Appendix A

Appendix

A-2.1: Proof of proposition-2.1

Using Table-2.1,

\[
\frac{d\pi_r}{da} = \frac{(2 - 2k - \alpha k - \alpha k^2)(a - b)\alpha}{b(1 - \alpha - \alpha k)^3}
\]

Now, \(d\pi_r/da > 0\) if \(2 - 2k - \alpha k - \alpha k^2 > 0\), i.e., if \(\alpha < 2(1 - k)/k(1 + k) = f(k)\), say. Interestingly, \(df(k)/dk = -[k(2 - k) + 1]/k^2(1 + k)^2 < 0\), i.e., \(f(k)\) is a decreasing function of \(k\). Since \(k \in (0, 1)\), it attains maximum value at 0 and minimum value at 1. Moreover, \(f(k) > 1\), if \(k^2 + 3k - 2 < 0\), i.e., if \(k < 0.561\). Thus, for any \(k \in (0, 0.561)\), \(\pi_r\) is increasing if \(\alpha \in (0, f(k))\) and decreasing for \(\alpha \in (f(k), 1)\).

Further, for any \(k \in (0, 0.561)\), \(f(k) > 1\). As long as \(f(k)\) is bounded above by 1, \(\pi_r\) is increasing because \(\alpha\) is bounded above by 1. That is, \(\pi_r\) is increasing for all \(\alpha \in (0, 1)\) if \(k \in (0, 0.561)\) (see fig-2.1). Since, the channel first decides the intensity of CSR and then the channel members decide the share of CSR, the above results can be represented from the perspective of CSR as follows. For any \(k \in (0, 1)\) and \(\alpha \in (0, f(k))\), \(f(k) \leq 1\), \(\pi_r\) increases with increasing CSR of the channel.

Similarly, \(d\pi^{m*}/da = (4k - \alpha k - \alpha k^2)/(4 - \alpha - \alpha k)^3 > 0\) if \(4k - \alpha - \alpha k > 0\), i.e., if \(\alpha < 4k/(1 + k) = g(k)\), (say). Now \(dg(k)/dk = 4/(1 + k)^2 > 0\) for any \(k \in [0, 1]\). So, \(g(k)\) is an increasing function of \(k\) and \(g(k)\) attains maximum value at \(k = 1\) and minimum value at \(k = 0\). Now \(g(k) > 1\) if \(k > 0.33\). Since \(\alpha\) is bounded above by 1, \(\pi_m\) is increasing for \(\alpha \in (0, g(k))\) and \(k \in (0, 0.33)\). Also, for any \(k \in (0.33, 1)\) \(\pi^{m*}\) is increasing for \(\alpha \in (0, 1)\). Thus, combining these it may be concluded that \(\pi^{m*}\) is increasing for \(\alpha \in (0, g(k))\), \(g(k) \leq 1\), \(k \in (0, 1)\) and it is decreasing for \(\alpha \in (g(k), 1)\), \(g(k) \leq 1\), \(k \in (0, 1)\) (see fig-2.1).

Thus, for \(k \in (0, 1)\) \(\pi_r\) and \(\pi^{m*}\) are increasing if the channel performs CSR in same ranges. Obviously, both \(\pi^{m*}\) and \(\pi_r\) will increase if the CSR lies in the intersection of the two ranges. Hence, the pure profit of the manufacturer and the retailer, for any \(k \in (0, 1)\), are increasing if \(\alpha \in (0, \min\{f(k), g(k)\})\), \(f(k) \leq 1\), \(g(k) \leq 1\) and decreasing if \(\alpha \in (\max\{f(k), g(k)\}, 1)\), \(f(k) \leq 1\), \(g(k) \leq 1\). The pure profit of the manufacturer and the retailer behave inversely if \(\alpha \in (\min\{f(k), g(k)\}, \max\{f(k), g(k)\})\), \(f(k) \leq 1\), \(g(k) \leq 1\) (see fig-2.1).

A-2.2: Proof of Proposition-2.3

Assume that the retailer offers a wholesale price \(w^{m}_r\) to the manufacturer and wants to charge a retail price \(p_r\). If the retailer and the manufacturer wants \(L_r\) and \(L_m\) share of the profit in subgame perfect
equilibrium then the retailer has the following maximization problem.

\[
\begin{align*}
\text{Max } & v_r(p_r, \ w_r^m) = (p_r - \ w_r^m)(a - b p_r) + k \left[ \frac{\alpha}{2b} (a - b p_r)^2 \right] \\
\text{such that } & (w_r^m - c)(a - b p_r) + (1 - k) \left[ \frac{\alpha}{2b} (a - b p_r)^2 \right] \geq L_m
\end{align*}
\]  
(A1.1)

Since the retailer wants to maximize (A1.1), it must satisfy the manufacturers required profit share by providing minimum to it, i.e., \((w_r^m - c)(a - b p_r) + (1 - k) \left[ \frac{\alpha}{2b} (a - b p_r)^2 \right] = L_m\). Thus from (A1.2) \(w_r^m\) can be found as

\[
w_r^m = c + \frac{1}{(a - b p_r)} \left[ L_m - (1 - k) \left( \frac{\alpha}{2b} (a - b p_r)^2 \right) \right]
\]  
(A1.3)

Substituting (A1.3) in (A1.1) and simplifying, the retailers problem can be found as

\[
\begin{align*}
\text{Max } & v_r(p_r) = (p_r - c)(a - b p_r) - L_m + \frac{\alpha}{2b} (a - b p_r)^2
\end{align*}
\]  
(A1.4)

Using the necessary condition, \(\frac{d v_r(p_r)}{d p_r} = 0\), the optimal selling price of the retailer is

\[
p_r^* = \frac{(1 - \alpha)a + bc}{b(2 - \alpha)}
\]
(A1.5)

Using (A1.5) in (A1.3) \(w_r^{ms}\) can be found as

\[
w_r^{ms} = c + \frac{(2 - \alpha)L_m}{(a - bc)} + \frac{\alpha (1 - k)(a - bc)}{2b(2 - \alpha)}
\]
(A1.6)

Moreover, \(\frac{d^2 w_r^{m}}{d p_r^2} = -b (2 - \alpha) < 0\), i.e., \(p_r\), provides the global maximum of (A1.4).

Similarly, if the manufacturer offers \((p_m, \ w_m^m)\) retail price-wholesale price pair to the retailer than it has the following maximization problem

\[
\begin{align*}
\text{Max } & v_m(p_m, \ w_m^m) = (w_m^m - c)(a - b p_m) + (1 - k) \left[ \frac{\alpha}{2b} (a - b p_m)^2 \right] \\
\text{such that } & (p_m - w_m^m)(a - b p_m) + k \left[ \frac{\alpha}{2b} (a - b p_m)^2 \right] \geq L_r
\end{align*}
\]  
(A1.7)

As the manufacturer wants to maximize its profit share, it must provide least share to the retailer i.e., \((p_m - w_m^m)(a - b p_m) + k \left[ \frac{\alpha}{2b} (a - b p_m)^2 \right] = L_r\), Simplifying \(w_m^m\) is found as

\[
w_m^m = p_m - \frac{1}{(a - b p_m)} \left[ L_r - k \left( \frac{\alpha}{2b} (a - b p_m)^2 \right) \right]
\]
(A1.9)

Substituting \(w_m^m\) of (A1.9) in (A1.7) and using the necessary condition \(p_m^*\) can be found as

\[
p_m^* = \frac{(1 - \alpha)a + bc}{b(2 - \alpha)}
\]
(A1.10)

From (A1.9) \(w_m^{ms}\) can be found as

\[
w_m^{ms} = \frac{(1 - \alpha)a + bc}{b(2 - \alpha)} - \frac{(2 - \alpha)L_r}{(a - bc)} + \frac{ak(a - bc)}{2b(2 - \alpha)}
\]
(A1.11)

Moreover, \(\frac{d^2 w_m^{m}}{d p_m^2} = -b(2 - \alpha) < 0\). Hence, \(w_m^{ms}\) provides the global optimal solution.
A-2.3: Proof of Proposition-2.4

Figure A.1: A tree defining the largest share the retailer could obtain in a Subgame Perfect Equilibrium.

Figure A.1 depicts the subgame, where the retailer is the offering party and it gets maximum subgame perfect equilibrium share. Obviously, the manufacturer will get the minimum share. Following the procedure of Wu [147], traversing backward in the decision tree the maximum share of the channel surplus $\pi_s$ that the retailer can accrue in the subgame perfect equilibrium is

$$R_{\text{max}} = \pi_s - \frac{x}{2} \left( (1-x)O_m + \frac{x}{2} (\pi_s - R_{\text{max}} + M_{\text{min}}) + \pi_s - R_{\text{max}} \right) + (1-x)O_m$$  \hspace{1cm} (A1.12)

Similarly, replacing $R_{\text{max}}$ by $R_{\text{min}}$ and $M_{\text{min}}$ by $M_{\text{max}}$ the retailer’s minimum share of the channel surplus in the subgame perfect equilibrium is found as

$$R_{\text{min}} = \pi_s - \frac{x}{2} \left( (1-x)O_m + \frac{x}{2} (\pi_s - R_{\text{min}} + M_{\text{max}}) + \pi_s - R_{\text{min}} \right) + (1-x)O_m$$  \hspace{1cm} (A1.13)

Since, the manufacturer and the retailer are equally likely to propose an offer, following the same procedure, the manufacturer’s maximum and minimum share of the channel surplus can be found as

$$M_{\text{max}} = \pi_s - \frac{x}{2} \left( (1-x)O_r + \frac{x}{2} (\pi_s - M_{\text{max}} + R_{\text{min}}) + \pi_s - M_{\text{max}} \right) + (1-x)O_r$$  \hspace{1cm} (A1.14)

$$M_{\text{min}} = \pi_s - \frac{x}{2} \left( (1-x)O_r + \frac{x}{2} (\pi_s - M_{\text{min}} + R_{\text{max}}) + \pi_s - M_{\text{min}} \right) + (1-x)O_r$$  \hspace{1cm} (A1.15)

Solving (A1.12), (A1.13), (A1.14) and (A1.15) the optimal solutions can be found as

$$R_{\text{max}} = R_{\text{min}} = \pi_s - O_m - \frac{x^2}{2(2-x)} (\pi_s - O_m - O_r) = R, \quad \text{(say)}$$  \hspace{1cm} (A1.16)

$$M_{\text{max}} = M_{\text{min}} = \pi_s - O_r - \frac{x^2}{2(2-x)} (\pi_s - O_m - O_r) = M, \quad \text{(say)}$$  \hspace{1cm} (A1.17)

A-2.4: Proof of proposition-2.5
From equation (2.9), differentiating \( w_m^m \) with respect to \( \alpha \) gives

\[
\frac{d w_m^m}{d \alpha} = -\frac{(a-bc)}{2b} \left[ \frac{2(1 + k)}{(2 - \alpha)^3} + \frac{(1 + k)(2 - \alpha)(6 - 2\alpha - \alpha) - (4 - \alpha - \alpha)(2 + 2\alpha + 2 - 2\alpha - \alpha^2 k)}{(4 - \alpha - \alpha)^3} \right]
\]

For any \( k \in (0, 1) \) the right hand side of the above expression is negative, i.e., the optimal bargaining wholesale price of the manufacturer decreases with increasing CSR of the channel.

Again,

\[
w_m^m - c = \frac{(a-bc)}{2b} \left[ \frac{(2 - 2\alpha + \alpha)}{(2 - \alpha)} - \frac{(2 - \alpha)(6 - 2\alpha - \alpha)}{2(4 - \alpha - \alpha)^2} \right]
\]

Now, \( w_m^m - c < 0 \) if \( (4 + 4k - k^2 - 2k^3)\alpha^3 - 2\alpha^2(16 + 6k - 7k^2) + 4\alpha(19 - 4k) > 0 \). Simplifying the inequality and solving it for \( \alpha \), it is found that \( w_m^m < c \) if \( \alpha \in (\phi(k), 1) \), \( 0 \leq \phi(k) \leq 1 \). Where \( \phi(k) \) is given by

\[
\phi(k) = \frac{m}{3L} - \frac{2^{1/3}(-m^2 + 3Ll)}{3s} + \frac{s}{2^{1/3}3l}, \quad (A1.18)
\]

Where, \( l = 4 + 4k - k^2 - 2k^3, \quad m = 2(16 + 6k - 7k^2), \quad n = 76 - 16k \) and \( s = [1080l^2 + 2m^3 - 9lmn + \sqrt{4(-m^2 + 3ln)} + (1080l^2 + 2m^3 - 9lmn)^{1/3}] \)

Note that \( \phi(k) \) is a function of the CSR sharing fraction \( k \). Since, \( \alpha \) is bounded above by 1, \( \phi(k) \) must be bounded above by 1. If one plot (A1.18) with respect to \( k \) (see fig-2.3) then \( \phi(k) \leq 1 \) for any \( k \in (0, 0.425) \). Thus, \( w_m^m \leq 1 \) for any \( \alpha \in (\phi(k), 1), \quad 0 \leq k \leq 0.425 \). In particular \( \phi(0.425) = 0.73 \), i.e., the wholesale price of the manufacturer is less than its marginal production cost for \( \alpha \in (0.73, 1) \) and \( k \in (0, 0.425) \) (see fig-2.3).

