosine functions is best suited for approximating the coefficients. The DCT is purely real unlike discrete Fourier transform which is complex. DCT domain watermarking is a type of frequency domain watermarking which is similar to spatial domain watermarking in that the values of selected frequencies can be altered. Because high frequencies will be lost by compression or scaling, the watermark signal is applied to the lower frequencies, or better yet, applied adaptively to frequencies containing important elements of the original picture. Upon inverse transformation, watermarks
applied to frequency domain will be dispersed over the entire spatial image, so these methods are not as susceptible to defeat by cropping as the spatial techniques. However, the trade-off between invisibility and robustness is greater here. The DCT allows an image to be broken up into different frequency bands, making it much easier to embed watermarking information into the middle frequency bands of an image. The middle frequency bands are chosen such that they avoid the most visual important parts of the image (low frequencies) without over exposing themselves to removal through compression and noise attacks (high frequencies). The principle advantage of image transformation is the removal of redundancy between neighboring pixels. This leads to uncorrelated transform coefficients which can be encoded independently. DCT exhibits excellent energy compaction for highly correlated images. The uncorrelated image has its energy spread out, whereas the energy of the correlated image is packed into the low frequency region. The DCT does a better job of concentrating energy into lower order coefficients than does the DFT for image data. The inverse discrete transform is orthogonal and separable which gives it the much needed robustness towards external attacks.

The general equation for a 1D DCT is defined by the following equation:

$$X(u) = \left(\frac{2}{N}\right)^{\frac{1}{2}} \sum_{i=0}^{N-1} \nabla \cdot \cos \left[\frac{\pi}{2} \cdot \frac{u}{N} (2i + 1)\right] x(i)$$  \hspace{1cm} (4.1)

where $x(i)$ the input signal and $N$ is the number of samples.

and the corresponding inverse 1D DCT transform is simple $X^{-1}(u)$, where

$$\nabla = \begin{cases} \frac{1}{\sqrt{2}} & \text{for } \xi = 0 \\ 1 & \text{otherwise} \end{cases} \hspace{1cm} (4.2)$$

A 2 – D Discrete Cosine Transform is defined by the equation
\[ X(u, v) = \left( \frac{2}{N} \right)^{\frac{1}{2}} \left( \frac{2}{M} \right)^{\frac{1}{2}} \sum_{i=0}^{N-1} \sum_{j=0}^{M-1} \nabla (i) \nabla (j) \cos \left[ \frac{\pi}{N} \frac{u}{N} (2i + 1) \right] \cos \left[ \frac{\pi}{M} \frac{v}{M} (2j + 1) \right] x(i, j) \]

(4.3)

and the corresponding inverse 2D DCT transform is \( X^{-1}(u) \)

### 4.2.1 Embedding using Discrete Cosine Transform

The data embedding procedure in most of the frequency domain techniques are one and the same except for some minor modifications. To begin with the cover image, watermarks in the form of hospital logo and doctor’s signature are taken as shown in figure 4.1. The cover image is a MRI brain image of dimension 512 x 512 and divided into sub blocks of 32 x 32. To each of the 32 x 32 block the DCT is applied and the resulting image is shown in figure 4.2.

![figure 4.1](image)

(a) (b) (c)

**Figure 4.1 Input images and payload**

a. Cover MRI brain image  
   b. Hospital logo (watermark1)  
   c. Doctors signature (watermark2)
The DCT transforms the image into low, mid and high frequency bands. Since robustness is one of the key criteria, high frequency regions are selected as locations for embedding and the payload and watermarks are cast into the cover image as per the embed equation given below.

\[ C_{DCT_{new}}(i,j) = C_{DCT_{old}}(i,j) + \alpha W_{DCT(i,j)} \] (4.4)

Once the embedding is done, the inverse DCT is applied to get back the image in the spatial domain as shown in figure 4.3.

From figure 4.3, it can be seen that the embedded and original image are visually imperceptible as far as HVS is considered.
4.2.2 Extraction using Discrete Cosine Transform

The Extraction follows the reverse of the embedding process where the DCT is applied to the embedded image, followed by identification of the embedding location and then differencing it from the original image to get the watermarks and differencing from the original watermarks to get the cover image. The extracted cover image and the watermarks and payload are shown below in figure 4.4 and figure 4.5.