Similarly, \( w_m^m < 0 \) if \( [(4 + 4k - k^2 - 2k^3)(a-bc) + 4(1 + k)^2bc]\alpha^3 - [2(16 + 6k - 7k^2)(a-bc) + 8(5 + 6k + k^2)bc]\alpha^2 + 4\alpha[(19 - 4k)(a-bc) + 16(2 + k)bc] - (40a + 88bc) > 0 \), i.e., for \( \alpha \in (\psi(k), 1) \), where \( \psi(k) \) is the real root of the equation \((4 + 4k - k^2 - 2k^3)(a-bc) + 4(1 + k)^2bc\alpha^3 - [2(16 + 7k - 7k^2)(a-bc) + 8(5 + 6k + k^2)bc]\alpha^2 + 4\alpha[(19 - 3k)(a-bc) + 16(2 + k)bc] - (40a + 88bc) = 0 \) and it is given by

\[
\psi(k) = \frac{M}{3L} - \frac{2^{1/3}(-M^2 + 3LN)}{3LS} + \frac{S}{2^{1/3}3L}, \quad (A1.19)
\]

\( L = (4 + 4k - k^2 - 2k^3)(a-bc) + 4(1 + k)^2bc, \quad M = 2(16 + 6k - 7k^2)(a-bc) + 8(5 + 6k + k^2)bc, \quad N = 4(19 - 4k)(a-bc) + 64(2 + k)bc, \quad S = [27L^2T + 2M^3 - 9LMN + \sqrt{4(-M^2 + 3LN)} + (27L^2T + 2M^3 - 9LMN)^{1/3}] \)

And \( T = 40a + 88bc \). In this case also \( \psi(k) \) is a function of \( k \) but the range of \( k \) for which \( \psi(k) \) lies in its range (0, 1) that depends on the system parameters. In particular, suppose \( a = 200, \quad b = 0.5, \quad c = 50 \). Now \( \psi(k) \leq 1 \) when \( k \in (0, 0.174) \), also the minimum value of \( \alpha \) for which wholesale price of the manufacturer is less zero is 0.917. Thus the range of \( \alpha \) is \( \alpha \in (0.917, 1) \).

A-2.5: Proof of proposition-2.6

(i) From Table 2.4,

\[
\frac{d\pi_m^g}{d\alpha} = T[-128 + 288k - \alpha(104 + 24k + 192k^2) + \alpha^2(132 + 84k - 24k^2 - 48k^3) - \alpha^3(40 + 56k - 6k^2 - 14k^3 + k^4) - \alpha^4(4 + 8k + 3k^2 - 3k^3 - k^4)]
\]

\( \alpha \leq 0 \) if \(-128 + 288k - \alpha(104 + 24k + 192k^2) + \alpha^2(132 + 84k - 24k^2 - 48k^3) - \alpha^3(40 + 56k - 6k^2 - 14k^3 + k^4) + \alpha^4(4 + 8k + 3k^2 - 3k^3 - k^4) > 0 \). Simplifying and solving the inequality for \( \alpha \), provides

\[
\beta(k) = \frac{c_2}{4c_1} - \frac{G}{2} - \frac{\sqrt{G - \left( \frac{c_2^2}{c_1^2} + \frac{4c_2c_3}{c_1^2} - \frac{8c_4}{c_1} \right)}}{2}, \quad (A1.20)
\]
Where \( G = \sqrt{\frac{c_2^3}{4e_1} - \frac{2c_3}{3c_1} + \frac{\sqrt{\frac{1}{3}}(c_3^2 - 3c_2^2c_4 + 12c_1c_5 + k_2^3)}{3c_1F_1^{1/3}}} + \frac{\sqrt[3]{\frac{1}{3}}}{3c_12^{1/3}}, \) \( F = 2c_3^2 - 9c_2c_3c_4 + 27c_1c_2^3 + 27c_2^3c_5 - 72c_1c_3c_5 + \sqrt{-4(c_2^3 - 3c_2c_4 + 12c_1c_5)^3} + (2c_2^3 - 9c_2c_3c_4 + 27c_1c_2^3 + 27c_2^3c_5 - 72c_1c_3c_5)^2, \) \( c_1 = 4 + 8k + 3k^2 - 3k^3 + 2k^4, \) \( c_2 = -160k - 6k^2 - 14k^3 + 4k^4, \) \( c_3 = 132 + 84k - 24k^2 + 8k^3, \) \( c_4 = -104 + 24k + 192k^2, \) \( c_5 = -128 + 288k. \) The pure profit of the manufacturer increases for \( \alpha < \beta(k), \) but \( \beta(k) \) is a function of \( k \) and \( 0 \leq k, \beta(k) \leq 1. \) The equation (A.20) gives \( \beta(k) \geq 0 \) if \( k \geq 0.445 \) and \( \beta(k) \leq 1 \) if \( k \leq 0.811. \) Thus, pure profit of the manufacturer is increasing when 0.445 < \( k \leq 0.811 \) and \( \alpha < \beta(k). \) But, if \( k > 0.811 \) then \( \beta(k) > 1 \) and obviously \( \pi_{m/g} \) is increasing for all \( \alpha \in (0, 1). \)

(ii) The table-2.4 gives
\[
\frac{d\pi_{r/g}}{da} = \frac{6}{3} \left[ (152 - 24k - 128k^2) + \alpha^2(60 + 108k + 24k^2 - 48k^3) - \alpha^3(8 + 40k + 54k^2 + 14k^3 - 4k^4) \right]^{(4(\alpha - \alpha))^3(4 - \alpha - \alpha)}
\]
Now, \( \frac{d\pi_{r/g}}{da} > 0 \) if 128 - 288k - \( \alpha(152 - 24k - 192k^2) + \alpha^2(60 + 108k + 24k^2 - 48k^3) - \alpha^3(8 + 40k + 54k^2 + 14k^3 - 4k^4) \) > 0. Simplifying and solving the inequality for \( \alpha, \) provides
\[
\eta(k) = \frac{c_2}{4e_1} - \frac{H}{2} - \frac{\sqrt{H - \frac{1}{3} \left( \frac{-c_1e_1^2}{e_1} + 4c_2e_1 \right)^3 + \frac{8e_1^2}{e_1}}} {2} \tag{A1.21}
\]
Where \( H = \sqrt{\frac{c_2^3}{4e_1} - \frac{2c_3}{3c_1} + \frac{4^{1/3}(c_3^2 - 3c_2^2c_4 + 12c_1c_5 + k_2^3)}{3c_1F_1^{1/3}}} + \frac{\sqrt[3]{\frac{1}{3}}}{3c_12^{1/3}}, \) \( F_1 = 2c_3^3 - 9c_2c_3c_4 + 27c_1c_2^3 + 27c_2^3c_5 - 72c_1c_3c_5 + \sqrt{-4(c_2^3 - 3c_2c_4 + 12c_1c_5)^3} + (2c_2^3 - 9c_2c_3c_4 + 27c_1c_2^3 + 27c_2^3c_5 - 72c_1c_3c_5)^2, \) \( c_1 = 4 + 9k + 7k^2 + 2k^4, \) \( c_2 = -160k - 6k^2 - 14k^3 + 4k^4, \) \( c_3 = 132 + 84k - 24k^2 + 8k^3, \) \( c_4 = -104 + 24k + 192k^2, \) \( c_5 = -128 + 288k. \) The pure profit of the retailer is increasing if \( \alpha < \eta(k), \) but \( \eta(k) \) is a function of \( k \) and \( 0 \leq k, \eta(k) \leq 1. \) The equation (A21) gives \( \eta(k) \geq 0 \) if \( 0.445 < k \leq 0.811 \) and \( \eta(k) \leq 1 \) if \( 0 \leq k < 0.17. \) Thus, it follows that for any \( \alpha \in (0, \min \{\eta(k), 1\}) \) and \( k \in (0, 0.445), \) the pure profit of the retailer increasing otherwise it decreases.

(iii) From (i) and (ii) one can easily realize that the equilibrium pure profits of the manufacturer and the retailer decrease for \( \alpha \in ((\min \{\beta(k), 1\}, 1) \cup (\min \{\eta(k), 1\}, 1)), k \in (0.17, 0.811) \)
(iv) The pure profit of the manufacturer is negative if \( [(20 - 12\alpha - 80k + 2\alpha^2 + 2\alpha^2k + \alpha^2k^2)/2(4 - \alpha - \alpha - \alpha - \alpha)] \) - \( \alpha(1-k)/(2-\alpha) < 0, \) i.e., if \( (4 + 4k - k^2 - 2k^3)\alpha^3 - 2\alpha^2(16 + 6k - 7k^2) + 4\alpha(19 - 4k) - 40 > 0. \) It is the same condition as \( w_c^m < c \) in proposition-2.5.

(v) The pure profit of the retailer is negative if \( [(6 - 2\alpha - \alpha - \alpha - \alpha - \alpha)/2(4 - \alpha - \alpha - \alpha - \alpha)] \) - \( \alpha(1-k)/(2-\alpha) < 0, \) i.e., if \( (4k + 5k^2 + 2k^3)\alpha^3 - 2\alpha^2(2 + 14k + 9k^2) + 4\alpha(5 + 12k) - 24 > 0. \) Simplifying the inequality and solving it for \( \alpha, \) it is found that the pure profit of the retailer is negative if \( \alpha \in (\xi(k), 1), 0 \leq \xi(k) \leq 1. \) Where \( \xi(k) \) is given by
\[
\xi(k) = \frac{m_1}{3l_1} - \frac{2^{1/3}(l_1^2 + 3l_1n_1)}{3l_1s_1} + \frac{s_1}{2^{1/3}l_1} \tag{A1.22}
\]
Where, \( l_1 = 4 + 4k - k^2 - 2k^3, m_1 = 2(16 + 6k - 7k^2), n_1 = 76 - 16k, \) and \( s_1 = \sqrt{68l_1^2 + 2m_1^2 - 9l_1m_1n_1 + \sqrt{4(-m_1^3 + 3l_1n_1)^3 + 6(48l_1^2 + 2m_1^2 - 9l_1m_1n_1)^2}}/3. \) Note that \( \xi(k) \) is a function of the CSR sharing fraction \( k. \) Also, \( \xi(k) \leq 1. \) One can easily obtain \( \xi(k) \leq 1 \) when \( k \in (0.425, 1). \)

A-2.6: Proof of proposition-2.8

Using the wholesale price of (2.14) the equilibrium pure profits of the retailer and the manufacturer are respectively found as
\[
\pi_{r/g}|_{k=0.5} = \left( \frac{48 - 88\alpha + 45\alpha^2 - 7\alpha^3}{(2 - \alpha)(8 - 3\alpha)^2} \right) \pi_c \tag{A1.23}
\]
\[ \pi_{m/g|k=0.5} = \left( \frac{80 - 136\alpha + 69\alpha^2 - 11\alpha^3}{(2 - \alpha)(8 - 3\alpha)^2} \right) \pi_c \quad (A1.24) \]

When the manufacturer is socially responsible using table-2.4

\[ \pi_{m/r} > \pi_{m/m} \]

i.e., \( \pi_{m/r} > \pi_{m/m} \) for any \( \alpha \in (0, 1) \). Also, using table-2.4

\[ \pi_{m/m} - \pi_{m/g|k=0.5} = \frac{(576\alpha - 756\alpha^2 + 374\alpha^3 - 83\alpha^4 + 7\alpha^5)\pi_c}{(2 - \alpha)(4 - \alpha)^2(8 - 3\alpha)^2} \leq 0 \quad (A1.26) \]

i.e., \( \pi_{m/m} < \pi_{m/g|k=0.5} \) for any \( \alpha \in (0, 1) \). Finally,

\[ \pi_{m/r} - \pi_{m/g|k=0.5} = \frac{(208\alpha - 247\alpha^2 + 98\alpha^3 - 13\alpha^4)\pi_c}{2(2 - \alpha)^2(8 - 3\alpha)^2} \geq 0 \quad (A1.27) \]

i.e., \( \pi_{m/r} > \pi_{m/g|k=0.5} \) for any \( \alpha \in (0, 1) \). Hence using (A1.25), (A1.26) and (A1.27) one can conclude that \( \pi_{m/m} \leq \pi_{m/g|k=0.5} \leq \pi_{m/r} \) for all \( \alpha \in (0, 1) \).

Similarly, when the retailer is CSR using table-2.4

\[ \pi_{r/r} - \pi_{r/m} = \frac{(88\alpha - 87\alpha^2 + 28\alpha^3 - 3\alpha^4)\pi_c}{2(2 - \alpha)^2(4 - \alpha)^2} < 0 \quad (A1.28) \]

i.e., \( \pi_{r/r} < \pi_{r/m} \) for any \( \alpha \in (0, 1) \). Also, using table-2.4

\[ \pi_{r/m} - \pi_{r/g|k=0.5} = \frac{(576\alpha - 756\alpha^2 + 374\alpha^3 - 83\alpha^4 + 7\alpha^5)\pi_c}{(2 - \alpha)(4 - \alpha)^2(8 - 3\alpha)^2} \geq 0 \quad (A1.29) \]

i.e., \( \pi_{r/m} > \pi_{r/g|k=0.5} \) for any \( \alpha \in (0, 1) \) and

\[ \pi_{r/r} - \pi_{r/g|k=0.5} = \frac{(208\alpha - 247\alpha^2 + 98\alpha^3 - 13\alpha^4)\pi_c}{2(2 - \alpha)^2(8 - 3\alpha)^2} \leq 0 \quad (A1.30) \]

i.e., \( \pi_{r/r} < \pi_{r/g|k=0.5} \) for any \( \alpha \in (0, 1) \). Thus, using (A1.28), (A1.29) and (A1.30) one can conclude that \( \pi_{r/m} \geq \pi_{r/g|k=0.5} \geq \pi_{r/r} \) for all \( \alpha \in (0, 1) \).

A-2.7: Proof of proposition-2.17

The profits of the jth retailer, the jth distributor and the manufacturer in the proposed mechanism must be greater than or equal to their respective decentralized profit. That is, \( \pi_{i/r}^{e/r} > \pi_{i/r}^{*} \), \( \pi_{d/r}^{d/r} > \pi_{d/r}^{*} \), and \( v_{m/r}^{e/r} > v_{m/r}^{*} \). The first inequality gives

\[ \phi_{ij} \geq \left( \frac{(b + \theta)(2 - \alpha)^2\pi_{ij}^{*}}{a_{ij} + \theta P - (b + \theta)c_i} \right) \pi_{ij}^{d/r} = \phi_{ij}^{d/r} \quad (A1.31) \]

Decentralized profit of the jth-distributor and decentralized total profit of the manufacturer corresponding to the jth-retailer are given by

\[ \pi_{ij}^{d/r} = (D_{ij}^{*} / \sum_{i=1}^{n_j} D_{ij}^{*}) \pi_{ij}^{d/r} \quad (A1.32) \]
On the other hand, profit of the jth-distributor and total profit of the manufacturer corresponding to the ith-retailer under RS contract is given by

\[ \pi_{d/rs}^{ij} = \mu_{ij}(1 + \phi_{ij})[a_{ij} + \theta P - (b + \theta)c]^2 \]

\[ \phi_{ij} + \mu_{ij} = 1 - \frac{(b + \theta)(2 - \alpha)^2 v_{ij}^{m*}}{[a_{ij} + \theta P - (b + \theta)c]^2} \]  

Using the inequality (A1.36) in (A1.37), we get the maximum value of \( \phi_{ij} \) as

\[ \phi_{ij} = 1 - \frac{(b + \theta)(2 - \alpha)^2 (v_{ij}^{m*} + \pi_{ij}^e)}{[a_{ij} + \theta P - (b + \theta)c]^2} \]

From (A1.36), one can get the minimum value of \( \mu_{ij} \) when \( \phi_{ij} = \bar{\phi}_{ij} \) and is given by

\[ \mu_{ij} = \frac{(b + \theta)(2 - \alpha)^2 \pi_{ij}^e}{(1 + \phi_{ij})[a_{ij} + \theta P - (b + \theta)c]^2} \]

Simplification yields

\[ \mu_{ij} = \frac{2(b + \theta)(2 - \alpha)^2 \pi_{ij}^e}{(4 - \alpha)[a_{ij} + \theta P - (b + \theta)c]^2 - 2(b + \theta)(2 - \alpha)^2(v_{ij}^{m*} + \pi_{ij}^e)} \]

Thus,

\[ \mu_{ij} = \frac{\sum_{i=1}^{n_j} [D_{ij}^{e/rs} \mu_{ij}]}{\sum_{i=1}^{n_j} [D_{ij}^{e/rs}]} \]

Similarly, from (A1.37), one can get the maximum value of \( \mu_{ij} \) when \( \phi_{ij} = \bar{\phi}_{ij} \) and is found as follows

\[ \mu_{ij} = \frac{1}{1 + \phi_{ij}} \left[ 1 - \frac{\phi_{ij} - \alpha}{[a_{ij} + \theta P - (b + \theta)c]^2} \right] \]

Simplification yields

\[ \mu_{ij} = \frac{[a_{ij} + \theta P - (b + \theta)c]^2}{[a_{ij} + \theta P - (b + \theta)c]^2 + (b + \theta)(2 - \alpha)^2 \pi_{ij}^e} \left[ 1 - \frac{(b + \theta)(2 - \alpha)^2 (\pi_{ij}^* + v_{ij}^{m*})}{[a_{ij} + \theta P - (b + \theta)c]^2} \right] \]
Thus,
\[
\bar{\mu}_j = \frac{\sum_{i=1}^{n_j} [D_{ij}^{rs} \bar{\mu}_{ij}]}{\sum_{i=1}^{n_j} [D_{ij}^{rs}]} \tag{A1.42}
\]

Hence, if \(\phi_{ij} \in (\phi_{ij}, \bar{\phi}_{ij})\) and \(\mu_{ij} \in (\mu_{ij}, \bar{\mu}_{ij})\) for \(i = 1, 2, \ldots, n_j; j = 1, 2, \ldots, n\), then the RS contract not only can be successfully implemented to coordinate the channel but also it provides win-win opportunity to the channel members. From (2.56) and (2.58) win-win range of wholesale price of the jth distributor and the manufacturer corresponding to the ijth retailer are found as

\[
w^{d,rs}_{ij} = \frac{(b + \theta)(2 - \alpha)}{T_{ij}^2} \left[ 2c - \frac{\alpha(a_{ij} + \theta P)}{b + \theta} \right] \pi_{ij}^{rs} \tag{A1.43}
\]

\[
w^{d,rs}_{ij} = \frac{\left( \frac{1}{2} - \frac{(b + \theta)(2 - \alpha)(\pi_{ij}^{rs} + v_{ij}^{rs})}{T_{ij}} \right) \left[ 2c - \frac{\alpha(a_{ij} + \theta P)}{b + \theta} \right]}{T_{ij}[(4 - \alpha)T_{ij}^2 - (2(b + \theta)(2 - \alpha)(\pi_{ij}^{rs} + v_{ij}^{rs}))]} \tag{A1.44}
\]

\[
w^{d,rs}_{ij} = \left( \frac{2(2 - \alpha)\pi_{ij}^{rs}}{2T_{ij}^2 - (2(b + \theta)(2 - \alpha)(\pi_{ij}^{rs} + v_{ij}^{rs}))} \right) \left[ 2(b + \theta)c - \frac{\alpha(a_{ij} + \theta P)}{(b + \theta)(2 - \alpha)} - \frac{2(2 - \alpha)\pi_{ij}^{rs}}{T_{ij}} \right] \tag{A1.45}
\]

\[
w^{d,rs}_{ij} = \left( \frac{(2 - \alpha)T_{ij}^2 - (2(b + \theta)(2 - \alpha)(\pi_{ij}^{rs} + v_{ij}^{rs}))}{2T_{ij}^2 - (2(b + \theta)(2 - \alpha)(\pi_{ij}^{rs} + v_{ij}^{rs}))} \right) \left[ 2(b + \theta)c - \frac{\alpha(a_{ij} + \theta P)}{(b + \theta)(2 - \alpha)} - \frac{2(2 - \alpha)\pi_{ij}^{rs}}{T_{ij}} \right] \tag{A1.46}
\]

Where \(T_{ij} = a_{ij} + \theta P - (b + \theta)c\)

### Appendix-3.1: Solution procedure to find optimal solution

**Input parameter values**

**Procedure 1** // Decentralized decision //

- \(TCD = 0, D_m = 0\)
- for \(j = 1 \text{ to } n\)
- \(TCR_j = 0, D_j^j = 0\)
- for \(i = 1 \text{ to } n_j\)
  - calculate \(T_{ij}^{rs}, Q_{ij}^{rs}\) and \(C_{ij}^{rs}\) using equations (3.40), (3.41) and (3.42)
  - \(T_{ij}^{rs} = TCR_j + C_{ij}^{rs}\)
  - \(D_j^j = D_j^j + Q_{ij}^{rs} / T_{ij}^{rs}\)
  - write output \(T_{ij}^{rs}, Q_{ij}^{rs}, C_{ij}^{rs}\) and \(TCR_j\)
- \(TCD = TCD + C^{rs}\)
- \(D_m = D_m + Q_{ij}^{rs} / T_{ij}^{rs}\)
- write output \(T_{ij}^{rs}, Q_{ij}^{rs}, C_{ij}^{rs}\) and \(TCD\)

**Procedure 2** // Semi-centralized decision //

- calculate \(T_{ij}^{rs}, Q_{ij}^{rs}\) and \(C_{ij}^{rs}\) using equations (3.51), (3.52) and (3.50)
- \(N_\alpha = 1, A = 0\)
- repeat
  - calculate \(C_{ij}^{rs}\) using (3.53)
  - \(A = C_{ij}^{rs}\)
  - \(N_\alpha = N_\alpha + 1\)
- until \((C_{ij}^{rs} > A)\)
- write output \(N_\alpha, A\) as distributors’ cost
\( N_\beta = 1, B = 0 \)
repeat
  calculate \( C^{m*} \) using (3.54)
  \( \text{B} = C^{m*} \)
  \( N_\beta = N_\beta + 1 \)
until \( (C^{m*} > B) \)
write output \( N_\beta, B \) as manufacturer’s cost

Procedure 3 // Centralized decision //
calculate \( T^*, Q_{ij}^{rc*}, Q_{ij}^{dc*}, C_{ij}^{rc*}, C_{ij}^{dc*} \) and \( C^{mcc*} \) using equations (3.57) to (3.62)

Procedure 4 // Minimum and maximum contract limits for retailers and manufacturer //
for \( (j = 1 \text{ to } n) \)
  for \( (i = 1 \text{ to } n_j) \)
    calculate \( \lambda_{ij} \) using (3.64)
    calculate \( C^{m*}, C^{mcc*} \) and \( \overline{p}_j \) using equations (3.66), (3.67) and (3.68)
Procedure 5 // Backward contract-bargaining process //
for \( (j = 1 \text{ to } n) \)
  \( C^{db}_j = s_{ij}/T^* \)
for \( (i = 1 \text{ to } n_j) \)
  calculate \( C_{ij}^{dc*}, C_{ij}^{d*} \) and \( \overline{\lambda}_{ij} \) using (3.69), (3.70) and (3.71)
  \( \overline{C^{db}}_j = C^{db}_j + \overline{\lambda}_{ij} DC_{ij}(T^*) \)
  determine \( \lambda_{ij} \) using (3.75)
determine \( \rho_j^b \) and \( \rho_j^f \) using (3.77) and (3.79)
for \( (j = 1 \text{ to } n) \)
  for \( (i = 1 \text{ to } n_j) \)
    calculate \( C^{r*b*} \) using (3.80)
    calculate \( C_j^{db*} \) using (3.81)
calculate \( C^{m*b*} \) using (3.82)
write output \( \lambda_{ij}^b, C_j^{rb*}, C_j^{db*}, C_j^{m*b*} \)
Procedure 6 // Forward contract-bargaining process //
for \( (j = 1 \text{ to } n) \)
  calculate \( \rho_j^f, \rho_j^f \) and \( C_j^{df} \) using (3.83), (3.85) and (3.86)
for \( (j = 1 \text{ to } n) \)
  for \( (i = 1 \text{ to } n_j) \)
    calculate \( C_{ij}^{df*}, \overline{\lambda}_{ij} \) and \( \lambda_{ij}^f \) using (3.87), (3.88) and (3.89)
for \( (j = 1 \text{ to } n) \)
  for \( (i = 1 \text{ to } n_j) \)
    calculate \( C_{ij}^{rf*} \) using (3.91)
    calculate \( C_j^{df*} \) using (3.92)
calculate \( C^{m*f*} \) using (3.93)
write output \( \lambda_{ij}^f, C_{ij}^{rf*}, C_j^{df*}, C_j^{m*f*} \)

Appendix-3.2: Decentralized Collusion Solution

The necessary conditions to obtain the optimal total expected profit of the downstream retail market yield (i.e., \( \frac{\partial \pi}{\partial p_1} = 0 \) and \( \frac{\partial \pi}{\partial p_2} = 0 \))

\[
p_1 = \frac{1}{2} \left[ \frac{a_2 \alpha_1 + \beta \alpha_2}{a_1 a_2 \beta^2 + c_{t_1} + w_d} \right] \\
\]

(A3.1)
and
\[ p_2 = \frac{1}{2} \left\{ a_1 \alpha_2 + \beta \alpha_1 + c_{r2} + w_{dp} \right\}. \] (A3.2)

Taking the second-order partial derivatives of \( \pi_r \) with respect to \( p_1 \) and \( p_2 \)
\[ \partial^2 \pi_r / \partial p_1^2 = -2a_1 \text{ and } \partial^2 \pi_r / \partial p_2^2 = -2a_2 \]
and \( \partial^2 \pi_r / \partial p_1 \partial p_2 = \partial^2 \pi_r / \partial p_2 \partial p_1 = 2 \beta \)

Let \( \Delta^1_r \) and \( \Delta^2_r \) denote respectively the first and second-order principal minors of Hessian matrix of the total channel profit, \( \pi_r \). Then

\[ \Delta^1_r = -2a_1 < 0 \text{ and } \Delta^2_r = 4a_1 a_2 - 4 \beta^2 > 0 \]

Hence, the Hessian matrix of the total profit of the retailers, is negative definite. Thus, For any given \( w_{dp} \), the total expected profit for the downstream retail market, \( \pi_r \) is a concave function of \( p_1 \) and \( p_2 \).

Total demand of the downstream retail market is
\[ D_1 + D_2 = A_1 - \frac{1}{2} (a_1 + a_2 - 2 \beta) w_{dp} \] (A3.3)
where \( A_1 = \alpha_1 + \alpha_2 - (\alpha_1 - \beta)[(a_2 \alpha_1 + \beta \alpha_2)/(a_1 a_2 - \beta^2) + c_{r1}] / 2 - (a_2 - \beta)[(a_1 \alpha_2 + \beta \alpha_1)/(a_1 a_2 - \beta^2) + c_{r2}] / 2 \).