![Fig 4.4 Original and extracted image using DCT](image)

Figure 4.4 Original and extracted image using DCT

![Fig 4.5 Extracted payloads (DCT)](image)

Doctors Signature

(a)  (b)

Figure 4.5 Extracted payloads (DCT)

a. Extracted hospital logo (watermark1)  b. Extracted doctor’s signature (watermark 2)

From figure 4.4 and 4.5 it can be seen that the original and extracted images have no visual differences thus satisfying the property of visual imperceptibility.
However, in the presence of noise in the channel or when attacked, they may not exhibit the same response. Hence the metric of robustness comes into picture which is experimented and discussed in later chapters.

### 4.3 DISCRETE WAVELET TRANSFORM

The discrete wavelet transform is an important class of multi resolution transforms and computed by successive low pass and high pass filtering of the discrete time-domain signal as shown in figure 4.6. This is called the Mallat algorithm or Mallat-tree decomposition.

![Figure 4.6 A 3 level DWT decomposition filter bank structure](image)

The above figure illustrates a 3 level decomposition filter bank structure where the input discrete time signal x(n) is passed through a low pass filter (LPF) and a high pass filter (HPF) followed by a down sampling by 2 to generate an approximation image giving the approximation coefficients (AC) and a directional sub band giving the directional coefficients (DC). Three directional sub bands are generated at every stage known as the horizontal sub band, vertical sub band and diagonal sub band. The
approximation image contains the low frequency components while the other three contain the high frequency components like edges etc. The transform at high frequencies, yields good time resolution and poor frequency resolution, while at low frequencies, gives good frequency resolution and poor time resolution. It is just a sampled version of continuous wavelet transform (CWT) and its computation may consume significant amount of time and resources, depending on the resolution required. Once the required number of decomposition levels is obtained, the required processing is done either with the approximation or detailed sub bands and then reconstructed back to get the original time domain signal through the inverse wavelet transform. The same number of reconstruction levels is used as in the decomposition phase. The filters used in the decomposition phase are known as the analysis filters while those in the reconstruction phase are known as the synthesis filters. Figure 4.7 shows the reconstruction of the original signal from the wavelet coefficients comprising of approximation coefficients (AC) and directional coefficients (DC_n) where ‘n’ is the decomposition level.

![Figure 4.7 A 3 level DWT reconstruction filter bank structure](image)

Basically, the reconstruction is the reverse process of decomposition. The approximation and detail coefficients at every level are up sampled by two, passed
through the low pass and high pass synthesis filters and then added. This process is continued through the same number of levels as in the decomposition process to obtain the time domain signal x(n).

Based on the application, wavelets are classified into orthogonal wavelets whose coefficients are real and biorthogonal wavelets whose coefficients may be real or contain integers. Further, in biorthogonal wavelets, the LPF is symmetric while the HPF may be symmetric or anti symmetric. The mother wavelet produces all wavelet functions used in the transformation. Haar wavelet is one of the oldest and simplest wavelet. Daubechies wavelets are the most popular wavelets. They represent the foundations of wavelet signal processing and are used in numerous applications. These are also called Maxflat wavelets as their frequency responses have maximum flatness at frequencies 0 and π. This is a very desirable property in some applications. The Haar, Daubechies, Symlets and Coiflets are compactly supported orthogonal wavelets. These wavelets along with Meyer wavelets are capable of perfect reconstruction. The Meyer, Morlet and Mexican Hat wavelets are symmetric in shape. The wavelets are chosen based on their shape and their ability to analyze the signal in a particular application.

There is a wide range of applications for wavelet transforms. They are applied in different fields ranging from signal processing to biometrics, and the list is still growing. One of the prominent applications is in compression for storage in data banks. Wavelets also find application in speech compression, which reduces transmission time in mobile applications. They are used in denoising, edge detection, feature extraction, speech recognition, echo cancellation and others. They are very promising for real time audio and video compression applications. Wavelets also have numerous applications in digital communications. Orthogonal frequency division multiplexing (OFDM) is one of them. Wavelets are used in biomedical imaging. For example, the electro cardiogram (ECG) signals, measured from the heart, are analyzed using wavelets or compressed for storage.
The popularity of wavelet transform is growing because of its ability to reduce distortion in the reconstructed signal while retaining all the significant features present in the signal.