The distributor’s expected profit function is
\[ E(\pi^d) = [A_1 - \frac{1}{2} (a_1 + a_2 - 2 \beta) w_{dp}] [w_{dp} + E(r)w_{dl} - (1 + E(r))(s + c^d + w_m)]. \] (A3.4)

Now, \( \frac{d^2 \pi^d}{dw_{dp}^2} = -(a_1 + a_2 - 2 \beta) < 0 \) (due to \( \beta < a_i; i = 1, 2 \)), the distributor’s expected profit, \( \pi^d \), is a concave function of \( w_{dp} \). The distributor’s optimal wholesale price is (solving \( \frac{d \pi^d}{dw_{dp}} = 0 \))
\[ w_{dp}^c = \frac{A_1}{a_1 + a_2 - 2 \beta} + \frac{(1 + E(r))(s + c^d + w_m) - E(r)w_{dl}}{2}. \] (A3.5)

Using (3.117) and (3.115) in (3.98) yields the expected profit function of the manufacturer \( \pi^m \). \( \pi^m \) is a concave function of \( w^m \) (since, \( \frac{d^2 \pi^m}{dw_{dm}^2} = -(1 + E(r))^2(a_1 + a_2 - 2 \beta)/2 < 0 \) due to \( \beta < a_i \)). The necessary condition, i.e., \( \frac{d \pi^m}{dw_{dm}} = 0 \), to obtain the maximum profit of the manufacturer yields
\[ w_{m, max}^c = c + \frac{1}{1 + E(r)} \left\{ \frac{A_1}{(a_1 + a_2 - 2 \beta)} - \frac{(1 + E(r))(s + c^d + c) - E(r)w_{dl}}{2} \right\}. \] (A3.6)

Substituting (A3.6) in (A3.5), the distributor’s optimal wholesale price of the perfect quality products found as follows:
\[ w_{dp}^c = \frac{3A_1}{2(a_1 + a_2 - 2 \beta)} + \frac{1}{4}[(1 + E(r))(s + c^d + c) - E(r)w_{dl}]. \] (A3.7)

Using (A3.7), the optimal selling prices of the perfect quality product at the retailer’s end are
\[ p_{1c} = \frac{1}{2} \left\{ a_2 \alpha_1 + \beta \alpha_2 + c_{r1} + \frac{3A_1}{2(a_1 + a_2 - 2 \beta)} + \frac{1}{4}[(1 + E(r))(s + c^d + c) - E(r)w_{dl}] \right\}. \] (A3.8)
and
\[
p_{2}^{\text{opt}} = \frac{1}{2} \left[ \frac{a_{1}a_{2} + \beta a_{1}}{a_{2}a_{2} - \beta^2} + c_{r_{2}} + \frac{3A_1}{2(a_1 + a_2 - 2\beta)} + \frac{1}{4}[(1 + E(r))(s + c^d + c) - E(r)w_{dp}] \right]. \quad (A3.9)
\]

Using (A3.6), (A3.7), (A3.8) and (A3.9), the expected profits of the manufacturer, the distributor and the ith \((i = 1, 2)\) retailer are displayed in table-3.6.

**Appendix-3.3: Decentralized Stackelberg Solution**

For any given \(w_{dp}\) and \(p_{1}\), the maximizing-profit of retailer-2 responds with the retail price of \(p_2 = [a_2 + \beta p_1 + a_2(c_{r_2} + w_{dp})] / (2a_2)\) (solving \(\frac{d\pi_2}{dp_2} = 0\)). Substituting retailer 2’s reaction function in retailer 1’s profit function
\[
\pi_{r_1} = (p_1 - w_{dp} - c_r)[2a_2a_1 - (2a_1a_2 - \beta^2)p_1 + \beta(a_2 + a_2c_{r_2} + a_2w_{dp})] / (2a_2). \quad (A3.10)
\]
Retailer-1’s profit is a function of \(p_1\) alone and a concave function of \(p_1\) (as \(\frac{d^2\pi_2}{dp_1^2} = -2(2a_1a_2 - \beta^2) < 0\) due to \(\beta < a_1\)). Thus, for any \(w_{dp}\) set by the distributor, retailer-1 who acts as the leader can obtain his optimal retail price below, setting \(\frac{d\pi_1}{dp_1} = 0\),
\[
p_1^{\text{opt}} = F_1 + F_3w_{dp} \quad (A3.11)
\]
where \(F_1 = [2a_2a_1 + \beta a_2 + (2a_1a_2 - \beta^2)c_{r_1} + a_2c_{r_2}] / [2(2a_1a_2 - \beta^2)]\) and \(F_3 = [(2a_1a_2 - \beta^2 + a_2\beta)] / [2(2a_1a_2 - \beta^2)]\). This optimal retail price will give the maximum profit of the retailer-1 as
\[
\pi_{r_1}^{\text{opt}} = [2a_2a_1 + \beta a_2 - (2a_1a_2 - \beta^2)(w_{dp} + c_{r_1}) + a_2\beta(w_{dp} + c_{r_2})] / [8a_2(2a_1a_2 - \beta^2)]. \quad (A3.12)
\]
Retailer-2’s optimal sales price and maximum profit from follower-ship can be determined by substituting retailer-1’s optimum leadership retail price in retailer-2’s reaction function. Then, the relevant expressions are
\[
p_{2}^{\text{opt}} = F_2 + F_4w_{dp}, \quad (A3.13)
\]
where \(F_2 = [2a_2a_1 + a_2(4a_1a_2 - \beta^2) + \beta(2a_1a_2 - \beta^2)c_{r_1} + a_2(4a_1a_2 - \beta^2)c_{r_2}] / [4a_2(2a_1a_2 - \beta^2)]\) and \(F_4 = [\beta(2a_1a_2 - \beta^2) + a_2(4a_1a_2 - \beta^2)] / [4a_2(2a_1a_2 - \beta^2)]\). Thus the maximum profit of the retailer-2 is
\[
\pi_{r_2}^{\text{opt}} = \frac{[2a_2a_1 + a_2(4a_1a_2 - \beta^2) + \beta(2a_1a_2 - \beta^2)(w_{dp} + c_{r_1}) - a_2(4a_1a_2 - 3\beta^2)(w_{dp} + c_{r_2})]}{16a_2(2a_1a_2 - \beta^2)^2}. \quad (A3.14)
\]
The distributor follows the retailers’ reaction functions \(p_{1}^{\text{opt}}\) and \(p_{2}^{\text{opt}}\) for any \(w_{dp}\) value she sets for perfect quality products. The profit function of the distributor is
\[
\pi^d = (D_1 + D_2)w_{dp} + E(r)(D_1 + D_2)w_{dl} - (1 + E(r))(D_1 + D_2)(s + c^d + w^m). \quad (A3.15)
\]
The total downstream demand is thus \(D_1 + D_2 = F - F_5w_{dp}\) where \(F = a_1 + a_2 - (a_1 - \beta)F_1 - (a_2 - \beta)F_2\) and \(F_5 = (a_1 - \beta)F_3 + (a_2 - \beta)F_4\).
Since \(\frac{d^2E(\pi^d)}{dw_{dp}^2} = -2N_5 < 0\) (due to \(\beta < a_1\) and \(F_3 > 0, F_4 > 0\)). Thus, distributor’s expected profit, \(E(\pi^d)\), is a concave function of \(w_{dp}\). Distributor’s optimal wholesale price is obtained by solving
\[
\frac{dE(s^d)}{dw_{dp}} = 0 \quad \text{as follows:}
\]
\[
w_{dp}^{sg} = \frac{F}{2F_5} + \frac{1}{2}(1 + E(r))w^m + \frac{(1 + E(r))(s + c^d) - E(r)w_{di}}{2}.
\]
(A3.16)

As the manufacturer knows the distributor’s reaction function \(w_{dp}^{sg}\) for any \(w^m\) value he/she can sets to maximize the profit:
\[
\pi^m = (1 + E(r))(D_1 + D_2)(w^m - c).
\]
(A3.17)

The manufacturer’s expected profit, \(E(\pi^m)\), is also a concave function of \(w^m\) (as \(\frac{d^2\pi^m}{dw^2} = -(1 + E(r))^2F_5 < 0\) due to \(\beta < a_1\), \(F_3 > 0\), and \(F_4 > 0\). Therefore, when the duopolistic retailers play Stackelberg game in the downstream retail market, the manufacturer will set an optimal wholesale price as ( solving \(\frac{d\pi^m}{dw^m} = 0\) )
\[
w^m = \frac{1}{2}(1 + E(r)) \left[ \frac{F}{F_5} - (1 + E(r))(s + c^d + c) + E(r)w_{di} \right].
\]
(A3.18)

\[
w_{dp}^{sg} = \frac{1}{4} \left[ \frac{3F}{F_5} + (1 + E(r))(s + c^d + c) - E(r)w_{di} \right]
\]
(A3.19)

\[
p_{1}^{sg} = F_1 + F_5 \left[ \frac{3F}{F_5} + (1 + E(r))(s + c^d + c) - E(r)w_{di} \right]
\]
(A3.20)

and
\[
p_{2}^{sg} = F_2 + F_4 \left[ \frac{3F}{F_5} + (1 + E(r))(s + c^d + c) - E(r)w_{di} \right].
\]
(A3.21)

The expected profits are given in table-1.

Appendix-3.4 Comparison of the wholesale prices

Comparison of the wholesale price of the manufacturer and the distributor under three scenario are as follows.
\[
w^{mct} - w^{mcn} = \frac{(a_2 - a_1)\beta\Psi}{2B(1 + E(r))(a_1 + a_2 - 2\beta)(4a_1a_2 - \beta^2)}
\]
(A3.22)

Where \(\Psi = [(a_2 - \beta)a_1 - (a_1 - \beta)a_2 - (a_1a_2 - \beta^2)](c_1 - c_2)\). Clearly, \(w^{mct} = w^{mcn}\) when \(\beta = 0\) or \(a_2 = a_1\) or \(\Psi = 0\). But \(w^{mct} > w^{mcn}\) if \((a_2 - a_1)\Psi > 0\) i.e., either \(a_2 > a_1\) & \(\Psi > 0\) or \(a_2 < a_1\) & \([a_2 - \beta)a_1 - (a_1 - \beta)a_2 - (a_1a_2 - \beta^2)](c_1 - c_2) < 0\). Otherwise \(w^{mct} < w^{mcn}\).

\[
w^{mct} - w^{msg} = \frac{\beta^2(2a_1a_2 - \beta)(a_2 + \beta)}{8B(1 + E(r))F_5(8a_1^2a_2^2 - 6a_1a_2\beta^2 + \beta^4)}
\]
(A3.23)

Now, \(w^{mct} = w^{msg}\) if \(\beta = 0\) or \(\Psi = 0\). If \(\Psi > 0\) then \(w^{mct} > w^{msg}\) otherwise \(w^{mct} > w^{msg}\). Finally comparing the wholesale price of the manufacturer between collusion and stackelberg scenario yields,
\[
w^{mcn} - w^{msg} = \frac{\beta(2a_1a_2 - 2a_2^2 + a_2\beta - \beta^2)}{8(1 + E(r))F_5(a_1 - 2\beta)a_2(2a_1a_2 - \beta^2)}
\]
(A3.24)

That is, \(w^{mcn} = w^{msg}\) if \(\beta = 0\) or \((2a_1a_2 - 2a_2^2 + a_2\beta - \beta^2) = 0\) or \(\Psi = 0\). \(w^{mcn} > w^{msg}\) if \((2a_1a_2 - 2a_2^2 + a_2\beta - \beta^2)\) and \(\Psi > 0\) have same sign i.e., either both are positive or both negative. If \((2a_1a_2 - 2a_2^2 + a_2\beta - \beta^2)\) and \(\Psi = 0\) are alternate in sign then \(w^{mcn} < w^{msg}\).

Now Comparing the wholesale price of the distributor under three scenario
\[
w^{ct}_{dp} - w^{cn}_{dp} = \frac{3(a_2 - a_1)\beta\Psi}{4B(a_1 + a_2 - 2\beta)(4a_1a_2 - \beta^2)}
\]
(A3.25)
w^{ct}_{dp} - w^{ct}_{dp} = \frac{3\beta^2(2a_1a_2 - \beta(a_2 + \beta))\Psi}{16BF_5(8a_1^2a_2^2 - 6a_1a_2\beta^2 + 3\beta^4)} \tag{A3.26}

w^{cn}_{dp} - w^{cn}_{dp} = \frac{3\beta(2a_1a_2 - 2a_2^2 + a_2\beta - \beta^2)\Psi}{16F_3(a_1 + a_2 - 2\beta)a_2(2a_1a_2 - \beta^2)} \tag{A3.27}

Hence, one can summarize that if \( \beta = 0 \) and/or \( \Psi = 0 \) then \( w^{mct} = w^{msg} = w^{mcn} \) & \( w^{ct}_{dp} = w^{ct}_{dp} = w^{cn}_{dp} \) and other results are as follows.

(i) \( w^{mcn} > w^{msg} > w^{mct} \) if \( \Psi < 0 \) and \( (2a_1a_2 - 2a_2^2 + a_2\beta - \beta^2) < 0 \).

(ii) \( w^{msg} > w^{mct} > w^{mcn} \) if \( \Psi < 0 \), \( (2a_1a_2 - 2a_2^2 + a_2\beta - \beta^2) > 0 \) and \( a_2 < a_1 \).

(iii) \( w^{msg} > w^{mcn} > w^{mct} \) if \( \Psi < 0 \), \( (2a_1a_2 - 2a_2^2 + a_2\beta - \beta^2) > 0 \) and \( a_2 > a_1 \).

(iv) \( w^{mct} > w^{mcn} > w^{msg} \) if \( \Psi > 0 \), \( (2a_1a_2 - 2a_2^2 + a_2\beta - \beta^2) > 0 \) and \( a_2 > a_1 \).

(v) \( w^{mcn} > w^{mct} > w^{msg} \) if \( \Psi > 0 \), \( (2a_1a_2 - 2a_2^2 + a_2\beta - \beta^2) > 0 \) and \( a_2 < a_1 \).

(vi) \( w^{mct} > w^{msg} > w^{mcn} \) if \( \Psi > 0 \) and \( (2a_1a_2 - 2a_2^2 + a_2\beta - \beta^2) < 0 \).

The wholesale prices of the distributor have same outcome as wholesale price of the manufacturer and hence omitted.

**Appendix-3.5: Effect of the rate of imperfect quality product**

\[
\frac{d\pi^{mcn}}{dE(r)} = -\frac{(a_1 + a_2 - 2\beta)(s + c^d + c - w_{adt})}{8} \bigg[ \frac{2A_1}{(a_1 + a_2 - 2\beta)} - X_1 \bigg] \tag{A3.34}
\]

\[
\frac{d\pi^{msg}}{dE(r)} = -\frac{B(s + c^d + c - w_{adt})}{4} \bigg[ \frac{F}{F_5} - X_1 \bigg] \tag{A3.35}
\]

\[
\frac{d\pi^{mcn}}{dE(r)} = -\frac{(a_1 + a_2 - 2\beta)(s + c^d + c - w_{adt})}{16} \bigg[ \frac{2A_1}{(a_1 + a_2 - 2\beta)} - X_1 \bigg] \tag{A3.36}
\]

\[
\frac{d\pi^{ct}_{ci}}{dE(r)} = -\frac{a_i(2a_1a_{a3-i} - a_{a3-i}\beta - \beta^2)(s + c^d + c - w_{adt})}{2(4a_1a_{a3-i} - \beta^2)}, \quad (i = 1, 2) \tag{A3.39}
\]

\[
\frac{d\pi^{cn}_{ci}}{dE(r)} = -\frac{(s + c^d + c - w_{adt})}{8} \bigg[ c_{ci}^{cn} + (a_i - \beta)(p_{ci}^{cn} - w_{dp}^{cn} - c_{ri}) \bigg], \quad (i = 1, 2) \tag{A3.40}
\]
Proof of proposition 4.1

For given $n^r$

\[
\frac{\partial \pi_i^{r/ds}}{\partial p_i^{r/ds}} = \frac{L}{n^r} \left[ (1 - k_1)a + r \frac{ak_1 + (b + r) \frac{Lh^r}{2n^r}}{2(b + r)} - \frac{r}{2} C_i - \left[ C_i + \frac{Lh^m}{2n^r} \right] \right] + (b + r) \left( \frac{r^2}{2(b + r)} - w_i \right), \quad i = 1, 2, \ldots, n^r \tag{A4.2} \]

\[
\frac{\partial^2 \pi_i^{m/ds}}{\partial p_i^{d/ds} \partial w_i} = 2 \left( \frac{r^2}{2(b + r)} - (b + r) \right) \tag{A4.3} \]

\[
\frac{\partial^2 \pi_i^{m/ds}(w_i, p_i^{d/ds})}{\partial p_i^{d/ds} \partial w_i} = -(b + r) \left( \frac{r^2}{2(b + r)} - w_i \right) \tag{A4.4} \]

\[
\frac{\partial^2 \pi_i^{m/ds}(w_i, p_i^{d/ds})}{\partial w_i^2} = \left( \frac{r^2}{2(b + r)} - w_i \right) \tag{A4.5} \]

\[
\frac{\partial^2 \pi_i^{m/ds}(w_i, p_i^{d/ds})}{\partial p_i^{d/ds} \partial w_i} = -r \left( \frac{r^2}{2(b + r)} - w_i \right) \tag{A4.6} \]

\[
\frac{\partial^2 \pi_i^{m/ds}(w_i, p_i^{d/ds})}{\partial w_i \partial p_i^{d/ds}} = 2 \left( \frac{r^2}{2(b + r)} - w_i \right) \tag{A4.7} \]

Equating (A4.2) and (A4.3) to zero and solving for $p_i^{d/ds}$ and $w_i$ the result can be realized. Substituting the optimal values of $p_i^{d/ds}$ and $w_i$ in Eq. (4.9), one can get the retailer's optimal selling price in $i$th, $i = 1, 2, \ldots, n^r$ replenishment cycle displayed in Eq. (4.10). Substituting the optimal values of $p_i^{d/ds}$ and $p_i^{r/ds}$ in Eq. (4.1) and Eq. (4.2), one can get the demand of the product in retail and direct channel per unit time for $i$th, $i = 1, 2, \ldots, n^r$ replenishment cycle and multiplying these with cycle length one can get the amount of quantity sold through retail and online channel respectively which are displayed in table-4.1. Also, the order quantity that the manufacturer faces is equal to the product of total demand and cycle length, the profit functions of the manufacturer and the retailer in
Proof of proposition 4.2

For given $n^r$, $c(t) = u_1 - u_2t$. Thus for $i = 1$,

$$p_i^{d/ds} - p_{i+1}^{d/ds} = \frac{1}{2} [c(0) - c(L/n^r)] = \frac{u_2 L}{2n^r} > 0$$

that is, $p_1^{d/ds} > p_2^{d/ds}$.

For $i = 2$,

$$p_i^{d/ds} - p_{i+1}^{d/ds} = \frac{1}{2} [c(L/n^r) - c(2L/n^r)] = \frac{u_2 L}{2n^r} > 0$$

that is, $p_2^{d/ds} > p_3^{d/ds}$.

For $i = m$,

$$p_i^{d/ds} - p_{i+1}^{d/ds} = \frac{1}{2} [c((m-1)L/n^r) - c((m)L/n^r)] = \frac{u_2 L}{2n^r} > 0$$

that is, $p_m^{d/ds} > p_{m+1}^{d/ds}$.

Hence, one can say $p_i^{d/ds} > p_{i+1}^{d/ds}$ for all $i = 1, 2, \ldots, n^r$ i.e., $p_1^{d/ds} > p_2^{d/ds} > \ldots > p_{n^r}^{d/ds}$.

Other results can be obtained in similar way and hence omitted.

Proof of proposition 4.4

Comparing the selling prices of the product in retail channel and direct channel in $i$th, $i = 1, 2, \ldots, n^r$ replenishment cycle one can get

$$p_r^{r/ds} - p_{r+1}^{r/ds} = \frac{1}{2} [c(L/n^r) - c(L/n^r)] = \frac{u_2 L}{2n^r} > 0$$

Again comparing $k_{1i}^{dc}$ with $k_{1i}^{max}$ gives,

$$k_{1i}^{max} - k_{1i}^{dc} = \frac{(b+2r)(a-2bc)}{2(5b+6r)a} + \frac{(b+2r)(7b+8r)h^r + (b+4r)h^m L}{4an'(5b+6r)} > 0$$

and comparing $k_{1i}^{dc}$ with $k_{1i}^{min}$ provides

$$k_{1i}^{dc} - k_{1i}^{min} = \frac{2(b+r)(a-2bc)}{(5b+6r)a} + \frac{(b+r)(7b+8r)h^r + (b+4r)h^m L}{an'(5b+6r)} > 0$$

Hence, $k_{1i}^{dc} \in (k_{1i}^{min}, k_{1i}^{max})$. Thus one can easily realize that $p_i^{r/ds} > p_i^{d/ds} > w_i^*$ if $k_1 \in (k_{1i}^{dc}, k_{1i}^{max})$ and $p_i^{d/ds} > p_i^{r/ds} > w_i^*$ if $k_1 \in (k_{1i}^{min}, k_{1i}^{dc})$.

Proof of proposition 4.6
Using Cardano’s method for solving the cubic equation yields
\[ n_0^* = \left\{ \begin{array}{ll}
[n_0] & \text{if } \pi^{r/ds}(\lceil n_0 \rceil) > \pi^{r/ds}(\lfloor n_0 \rfloor) + 1 \text{ otherwise}
\end{array} \right. \]

Proof of proposition 4.7

From equation (4.16) \( \frac{\partial \pi^r}{\partial p^r} = 0 \) gives
\[ 2r p^d_i - 2(b + r) p^r_i + k_1 a + \frac{(b + r) h^r T^c}{2} - \frac{r h^m T^c}{2} + b c [(i - 1) T^c] = 0 \]
and \( \frac{\partial n^r}{\partial p^r} = 0 \) gives
\[-2(b + r)p^d_i + 2r p^r_i + (1 - k_1) a - \frac{r h^r T^c}{2} + \frac{(b + r) h^m T^c}{2} + b c [(i - 1) T^c] = 0 \]
Solving one can obtain the selling price of the product for retail and direct channel in \( i \)th, \( i = 1, 2, \ldots, n^r \) replenishment cycle.
Again,
\[ \frac{\partial^2 \pi^r_i(p_{i}^{rc}, p_{i}^{dc})}{\partial p_{i}^{rc}} = -2(b + r) \]
\[ \frac{\partial^2 \pi^r_i(p_{i}^{rc}, p_{i}^{dc})}{\partial p_{i}^{dc}} = -2(b + r) \]
\[ \frac{\partial^2 \pi^{m/ds}_i(p_{i}^{rc}, p_{i}^{dc})}{\partial p_{i}^{dc} \partial p_{i}^{rc}} = 2r \frac{\partial^2 \pi^{m/ds}_i(p_{i}^{rc}, p_{i}^{dc})}{\partial p_{i}^{dc} \partial p_{i}^{rc}} \]
\[ \frac{\partial^2 \pi^r_i(p_{i}^{rc}, p_{i}^{dc})}{\partial p_{i}^{rc}} \times \frac{\partial^2 \pi^r_i(p_{i}^{rc}, p_{i}^{dc})}{\partial p_{i}^{dc}} - \left( \frac{\partial^2 \pi^{m/ds}_i(p_{i}^{rc}, p_{i}^{dc})}{\partial p_{i}^{dc} \partial p_{i}^{rc}} \right)^2 = 4[(b + r)^2 - r^2] > 0 \]
That is, \( \pi^r_i \) is a concave function of \( p_{i}^{rc} \) and \( p_{i}^{dc} \).

Proof of proposition 4.8
c(t) = u_1 - u_2t. Thus for i = 1,

\[ p_{1}^{rc} - p_{2}^{rc} = \frac{1}{2} [c(0) - c(L/n^c)] = \frac{u_2 L}{2n^c} > 0 \]

that is, \( p_{1}^{rc} > p_{2}^{rc} \).

For \( i = 2 \),

\[ p_{2}^{rc} - p_{3}^{rc} = \frac{1}{2} [c(L/n^c) - c(2L/n^c)] = \frac{u_2 L}{2n^c} > 0 \]

that is, \( p_{2}^{rc} > p_{3}^{rc} \).

For \( i = m \),

\[ p_{m}^{rc} - p_{m+1}^{rc} = \frac{1}{2} [c((m-1)L/n^c) - c((m)L/n^c)] = \frac{u_2 L}{2n} > 0 \]

that is, \( p_{m}^{rc} > p_{m+1}^{rc} \).

Hence, one can say \( p_{1}^{rc} > p_{2}^{rc} > \cdots > p_{n}^{rc} \) for all \( i = 1, 2, \ldots, n^c \) i.e., \( p_{1}^{rc} > p_{2}^{rc} > \cdots > p_{n}^{rc} \).

Other results can be obtained in similar way and hence omitted.
Channel coordination and profit distribution in a social responsible three-layer supply chain

S. Panda a,⁎, N.M. Modak b, M. Basu b, S.K. Goyal c

a Department of Mathematics, Bengal Institute of Technology, 2nd Floor, Civil Lines, Kolkata 700050, West Bengal, India
b Department of Supply Chain Management, John Molson School of Business, Concordia University, Montreal, Canada H3C1M8

Article Info
Article history:
Received 14 April 2014
Accepted 24 June 2015

Abstract
This paper analyzes coordination of a manufacturer-distributor-retailer supply chain, where the manufacturer exhibits corporate social responsibility (CSR). In manufacturer-stackelberg game setting, the paper proposes a contract-bargaining process to resolve channel conflict and to distribute surplus profit among the channel members. The contract-bargaining process consists of two wholesale price discount-Nash bargaining. One between the distributor and the retailer based on the outcome of that between the distributor and the manufacturer. Although the contract-bargaining process cuts out channel conflict and distributes surplus profit, the wholesale prices are quite different from those of a pure profit maximizing supply chain. The wholesale price of the manufacturer is less than its marginal production cost above a threshold of CSR. Even it is negative for the manufacturer’s heavy CSR practice. So, the manufacturer’s profit may be negative. The behavior of the wholesale price of the distributor is same as that of the manufacturer but for higher threshold of CSR.