### 4.3.1 Embedding in Wavelet Domain

**Step 1:** A 3 level DWT is applied on the cover medical image using the ‘haar’ wavelet function, resulting in 1 approximation sub band (CA) and 3 directional sub bands (CH, CV and CD) as shown in figure 4.8

![Figure 4.8 A 3 level decomposed cover MR brain image](image)

**Step 2:** Three sub bands for each of the watermarks and payload are selected as the embedding location and decomposed into sub blocks to match the size of the watermark and payload.

**Step 3:** The DCT encapsulated watermarks are cast into the corresponding pre identified sub bands and the payload into its appropriate sub band and
the inverse DCT and DWT are computed to get back the spatial domain image as shown in figure 4.9

![Image](image1.png)

**Figure 4.9 Original and embedded image in wavelet domain**

### 4.3.2 Data Retrieval in Wavelet Domain

The extraction follows the reverse of the embedding process where the DWT is applied to the embedded image and decomposed to ‘n’ levels where n = 3 in the current case, followed by identification of the embedding location and performing the DCT over the embedded location and then differencing it from the original image to get the watermarks and differencing from the original watermarks to get the cover image. The extracted cover image and the watermarks and payload are shown below in figure 4.10 and figure 4.11.

![Image](image2.png)

**Figure 4.10 Original and extracted image in wavelet domain**
4.4 THE CONTOURLET TRANSFORM

A filter bank structure that can deal effectively with piecewise smooth images with smooth contours, was proposed by Minh N Do and Martin Vetterli. The resulting image expansion is a directional multi resolution analysis framework composed of contour segments, and thus is named contourlet. This will overcome the challenges of wavelet and curvelet transform. contourlet transform is a double filter bank structure. It is implemented by the pyramidal directional filter bank (PDFB) which decomposes images into directional sub bands at multiple scales. In terms of structure the contourlet transform is a cascade of a laplacian pyramid and a directional filter bank. In essence, it first uses a wavelet-like transform for edge detection, and then a local directional transform for contour segment detection. The contourlet transform provides a sparse representation for two-dimensional piecewise smooth signals that resemble images.

Efficient representations of signals require that coefficients of functions, which represent the regions of interest, are sparse.

Wavelets can pick up discontinuities of one dimensional piecewise smooth functions very efficiently and represent them as point discontinuities, but cannot recognize smoothness along contours. Do and Vetterli proposed the pyramidal directional filter bank (PDFB), which overcomes the block-based approach of curvelet transform by
a directional filter bank, applied on the whole scale, also known as contourlet transform. It has been developed to offer the directionality and anisotropy to image representation that are not provided by separable wavelet. Contourlet transform is a multiscale and directional decomposition of a signal, using a combination of a modified laplacian pyramid and a directional filter bank. In terms of digital watermarking, contourlet transform has many key features, in the sense; it offers a wide range of flexibility in the choice of embedding locations. For example, a 3 level CT generates 8 directional sub bands out of which the user can decide upon the embedding location based on specific criteria. It also offers the necessary resistance towards high frequency attacks, as the 8 sub bands are all high frequency sub bands. A general decomposition structure of a 3 level contourlet structure is illustrated in figure 4.12 where the input signal \( x(i) \) is given to a laplacian pyramid filter bank (LP) which generates a low frequency band and a band pass image. The low frequency sub band is given to stage 2 LP filter bank which generates another low pass image and 4 directional sub bands and so on.

Figure 4.12 A three level Contourlet decomposition filter bank structure
There are two stages in the proposed method; data embedding and data recovering stages. The watermark embedding and extraction algorithm is described here.

4.4.1 Data Embedding using Contourlet transform

The data embedding process consists of the following steps which is elucidated below in figure 4.13

**Step 1: Contourlet decomposition**

4 level Contourlet decomposition is applied to the original cover image which generates a low pass image and 16 directional sub bands as shown below in figure 4.13.

![Figure 4.13 Sixteen directional sub bands for a 4 level Contourlet transform decomposition](image)
Step 2: Energy computation

Following the generation of the 16 sub bands, the energy level of each of the sub bands is computed and plotted. The plots for a MR brain image and a CT image are shown in figures 4.14 and 4.15.

Figure 4.14 Energy plot of 4 level Contourlet transform sub bands of MR brain image

Figure 4.15 Energy plot of 4 level Contourlet transform sub bands of CT brain image
From figure 4.14 and 4.15, it can be seen that sub bands 4, 5, 3 and 13 and Sub bands 4, 3, 5 and 6 have high energy values in descending order for a MR brain image and CT brain image respectively. Now these high energy sub bands could be the ideal embedding locations for the multiple watermarks and payload.