1. Introduction
Coordination through cooperation is imperative for improving channel wide performance because it offers the potential to realize substantial profit benefit. To coordinate a supply chain, contracts are designed among the decentralized decision makers such that the difference between outcome of a centralized decision and a decentralized decision can be neutralized. The basic objective behind designing a coordination contract is to incentivize decentralized channel members to act coherently with one another. A variety of side-payment contracts (e.g. quantity discount, Li and Liu, 2008; two-part tariff, Coeling, 2012; Modak et al., 2015c; revenue sharing, Panda, 2013a, 2014a, sales rebate, Wong et al., 2009, buy back, Ding and Chen, 2008, credit option, Du et al., 2013, commitment to purchase quantity, Zhang et al., 2011, mail-in-rebate, Saha et al., 2015, etc. have been used in supply chains as the ways of cutting out channel conflict. These contracts differ by the contractual clauses among the channel members and are primarily concerned with quantity, time, quality and price.⁴

CSR is a form of corporate self-regulation that currently does not have unique definition. Broadly CSR can be defined as a doctrine that promotes expanded social stewardship by businesses and organizations. CSR suggests that corporations embrace responsibilities toward a broader group of stakeholders (customers, employees and the community at large) in addition to their customary financial obligations to stockholders. In the current global business environment CSR is now a determining factor in consumer and client choice, in which companies cannot afford to ignore. According to the results of a global survey in 2002 by Ernst and Young, 94% of companies believe the development of a CSR strategy can deliver real business benefits, however only 11% have made significant progress in implementing the strategy in their organization. Senior executives from 147 companies in a range of industry sectors across Europe, North America and Australasia have been interviewed for the survey. The survey has concluded that CEOs are failing to recognize the benefits of implementing CSR strategies, despite increased pressure to include ethical, social and environmental issues into their decision-making processes. For example, on social issue, largest apparel retailer GAP admits to charge of its substandard working conditions in as many as 3000 factories worldwide (Merrick, 2004). Nike is often accused for inhuman labor and business practices in Asian manufacturing factories (Amaeshi et al., 2008). For environmental issues, in 2009 a group of 186 institutional investors having assets of 13 trillion US dollars have signed a statement. It suggests directions to

Please cite this article as: Panda, S., et al., Channel coordination and profit distribution in a social responsible three-layer supply chain, International Journal of Production Economics (2015), http://dx.doi.org/10.1016/j.ijpe.2015.07.001
deal with global warming and greenhouse gases (Economist, 2009). The research has found that company CSR programs influence 70% of all consumer purchasing decisions, with many investors and employees also being swayed in their choice of companies. Recent empirical evidence shows that customers are willing to pay a higher price for products with CSR attributes (Ageron et al., 2012). Modak et al. (2015a) indicates that firms can strategically engage in socially responsible activities to increase private profits. Given that the firms’ stakeholders may value the firm’s social efforts, the firm can obtain additional benefits from enhancing the firm’s reputation and the ability to generate profits by differentiating its product. As a result, many leading international brands like Walmart, Nike, Adidas, GAP have been impelled to incorporate CSR in their complex supply chains by a code of conducts (Amaeshi et al., 2008).

This paper intends to merge two research areas, CSR and channel coordination in a three-echelon supply chain that consists of a manufacturer, a distributor and a retailer. A retailer pursues pure profit motive. The manufacturer has the intention to swell stakeholder’s welfare by exhibiting CSR. In a manufacturer-Stackelberg game setting the paper proposes a contract-bargaining process to resolve channel conflict and to distribute surplus profit among the channel members. In the contract-bargaining process the manufacturer first provides wholesale price discount to the distributor and bargains with the retailer for profit share. Based on the intermediate profit, the distributor provides wholesale price discount and bargains with the retailer for profit share. While formulating the model instead of considering the manufacturer’s CSR activities the paper considers the effect of CSR in the form of consumer surplus in the manufacturer’s profit. So, the socially responsible manufacturer maximizes its pure profit plus a share of consumer surplus that it accrues from its stakeholders (Lambertz and Tankperi, 2010; Goering, 2007, 2008; Kogel and Brand, 2012). The underlying principle of the paper is based on the classic paper of Vickers (1985) and hence the result supports the result of Vickers that non-profit maximizing firm may earn higher profits than would profit-maximizers. Here the objective of the manufacturer is to engage in CSR and to find the effects that CSR tends to bring about. The outcome of the paper indicates that when the manufacturer concentrates more on CSR than profit, its total profit is always higher than pure profit. On the other hand, the channel behaves more competitively than a pure profit maximizing supply chain by exhibiting CSR because it generates higher output by setting lower price. That is, wholesale price of the manufacturer behaves differently from that of a pure profit maximizing supply chain. The CSR has considerable impact on the wholesale price it may be less than marginal production cost or even negative for heavy CSR activity. Although total profit of the channel member increase, the pure profits may be zero of less, which is not desirable. Thus, for acceptable pure profit and for exhibition of social responsibility there must be a limit of CSR up to which a firm can practice CSR.

2. Literature review

Although use of coordination contract to cut out double marginization in two-echelon supply chain has been explored extensively, models dealt with resolving channel conflict in three-echelon supply chain are notably fewer. In practice it is more difficult to resolve channel conflict in a three-tire supply chain by applying coordination contract due to two tiers in supply chain. When the number of echelon increases, self-cost minimizing/profit maximizing objectives increase. As a result, division of the solution space increases and the channel coordination using contract becomes more complex. Also, many difficulties remain when it comes to carry out any coordination contract for channel members. For example geographical constraints, administrative problems, performance measurement and incentives at individual firms based on local perspective, dynamically interchanging products and the like (Kanda and Deshmukh, 2008). Focusing on multi-echelon supply chain (Munson and Rosenblatt, 2001) has developed a supplier-manufacturer-retailer chain and has explored channel coordination using quantity discount. Jaber et al. (2006) have extended Munson and Rosenblatt’s model by assuming profit function, discount dependent demand and Covington et al. (2010) have studied a three-echelon supply chain with learning based continuous improvement. Saha et al. (2013) have considered a three-echelon supply chain coordination problem, where demand is linear in price. They have used mail-in rebate and down-and-direct discount for channel coordination. Jaber and Goyal (2008) have investigated the coordination of order quantities in a three-tire supply chain, where they have allowed more than one member at each echelon. Ding and Chen (2008) have used flexible buy back contract to coordinate a three-level supply chain, where the profit is divided among the channel members freely. Panda et al. (2014a) have used disposal cost share and disposal cost contract for a manufacturer-distributor-retailer chain that deals with perishable product. They have assumed that the manufacturer and the distributor form a coalition and the coalition shares the retailer’s disposal cost. Mirdisk et al. (2015a) have considered a three-tire supply chain, where in the downstream two retailers play Cournot, Collusion and Stackelberg games. They have used two-part tariff and franchise for channel coordination and have performed a preference analysis for the channel members’ preference of game behavior.

Although there is a rich content on CSR consideration in individual firm in a supply chain, application of CSR in the entire supply chain has emerged in the last two decades. Murphy and Post (2002) have considered a CSR supply chain and have suggested a total responsibility approach by adding social issues to traditional economy. Through a case study and survey research Carter and Jennings (2004) have explained the necessity of CSR consideration in supply chain decision making. Using French sample data Agron et al. (2012) have found several conditions, which lead to a successful sustainable supply chain management. For an environmentally responsible supply chain network Cruz (2008) has traced equilibrium conditions by using a multi-criteria decision making approach. Cruz and Wåckolbiner (2008) have extended the model to multi-period setting for measuring long-term effects of CSR. Hsieh and Chang (2008) have considered a socially responsible supply chain network and have demonstrated that social responsibility sharing through monetary transfer leads to channel optimization. Panda et al. (2014b) have developed a two-echelon supply chain, where either the manufacturer or the retailer practices CSR and have used quantity discount to coordinate the chain. Savaskan et al. (2004) have focused on identifying a socially responsible close loop supply chain that is involved in product manufacturing and remanufacturing. Cruz (2009) has developed a decision support system framework for modeling and analysis of a CSR supply chain network. Ni et al. (2010) have developed a two-tire CSR supply chain by assuming that the dominant upstream channel member’s CSR cost is shared by the downstream channel member through wholesale price contract. Ni and Li (2012) have developed two-tire/echelon supply chain by assuming that each channel member has individual CSR cost. They have examined the effects of strategic interactions between the channel members under game theoretic setting. Hsieh (2014) has used a new revenue sharing contract to coordinate a CSR supply chain. Considering French sample data Crieu et al. (2014) have analyzed how different combinations of CSR affect economic performance and have compared the result based on quality of CSR and quantity of CSR. In this direction the works of Modak et al. (2015b), Chen and Storck (2015), Ding et al. (2015), and Subramanian and Gunasekaran (2015) are worthy mentioning.

Bargaining refers to situations where two or more players, who have the opportunity to collaborate from mutual benefit in

Please cite this article as: Panda, S., et al., Channel coordination and profit distribution in a socially responsible three-layer supply chain, International Journal of Production Economics (2015), http://dx.doi.org/10.1016/j.ijpe.2015.07.001
more than one way. There are two streams, axiomatic approach and strategic approach, of research and application of bargaining theory. The axiomatic approach requires the resulting solution should possess a set of axioms, whereas in the strategic approach the outcome is predicted by the concept of subgame perfect equilibrium. Bargaining in practice is the relationship that involves bargaining in the supply chain, in which a buyer and a seller negotiate the terms of a quantity discount contract in an EOQ setting by applying the approach of Kalai and Smorodinsky (1975). Kalai and Smorodinsky (1975) model suggests that both parties equally share the system surplus to achieve channel coordination. Sheu (2011) has used Nash bargaining framework for a green supply chain to investigate the problem of negotiations between producers and reverse-logistics suppliers for cooperative agreements under government intervention. Gan et al. (2005) have examined coordination contract in three different cases for a supply chain with risk adverse agent. They have explored that their contract yields the Nash bargaining solution for the cases where the supplier as well as the retailer maximizes their own expected utility. Summary of cooperative bargaining models in supply chain can be found in review article of Nagarajan and Soic (2008). In this direction the works of Panda (2013a, 2014b) and Etrogol and Wu (2001) are worth mentioning.

The objectives of the paper differ significantly from the prior works as follows. First, previous researches have explored CSR effects of CSR on supply chain and channel coordination, respectively. In contrast the present paper examines the double marginalization issues in a socially responsible supply chain. Although Hsu and Chang (2008) have used exogenous monetary transfer to coordinate a socially responsible supply chain network, this paper uses an endogenous procedure not only to coordinate the channel but also to distribute the surplus profit among the channel members. Second, in Ni et al. (2010) the supplier performs CSR and the downstream firm shares the CSR cost through wholesale price contract though channel coordination is not examined. Assuming each channel member has CSR cost Ni and Li (2012) have found win-win profits through strategic interaction. The present paper assumes that the upstream channel member has CSR cost and demonstrates a procedure that finds optimal channel profit through coordination. Third, in almost all the papers in traditional supply chain management the double marginalization is resolved by using variety of side payment contract. However, in these settings, ex post to coordination, the question of how the shares are determined is left unaddressed (Nagarajan and Soic, 2008). Besides channel coordination the present paper uses Nash bargaining product to distribute surplus profit among the channel members.

3. Model description and basic analysis

Notations: The following notations are used in developing the model.

- $p$: unit selling price of the retailer
- $w$: wholesale price of the distributor
- $w_0$: unit wholesale price of the manufacturer
- $p_c$: unit selling price of the supply chain in centralized decision making
- $c_c$: marginal production cost of the manufacturer
- $q$: order quantity of the product
- $PP_c$: pure profit of centralized channel
- $PP_d$: pure profit of decentralized channel
- $\pi$: profit function of the retailer in decentralized decision making
- $\pi_s$: profit function of the manufacturer in decentralized decision making
- $\pi_d$: profit function of the distributor in decentralized decision making
- $\pi_{sd}$: profit function of the manufacturer in decentralized decision making
- $\mu$: discount on wholesale price that the manufacturer provides to the distributor
- $\rho$: discount on wholesale price that the distributor provides to the retailer
- $\tau$: maximum discount on wholesale price that the retailer provides to the distributor
- $\tau_s$: maximum discount on wholesale price that the distributor provides to the retailer
- $\tau_d$: maximum discount on wholesale profit that the distributor provides to the retailer

Assume that the demand at the retailer's end is linear in retail price and is of the form $D(p) = a - bp$, where $a > 0$ is the market potential and $b > 0$ is the customers price sensitivity. For the non-negativity of the demand function assume $p \in (0, a/b)$. This demand function is fairly common in the literature. Shortages are not allowed at any stage of the channel. The lead time between the manufacturer and the distributor, and between the distributor and the retailer are zero because the demand is deterministic. The manufacturer follows lot-for-lot production policy. This simple channel structure allows us to analyze the effect of CSR on channel members profits. Also, assume that the manufacturer is the leader of the channel and takes decision independently. Other channel members make decision based on the decision of the manufacturer.