**Step 3:** DCT is applied to the watermarks and the payload and cast into the corresponding pre designated sub bands. The location of sub bands could themselves act as the key to the embedding and extraction process. The embedding process is carried according to the embed equation and the inverse transforms are computed to get the watermarked image in spatial domain as shown in figure 4.16

![Figure 4.16 Original and Embedded image using Contourlet transform](image)

**4.4.2 Data Extraction using Contourlet transform**

The extraction follows the reverse of the embedding process where the Contourlet transform is applied to the embedded image and decomposed to ‘n’ levels where n = 4 in the current case, followed by identification of the embedding location and performing the DCT over the embedded location and then differencing it from the original image to get
the watermarks and payload and differencing from the original watermarks to get the cover image. The extracted cover image and the watermarks and payload are shown below in figure 4.17 and figure 4.18

![Figure 4.17 Original and extracted cover image using Contourlet transform](image)

![Figure 4.18 Extracted payloads](image)

**Figure 4.17 Original and extracted cover image using Contourlet transform**

**Figure 4.18 Extracted payloads**  
(a) Extracted hospital logo (watermark1)  
(b) Extracted doctor’s signature (watermark 2)

## 4.5 HYBRID CONTOURLET TRANSFORM BASED DATA EMBEDDING

As briefed in the previous sections, Contourlet transform forms a pyramidal structure composed of two filter banks namely the Laplacian pyramid and the directional filter bank. It is a multi scale transform and provides high directionality properties which make it suitable for embedding data onto the directional high frequency sub bands.
To begin with four types of medical images \[104\] namely the MRI Brain Image (Axial and Sagittal), MRI Axial Neck Image, MRI Knee Image and a CT Brain image each of dimensions 512x 512 as shown in figure 4.19 are taken.

![Image](image1.png)

**Figure 4.19** Cover images a. MR Brain image (Axial) b. MR Brain image (Sagittal) c. MR Knee image d. CT Brain image

The watermarks taken in this work are multiple in nature, comprising of the hospital logo and doctor’s signature each of dimensions 32 x 32 and 256 x 256 respectively. The watermarks used are shown in figure 4.20

![Watermarks](image2.png)

**Figure 4.20** Watermarks a. Hospital Logo b. Doctor’s Signature

Contourlet Transform based data hiding is already explained with results in chapter 4. Since, the objective in this chapter is to evaluate the robustness, the embedding processes is revisited with a MRI Knee image as shown in figure 6.2 (c). As mentioned previously, a 4 level decomposition is done to generate \(2^d\) directional sub bands as shown
in figure 4.21. Further, it can be seen from the figure that the first 8 sub bands are horizontally oriented and the latter eight sub bands are vertically oriented. The low pass image contains much of the visual content of the image. Embedding in low pass is not much desirable especially with medical images as they get degraded easily when exposed to attacks.

![Low Pass Image](image)

**Figure 4.21 Knee MR Image a. Low Pass Image  b. Directional Sub bands**

Following the generation of sub bands, the next goal is to find the embedding locations. Energy plot is used as a means for identifying the sub bands with highest energy. The method of selection may vary from algorithm to algorithm. The energy plot of such a Knee MRI image is shown in figure 4.22 from which Sub bands 13, 14, 11 and 15 could be seen as the bands with high energy levels in decreasing order.
Once the sub bands are identified, the discrete cosine transform is applied to 4 x 4 blocks of the watermarks 1 and 2 as shown in figure 4.2. The pre identified high energy sub bands of the Cover Image and the needed sub block (4 x 4) is SVD transformed to obtain the singular values as shown equation 4.4.

\[
\begin{pmatrix}
64.7077 & 0 & 0 & 0 \\
0 & 20.9590 & 0 & 0 \\
0 & 0 & 9.3359 & 0 \\
0 & 0 & 0 & 4.6178 \\
\end{pmatrix}
\]

The above shown sample singular values of 4 x 4 sub block is modified with the coefficients of the watermarks according to the embed equation given by

\[
C_{CT_{svd}}(i, j) = C_{CT_{svd}}(i, j) + \propto W_{DCT}(i, j)
\]  

The modified values are updated and the inverse SVD transforms and the sub bands are reconstructed through the 4 levels where they were decomposed to using the
inverse Contourlet transform as shown in figure 4.23. It can be seen that they are perceptually imperceptible to the human visual system.