As indicated, many leading brands face intense pressure for socially responsible supply chain management (Armaeshi et al., 2008). A commonly noted response to this pressure is the primary firm introduces code of conduct to its partners business practices to be socially responsible (Pedersen and Andersen, 2006). As a result, other members of the channel involve in CSR practice. Also, it is widely observed the main target in supply chain is at the manufacturer's side (Armaeshi et al., 2008). Thus, we assume the manufacturer invests in CSR and aligns its CSR goal with channel performance. The cost associated with the CSR is shared by all the channel members through a transfer pricing (Grun, 2008). In modeling and analysis, we only consider effects of CSR in the form of consumer surplus rather than the CSR activities, in which the social responsible channel performs. It is obvious that when a firm practices CSR irrespective of its rival firms, its goodwill increases because it widely expresses the intention to enhance the stakeholders welfare. As a result customers will pay higher price than its base price for the firms produced product. Thus, it is quite reasonable to incorporate consumer surplus in profit maximization sense as the effect of CSR practice. Furthermore, it is well established that a firm’s CSR is accounted through the consumer.
surplus of its stakeholders (Lambertini and Tampieri, 2010; Goering, 2007, 2008; Kopel and Brand, 2012; Ni et al., 2010). The consumer surplus is the difference between the maximum price that consumers are willing to pay for a product and the market price that they actually pay for the product. Thus, the consumer surplus is

\[ \text{Consumer surplus} = \int_{p_i}^{p_m} \left( \frac{a}{b} \right) dp - \text{Consumer surplus} \]

(1)

If \( a \in [0,1] \) is the fraction of CSR that is the socially responsible manufacturer's concern then it incorporates \( a^k, b \) as consumer surplus in its profit, \( a = 0 \) implies the manufacturer is the pure profit maximizer and \( a = 1 \) represents the manufacturer is the perfect welfare maximizer. Since the manufacturer is socially responsible, its profit function consists of pure profit that is received by supplying the product to the distributor and the consumer surplus through CSR practice. Under this setting we first derive the centralized and decentralized decisions of the channel members.

3.1. Centralized decision

Assume that all the channel members are willing to cooperate and want to implement joint decision. So, there is a single marketing channel in which a product is produced in a single lot and is sold to the customers at a retail price \( p_r \). Also, the channel practices CSR. Thus, some consumer surplus is accumulated from the stakeholders in the channel. The profit function of the channel is

\[ s_c = (p_r - c(a - b)) \left( a^k - bp_r \right)^2 \]

(2)

Using the necessary condition, \( ds_c/dp_r = 0 \), for the existence of the optimal solution, optimal value of \( p_r \) can be found and is depicted in Table 1. Also, the optimal order quantity, pure profit, consumer surplus and total profit in the centralized channel are presented in Table 1. Moreover, \( d^2 s_c/dp_r^2 = b(2 - \theta) < 0 \), i.e., \( p_r^* \) provides global optimum to (2).

Note that \( d^2 s_c/p_r^2 = \left( a^k(2 - \theta)^2 / b^2 \right) > 0 \), i.e., optimal total profit increases when the manufacturer puts more weight on CSR. \( d^2 s_c/p_r^2 = \left( a^k(2 - \theta)^2 / b^2 \right) > 0 \), i.e., the optimal total profit of the centralized channel increases and consumer surplus increases when CSR increases. The increment of consumer surplus is higher than the decrement of pure profit. As a result, total profit of the centralized channel increases with increasing CSR. The pure profit is zero and consumer surplus is maximum in the centralized channel when the manufacturer is the perfect welfare maximizer. Also, \( d^2 s_c/p_r^2 = \left( a^k(2 - \theta)^2 / b^2 \right) > 0 \), i.e., retail price of the channel decreases and order quantity of the channel increases with increasing CSR of the manufacturer. When the manufacturer acts as the perfect welfare maximizer, the optimal retail price \( \left( b_1, a_1, c_1 \right) \) is equal to the marginal production cost of the centralized channel. So, the pure profit of the channel vanishes and the manufacturer accrues maximum consumer surplus \( k \) from its stakeholders. On the other hand, when \( a = 0 \), total profit of the channel is \( k \), which is less than total profit of the perfect welfare maximizing channel. Thus, the socially responsible supply chain performs more competitively than a pure profit maximizing supply chain. It actually encourages the consumers to purchase more by reducing the retail price.

3.2. Decentralized decision

When the channel members operate independently and optimize their individual goals, it is essentially a non-cooperative decision making process, where the manufacturer is the leader of the channel. The CSR goal is the manufacturer’s own. Through a code of conduct the manufacturer induces other members of the channel to involve in CSR practice. We consider the manufacturer-Stackelberg game, where the distributor is the manufacturer’s immediate follower and the retailer follows the distributor. It is a sequential move game, where the manufacturer enforces its own strategy on the distributor. Based on it the distributor finds its own strategy and enforces it on the retailer. Finally, depending on the optimal strategy of the distributor, the retailer identifies its own strategy. In fact the entire decision making process consists of two Stackelberg games. One between the manufacturer and the distributor and the other is between the distributor and the retailer. We use backward induction to find the sub-game perfect solution of the game. The profit functions of the channel members are

\[ s_m = (W_m - c(a - b)) \]

(3)

\[ s_d = (W_d - W_m)(a - b) \]

(4)

Total profit of the manufacturer is

\[ s_m = a^k(2 - \theta) \]

(5)

Using backward induction the optimal solution can be found and are presented in Table 1. Observe that \( ds_m^2/dp_d^2 = 4k/(8 - \theta) > 0 \), \( ds_d^2/dp_d^2 = 8k/(8 - \theta)^2 > 0 \), \( ds_m^2/dp_r^2 = k/(8 - \theta)^2 > 0 \), but \( ds_d^2/dp_r^2 = -2k/(8 - \theta)^3 < 0 \). Also, the manufacturer’s total profit, the distributor’s profit and the retailers profit increase but the manufacturer’s pure profit decreases with increasing CSR. Also, \( d^2 s_d/dp_d^2 = -(a - bc)(8 - \theta)^3 < 0 \), \( d^2 s_d/dp_r^2 = a(8 - \theta)^2 > 0 \), \( d^2 s_d / dp_r^2 = -(a - bc)(8 - \theta)^3 < 0 \) when the manufacturer puts more weight on CSR, it reduces its wholesale price. In response, the distributor also reduces its wholesale price. Finally, the retailer reacts to the upstream channel members’ activities by reducing retail price. Since, the retailprice of the channel is reduced, customers are encouraged to buy more. As a result, the order quantity of the retailer increases. Thus, the CSR attribute of the manufacturer influences all the downstream members of the channel. By exhibiting CSR, the manufacturer acts more competitively than a pure profit maximizing manufacturer. Although it loses some pure profit, it acquires some consumer surplus, which is more than the loss of pure profit. So, its total profit increases. The pure profit that the manufacturer loses due to CSR is accumulated at

Table 1

<table>
<thead>
<tr>
<th>Optimal</th>
<th>Centralized channel</th>
<th>Decentralized channel</th>
</tr>
</thead>
<tbody>
<tr>
<td>Manufacturer</td>
<td>Distributor</td>
<td>Retailer</td>
</tr>
<tr>
<td>Price</td>
<td>(4 - a + 6b/c)</td>
<td>(6 - a + 2bc)/8 - b</td>
</tr>
<tr>
<td>Order quantity</td>
<td>4b - 2bc/8 - b</td>
<td>4b - 2bc/8 - b</td>
</tr>
<tr>
<td>Pure profit (PP)</td>
<td>4b - 2bc/8 - b</td>
<td>4b - 2bc/8 - b</td>
</tr>
<tr>
<td>Consumer surplus (CS)</td>
<td>4b - 2bc/8 - b</td>
<td>4b - 2bc/8 - b</td>
</tr>
<tr>
<td>Total profit (PP + CS)</td>
<td>4b - 2bc/8 - b</td>
<td>4b - 2bc/8 - b</td>
</tr>
</tbody>
</table>

Please cite this article as: Panda, S., et al., Channel coordination and profit distribution in a socially responsible three-layer supply chain, International Journal of Production Economics (2015), http://dx.doi.org/10.1016/j.ijpe.2015.07.001
the distributor’s and the retailer’s sides in some proportion. Thus, the profits of the distributor and the retailer increase as the manufacturer’s CSR intensity increases. Note that \( x_1^* + x_2^* + y_1^* = (14 - \theta_1 k)/(8 - \theta_1^2) < x_1^* \), i.e., the channel conflict is not resolved. \( d(x_1^* + x_2^* + y_1^*)/\theta_1 = 16(28 - \theta_1^2)/(8 - \theta_1^2) > 0 \), i.e., the difference of centralized total profit and decentralized total profit increases with increasing CSR (see Fig. 1). This result is quite different from a pure profit maximizing supply chain. In a pure profit maximizing supply chain the double marginalization of the channel decreases when the retailer reduces unit selling price. The channel is coordinated when the selling price is equal to the centralized selling price. But in a socially responsible supply chain though the retail price decreases with the manufacturer’s increasing CSR, the double marginalization of the channel increases. Also, \( x_1^* + x_2^* + y_1^* \mid x_1 > 7k/32 + k/2 = x_1^* \mid x_1 = 1 < 13/k(\theta_1 + k - x_1^* \mid y_1 = 1 - 13/k(\theta_1 + k) < x_1^* \mid y_1 = 13/k(\theta_1 + k) - 1). \) That is, pure profit maximizing channel is not coordinated and the perfect welfare maximizing motive of the manufacturer does not resolve channel conflict.

The pure profit of the centralized channel is \( PP_c = 2(1 - \theta_1 k)/(8 - \theta_1^2) \) and the pure profit of the decentralized channel is \( PP_d = x_1^* + x_2^* + y_1^* = 2(7 - \theta_1 k)/(8 - \theta_1^2) \). Now \( PP_c - PP_d = 0 \) if \( \theta_1 = 8 \theta_2 + 6 < 0 \), i.e., \( \theta_1 < 4 \sqrt{10} \) (see Fig. 1). Thus, the pure profit of the centralized channel is higher when compared with decentralized pure profit if \( \theta_1 > 0 \). Otherwise, the decentralized pure profit is higher. Therefore, we have the following proposition.

Proposition 1. In a socially responsible supply chain (i) the manufacturer’s perfect welfare maximizing motive does not resolve channel conflict, (ii) double marginalization increases when the retailer’s unit selling price decreases and (iii) the pure profit of the decentralized channel is higher for \( \theta_1 \leq 4 \sqrt{10} \) when compared with centralized pure profit.

It turns out that, when the channel members do not cooperate the manufacturer should limit its CSR in \( (4 \sqrt{10}, 1) \). In such case, the retailer’s profit and the distributor’s profit are higher but the manufacturer’s pure profit is lower compared to centralized profits. But the consumer surplus that the manufacturer accrues from its stakeholders compensates its loss of pure profit. Otherwise, when the channel members cooperate, the best channel performance can be found through a transfer pricing policy.

3.3. Channel coordination, ranges of win–win opportunities and surplus pure profit distribution

As a socially responsible channel member, the manufacturer always wants to receive order of larger lot size from the retailer because in that case it can operate more competitively through CSR practice. But the retailer has no reason to order larger lot size because initially it is less profitable. The manufacturer provides a wholesale price discount to the distributor, who is its immediate downstream channel member. In response, by providing a wholesale price discount the distributor impels the retailer to increase the order quantity. The channel members provide and accept the wholesale price discounts under two restrictions. First, the retailer must order channel optimal order quantity. Second, under any form of wholesale price discount the channel members must get at least their decentralized profits.

Generally, in a multi-echelon supply chain a channel member interacts with other members of the channel in one-to-one basis, where it assumes that there is no other member in the channel. The manufacturer can provide wholesale price discount to the distributor only if its decentralized total profit is reserved. The retailer accepts the distributor’s wholesale price discount and orders centralized quantity as long as its loss of profit is subsidized through the wholesale price discount. In the entire transfer pricing policy, the distributor plays the central role because as an intermediator it maintains the incentive streaming from the manufacturer to the retailer and maintains lot streaming from the retailer to the manufacturer. By doing so it also receives some profit additional to its decentralized profit. Thus, when the wholesale price discount policy is applied aiming at resolving channel conflict the distributors decides (i) the manufacturer’s minimum wholesale price discount and (ii) maximum wholesale price discount that it can provide to the retailer. These two are interrelated and one definitely gets different limits when approaches from (a) the manufacturer to the retailer and (b) the retailer to the manufacturer. In the former case, the manufacturer and the distributor jointly decide the minimum and maximum discounts on the manufacturer’s wholesale price for channel coordination. Within this range they decide a particular discount that effectively divides the surplus profit between them. Based on the decentralized profit and the surplus profit share, the distributor and the retailer find win–win wholesale price discount range and bargain for benefit share. In the latter case, first the retailer deals with the distributor and settles benefit share. Based on decentralized profit and benefit share, it deals with the manufacturer. Since the surplus profits at the manufacturer’s end and at the retailer’s end are different, the results are different in these two cases. However, we consider the approach from the manufacturer to the retailer through the distributor because the manufacturer is the leader of the channel. Also, it is quite common in marketing practice that any discount flows from the manufacturer to the customers through different echelons and the move is initiated at the manufacturer’s side. Since, two coordination contracts and two bargains are involved in the entire process we term it as a contract-bargaining process. In this process the channel members operate in the following sequence.

Step 1: The manufacturer and the distributor find the wholesale price discount range for win–win profits subject to the condition that the distributor has to compensate the retailer’s loss due to changed order quantity.

Step 2: Within the discount range the manufacturer bargains with the distributor for a particular profit share. The decentralized profit plus the surplus profit is the manufacturer’s optimal profit. The decentralized profit plus surplus profit is the distributor’s intermediate profit.
Step 3: Depending on the intermediate profit, the distributer and the retailer determines the win-win range of wholesale price discount.

Step 4: The distributer and the retailer determine surplus profit share through bargaining. Decentralized profit plus surplus profit share is the retailer’s optimal profit. Intermediate profit plus surplus profit is the distributer’s optimal profit.