![Original and Embedded Knee MR Image](image)

**Figure 4.23 Original and Embedded Knee MR Image**

### 4.6 EVALUATION METRICS

Since robustness is a key criterion in any data embedding and extraction schemes, a number of metrics are used to evaluate the strength of the embedding technique after being exposed to a wide range of attacks mentioned in the previous sections. Figure 4.19 gives an illustration of how this work has progressed. It starts with the cover image along with the multiple watermarks and the payload which is the electronic patient information transformed into frequency domain by using a DCT, DWT and Contourlet transform. The watermarks are DCT encapsulated to provide the resistance towards external attacks. After the embedding process, the inverse transforms ICT (inverse Contourlet transform), IDCT (inverse discrete cosine transform) and IDWT (inverse discrete wavelet transform) are taken and then passed through the channel prevalent with intentional and unintentional attacks. At the receiver, the reverse process is done to extract the cover image, the watermarks and the payload. The extracted watermarks cover image and the payload are now evaluated for their strength of resistance (robustness) in terms of metrics namely MSE, PSNR, structural similarity index (SSIM), correlation coefficient (CC). The above mentioned metrics are briefly explained below. A general scheme of evaluation for the proposed technique is depicted in figure 4.19.
4.6.1 Mean Squared Error (MSE)

It is used to bring out the difference between values predicted by an estimator and the true values of the quantity being observed. It is basically an error function and denotes the average of mean square error. The MSE represents the cumulative squared error between the compressed and the original image. Taking the square root of MSE yields the root mean square error (RMSE).

\[ MSE = \frac{\sum_{M,N}[I_x(m,n) - I_y(m,n)]^2}{M*N} \]  \hspace{1cm} (4.7)

where \( M \) and \( N \) denote the number of rows and columns of the image and ‘\( m \)’ and ‘\( n \)’ denote the pixel coordinates.

4.6.2 Peak Signal to Noise Ratio (PSNR)

The PSNR computes the peak signal-to-noise ratio, in decibels, between two images. This ratio is often used as a quality measurement between the original and a compressed image. Higher the PSNR, better the quality of the compressed or reconstructed image. PSNR is usually expressed in terms of the logarithmic decibel scale. The PSNR is computed between the original cover image and the extracted cover image after having passed through the communication channel. A high value of PSNR indicates a good reconstruction due to lower content of noise with respect to the signal strength. Medical images exhibit a good PSNR ranges in the range of 40 – 60 dB.

\[ PSNR = 10 \log_{10} \left( \frac{M*N}{MSE} \right) \]  \hspace{1cm} (4.8)
where $M$ and $N$ denote the number of rows and columns of the image and MSE is the mean square error computed as in section 4.6.1.

### 4.6.3 Structural Similarity Index (SSIM)

It is yet another method for computing the similarity between two images. It is an improvement on conventional methods like PSNR and MSE, which have proved to be inconsistent with human eye perception. SSIM aims at detecting image degradation and make use of the spatial correlation between the pixels. These dependencies carry important information about the structure of the objects in the visual scene. The SSIM metric is calculated on various windows of an image. The measure between two windows $x$ and $y$ of common size $N \times N$ is:

$$
SSIM = \frac{(2\mu_x\mu_y + c_1)(2\sigma_{xy} + c_2)}{((\mu_x^2 + \mu_y^2 + c_1)(\sigma_x^2 + \sigma_y^2 + c_2))}
$$

(4.9)

Where $\mu_x$ and $\mu_y$ are the averages of ‘$x$’ and ‘$y$’, $\sigma_x^2$ and $\sigma_y^2$ are the variances and $c_1$ and $c_2$ stabilization variables.

### 4.6.4 Correlation Coefficient (CC)

The correlation coefficient is a number between 0 and 1. If there is no relationship between the predicted values and the actual values the correlation coefficient is 0 or very low. As the strength of the relationship between the predicted values and actual values increases so does the correlation coefficient. A perfect fit gives a coefficient of 1.0. Thus, higher the correlation coefficient, better the extracted watermark. In this work, the correlation coefficient is used to establish the relationship between the extracted watermark and the original watermark. After various attacks have been imposed on the watermark, a correlation coefficient of 1 indicates a good watermarking strategy and a 0 indicates poor strength of the watermarking algorithm.
\[
CC = \frac{\sum_m \sum_n (A_{mn} - \bar{A})(B_{mn} - \bar{B})}{\sqrt{\sum_m \sum_n (A_{mn} - \bar{A})^2} \sqrt{\sum_m \sum_n (B_{mn} - \bar{B})^2}} 
\]  
(4.10)

Table 4.1 depicts the various performance metrics for the MRI brain embedded in spatial and frequency domain techniques.

Table 4.1 Performance comparison between spatial, DCT, DWT and Contourlet domain data embedding for MR brain image

<table>
<thead>
<tr>
<th>Embedding Technique</th>
<th>Mean Square error (MSE)</th>
<th>Peak Signal to Noise Ratio (PSNR - dB)</th>
<th>Correlation Coefficient (CC)</th>
<th>Structural Similarity Index (SSIM)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spatial domain (Luminance based)</td>
<td>13.845</td>
<td>40.124</td>
<td>29.465</td>
<td>27.842</td>
</tr>
<tr>
<td>Discrete Cosine Transform (High frequency Band)</td>
<td>12.969</td>
<td>42.088</td>
<td>31.445</td>
<td>29.156</td>
</tr>
<tr>
<td>Discrete Wavelet Transform (Haar Wavelet)</td>
<td>12.514</td>
<td>43.047</td>
<td>33.566</td>
<td>31.868</td>
</tr>
<tr>
<td>Contourlet Transform (4\textsuperscript{th}, 5\textsuperscript{th} and 3\textsuperscript{rd} sub bands)</td>
<td>6.514</td>
<td>46.098</td>
<td>36.268</td>
<td>35.168</td>
</tr>
<tr>
<td>Hybrid Contourlet Transform (4\textsuperscript{th}, 5\textsuperscript{th} and 3\textsuperscript{rd} sub bands)</td>
<td>6.514</td>
<td>49.041</td>
<td>39.008</td>
<td>38.897</td>
</tr>
</tbody>
</table>
All the above metrics have been calculated in the absence of attacks in the transmission channel and hence they give a perfect reconstruction in terms of correlation coefficient. It can be seen that Contourlet transform is able to give a better PSNR for the cover image as the payload is cast into the directional sub bands which are of high frequency coefficients.

4.7 SINGULAR VALUE DECOMPOSITION FOR RST INVARIANCE

- A n x m matrix has a singular value decomposition of the form

\[ A = U S V^T \] (4.11)

where \( U \) is a orthonormal matrix with columns known as left singular vectors of \( A \) and \( V \) is a orthonormal matrix whose columns are known as right singular vectors of \( A \) and \( S \) is the singular matrix.

- SVD has some interesting properties in the sense that the singular matrix of the rectangular matrix of ‘\( A \)’ is equal to the square root of Eigen values of the matrix \( A^T A \). Further the rank of the matrix is equal to the number of positive singular values.

- Since, SVD is characterized by an important property that the diagonal singular value elements remain unchanged even if they are transposed, they find themselves very useful in data embedding applications to provide the cover image and the payload resistance towards translation, scaling and rotation attacks. For this purpose, the watermark bits or payload bits are modified in the singular values of the USV matrix and then added to the cover image.
Apart from this, they also find themselves applicable in solving homogenous linear equations, least squares minimization, low rank matrix approximation and also in the study of linear inverse problems. It also finds applications in Signal Processing, pattern recognition and Principal Component Analysis. It also plays active role in Quantum Information and Numerical Weather prediction.

4.8 SUMMARY

This chapter deals with the importance of data embedding in the frequency domain over spatial domain especially for medical images. A brief outline of the features of DCT, DWT and Contourlet transform is discussed. The embedding and the extraction algorithms for each of the above three transforms were elucidated and illustrations provided with the medical images which were a brain MR image and the watermarks to be a hospital logo and doctor’s signature and the payload to be the electronic patient information. The decomposition structures of each of the transforms are discussed and the extracted payload, watermarks and the cover image were evaluated in terms of some important metrics like PSNR, MSE, correlation coefficient and structural similarity index. It could be seen from the discussions that the Contourlet transform was able to outperform the other two frequency domain transforms in terms of the signal to noise ratio due to its high directional properties and the robustness properties of the DCT. The chapter concluded with a brief description on the significance and applications for singular value decomposition and its contribution to data hiding in medical images.