Suppose the manufacturer provides $\mu w_m^d$ ($\mu > 0$) discount on wholesale price to the distributor. The manufacturer’s total profit under the wholesale price discount is

$$v_m^d = (w_m^d - c)\theta a - b\theta p d + \frac{\theta}{12}Q_m^2 - \mu w_m^d Q_r^2 \quad (7)$$

The manufacturer can provide the discount on the wholesale price as long as its decentralized total profit is reserved, i.e., $v_m^d \geq v_m^d$. If $\mu$ is the maximum discount on wholesale price then simplifying the inequality $\mu$ can be found as

$$\mu = 1 - \frac{\theta}{bQ_m}(2 - 6\theta a - 2\theta c)d$$

Therefore, the minimum wholesale price that the manufacturer can offer to the distributer is

$$w_m = (1 - \mu)w_m = c + \frac{2(6\theta a + 2\theta c - a - bc)}{b(2 - \theta a - \theta c)} \quad (9)$$

Similarly, the distributer can consider the manufacturer’s wholesale price discount until its decentralized profit and minimum compensation that it provides to the retailer are reserved. If the distributer demands $\theta (\mu w_m^d)$ minimum discount from the manufacturer then

$$v_m^d - (w_m^d - \theta (\mu w_m^d)Q_d^2) - \mu w_m^d Q_r^2 = e_1 + [e_2 - (w_m^d - \theta (\mu w_m^d)Q_d^2)]$$

Simplifying the above expression, $\mu$ can be found as

$$\mu = \frac{2(6\theta a - bc)}{b(6\theta a - \theta c)}$$

Consequently, the manufacturer can demand the maximum wholesale price as

$$\theta w_m^d = (1 - \mu)w_m = c + \frac{(6\theta a - bc)}{b(2 - \theta a - \theta c)} \quad (11)$$

For any $w_m < \theta w_m^d$ the manufacturer’s profit is win-win and, after providing minimum compensation to the retailer, the distributer’s profit is also win-win. Within this range the manufacturer bargains with the distributer for a particular wholesale price that effectively divides the surplus profit between them. The bargaining outcome is based on the symmetric (Nash, 1950) that has been used in various contexts, is an axiomatic derivation of bargaining solution. The axiomatic derivation leaves out the actual process of negotiations while focusing on the expected outcome based on prespecified solution procedures. Also the axioms do not reflect the rationale of the agents or the process in which the agreement is reached. One of the important characteristics of the Nash solution concept is that the outcome is random because it depends on the participating players negotiation powers. In Nash bargaining model the objective function is the product of the players benefit from cooperation and it must be maximized. Each player’s benefit is the difference between the negotiated profit and the profit under decentralized decision making. The Nash bargaining product of the manufacturer and the distributer is

$$\max \{\theta w_m^d - c - \theta (\mu w_m^d)Q_d^2 - \mu w_m^d Q_r^2 \}$$

From (12) optimal value of $\mu$ can be found as

$$\mu = \frac{6\theta a + \theta c}{2(6\theta a - \theta c)}$$

Therefore, the bargaining wholesale price of the manufacturer is

$$w_m^d = (1 - \mu)w_m = c + \frac{(6\theta a - bc)}{b(2 - \theta a - \theta c)} \quad (13)$$

After first round of bargaining the distributer’s intermediate profit is

$$\rho = \frac{18\theta a + \theta c}{(2 - \theta a - \theta c)}$$

Based on the intermediate profit, the distributer and the retailer determine the range of wholesale price discount. If $\pi w_m^d$ is the maximum discount on wholesale price that the distributer can provide to the retailer then

$$\rho = \frac{18\theta a + \theta c}{(2 - \theta a - \theta c)}$$

Therefore, the minimum wholesale price that the distributer can demand from the retailer is

$$\theta w_m^d = (1 - \mu)w_m = c + \frac{(6\theta a - bc)}{b(2 - \theta a - \theta c)} \quad (17)$$

For any $w_m < \theta w_m^d$ the distributer’s profit and the retailer’s profit under $\theta w_m$ wholesale price discount are win-win. Within the range the distributer and the retailer bargain for particular wholesale price, which effectively divides the surplus profit between them. The Nash bargaining product is

$$\max \{\pi w_m^d - \mu w_m^d Q_r^2 - e_2 [e_2 - (w_m^d - \theta (\mu w_m^d)Q_d^2)] + \rho w_m^d Q_r^2 \}$$

From (20) the optimal value of $\rho$ can be found as

$$\rho = \frac{9\theta a - \theta c}{24(2 - \theta a - \theta c)}$$

Consequently, the distributer’s optimal wholesale price in the contract-bargaining process is

$$w_m^d = (1 - \rho)w_m = c + \frac{(6\theta a - bc)}{b(2 - \theta a - \theta c)} \quad (22)$$

Thus, in the contract-bargaining process the optimal profits of the channel members are

$$v_m^d = v_m^d + [\frac{18\theta a + \theta c}{(2 - \theta a - \theta c)}]$$

$$\rho = \frac{9\theta a - \theta c}{24(2 - \theta a - \theta c)}$$
\[ x_t^* = x_t^* + \frac{3k}{2} \frac{1}{(2-\delta)(1-\delta)} \]  \hspace{1cm} (25)

\[ x_n^* = \frac{(34 - 58\delta + 14\delta^2 - 5\delta^3 - 5\delta^2) a - bc^2}{bc(2-\delta)(1-\delta)} \]  \hspace{1cm} (26)

Note that \( x_t^* + x_n^* = k/(2-\delta) = x_t^* \), i.e., the channel conflict is resolved. All the channel members' profits are win-win. The manufacturer takes away half of the surplus profit, 36/(2-\delta), and divides the surplus equally between the manufacturer and the retailer equitably. Thus, we have the following proposition.

**Proposition 2.** The contract-bargaining process resolves channel conflict and distributes surplus profit among the channel members.

### 3.4. Effects of CSR

Using the contract-bargaining process it is possible to find win-win total profit for the manufacturer and win-win pure profits for the distributor and the retailer for any \( w_1 \in (w_{min}, w_{max}) \) and for any \( w_2 \in (w_{min}, w_{max}) \). Note that \( dw_{1} / dw = \frac{(\delta a - \delta c d) (4\delta - 2\delta^2)}{4\delta - 2\delta^2} > 0 \) and \( dw_{2} / dw = \frac{\delta a - \delta c d}{4\delta - 2\delta^2} > 0 \) if \( \delta > 0 \). The wholesale price of the manufacturer, when contract-bargaining process is used, decreases with increasing CSR. Further, \( w_{max} = c \in (2-6\delta + \delta^2, 2-\delta) \) and \( \theta > \delta \) if \( \theta > 0 \). Also, \( w_{max} = c = (4-\delta a - \delta c d) b/(b - \delta) \). The objective of the manufacturer is not to decrease its wholesale price when the contract-bargaining process is used, if \( \theta > 0 \). The discounted wholesale price of the channel manufacturer is non-negative for any \( \theta > 0 \). The contract-bargaining process resolves channel conflict and distributes surplus profit among the channel members.

### 3.5. The Marginal Production Cost of the Manufacturer

The marginal production cost of the manufacturer, when contract-bargaining process is used, decreases with increasing CSR. Further, \( \delta a - \delta c d \) if \( \theta > 0 \). Also, \( w_{max} = c = (4-\delta a - \delta c d) b/(b - \delta) \). The objective of the manufacturer is not to decrease its wholesale price when the contract-bargaining process is used, if \( \theta > 0 \). The discounted wholesale price of the channel manufacturer is non-negative for any \( \theta > 0 \). The contract-bargaining process resolves channel conflict and distributes surplus profit among the channel members.

### 3.6. Effect of CSR

Using the contract-bargaining process it is possible to find win-win total profit for the manufacturer and win-win pure profits for the distributor and the retailer for any \( w_1 \in (w_{min}, w_{max}) \) and for any \( w_2 \in (w_{min}, w_{max}) \). Note that \( dw_{1} / dw = \frac{(\delta a - \delta c d) (4\delta - 2\delta^2)}{4\delta - 2\delta^2} > 0 \) and \( dw_{2} / dw = \frac{\delta a - \delta c d}{4\delta - 2\delta^2} > 0 \) if \( \delta > 0 \). The wholesale price of the manufacturer, when contract-bargaining process is used, decreases with increasing CSR. Further, \( w_{max} = c \in (2-6\delta + \delta^2, 2-\delta) \) and \( \theta > \delta \) if \( \theta > 0 \). Also, \( w_{max} = c = (4-\delta a - \delta c d) b/(b - \delta) \). The objective of the manufacturer is not to decrease its wholesale price when the contract-bargaining process is used, if \( \theta > 0 \). The discounted wholesale price of the channel manufacturer is non-negative for any \( \theta > 0 \). The contract-bargaining process resolves channel conflict and distributes surplus profit among the channel members.

### Proposition 3

In the contract-bargaining process the CSR manufacturer's wholesale price is always (i) less than its marginal production cost for \( \theta > 0 \) (Mintzberg, 1983). Also, \( \theta < 0 \) and \( \theta > 3/4 \) if \( \theta > 0 \). The effect of the equation is that if \( \theta < 0 \) and \( \theta > 3/4 \), the channels are win-win. The effect of the equation is that if \( \theta < 0 \) and \( \theta > 3/4 \), the channels are win-win.
The bargaining process is negative if \( \theta < \theta_k \). Similarly, the optimal bargaining wholesale price of the distributor is less than the manufacturer’s marginal production cost if \( \theta < \theta_k \). Also the optimal discount on the wholesale price that the distributor provides not only nullifies the retailer’s loss due to deviation from the decentralized order quantity but also provides some extra profit.

The optimal bargaining wholesale price of the manufacturer decreases with increasing CSR and it is less than the marginal production cost if \( \theta^2 - 14\theta + 59 - 34 \leq 0 \) i.e. if \( \theta > 6.809 \). The optimal wholesale price of the manufacturer in the contract bargaining process is negative if \( \theta < (\theta_k + 1) \), where \( \theta_k \) is the real root of the equation \( 34\theta^2 + 94\theta - 350 = 0 \). 

The optimal wholesale price of the contract bargaining process lies in \( (W_{\theta}^m, W_{\bar{\theta}}^m) \). The limit of CSR intensity for non negative wholesale price is lower than over all non-negative CSR intensity, i.e. \( \theta_k \) is always less than \( \theta_k \). Similarly, the optimal bargaining wholesale price of the distributor is less than the manufacturer’s marginal production cost if \( 102 - 14\theta + 32\theta^2 - 2\theta^3 \geq 0 \) i.e. \( \theta > 0.87538 \). Also the optimal discounted wholesale price of the distributor is negative if \( \theta > \theta_k \), say, where \( \theta_k \) is the real root of the equation \( 102\theta^2 + 154\theta - (143\theta + 49\theta\theta^2 - 32\theta - 4\theta^2) \geq 0 \) (see Fig. 4). Thus, we have the following proposition.

**Proposition 5.** (a) The optimal bargaining wholesale price of the manufacturer is less than its marginal production cost if \( \theta > 0.87538 \) and is negative for \( \theta > \theta_k \). (b) The optimal bargaining wholesale price of the distributor is less than the manufacturer’s marginal production cost if \( \theta > 0.87538 \) and is negative for \( \theta > \theta_k \).

Note that \( dW_{\theta}^m / d\theta = (140 - 59\theta + 8\theta^2)(2 - \theta^2)(8 - \theta^2) > 0 \) and \( dW_{\bar{\theta}}^m / d\theta = (-164 + 78\theta - 12\theta^2)(2 - \theta^2)(8 - \theta^2) < 0 \) and \( dW_{\theta}^m / d\theta = (250 - 90\theta + 12\theta^2)(2 - \theta^2)(8 - \theta^2) > 0 \), i.e., the optimal bargaining pure profit of the manufacturer decreases but optimal total profit increases with increasing CSR practice of the manufacturer. Also, \( dW_{\theta}^m / d\theta = -164 + 78\theta - 12\theta^2(2 - \theta^2)(8 - \theta^2) < 0 \) and an increase in the optimal bargaining pure profit of the manufacturer decreases but total profit increases with increasing CSR (see Fig. 5). The pure profit is maximum at \( \theta = 0 \) and the total profit is maximum when the manufacturer is the perfect profit maximizer. The manufacturer’s pure profit is non-negative for \( \theta \geq 0.6809 \) because in this range its wholesale price is larger than the marginal production cost. Also, in this range the manufacturer’s total profit is win-win. So, CSR is purely a costly endeavor to the manufacturer. Obviously for any \( \theta > 0.6809 \) the manufacturer’s bargaining pure profit is negative. Thus, instead of selling the right of the product to the distributor, the

---

Please cite this article as: Panda, S., et al., Channel coordination and profit distribution in a socially responsible three-layer supply chain. International Journal of Production Economics (2015), http://dx.doi.org/10.1016/j.ijpe.2015.07.001
manufacturer encourages the distributor to sell more units by subordinizing the sell units. Even it pays the distributor \((w^n_d < 0)\) to sell additional units.

4. Summary and concluding remarks

In this paper we have discussed channel coordination issues in a socially responsible three-echelon supply chain by proposing a contract-bargaining process. It is assumed that the manufacturer, the leader of the channel, practices CSR and encourages its downstream channel members for CSR through a code of conduct. While formulating the model we have incorporated only the effect of CSR in the form of consumer surplus in the socially responsible firm's profit function rather than the activities, which it performs. A contract-bargaining process, which consists of two wholesale price discounts and two Nash bargaining products is used and profit maximization is done among the channel members. The proposed model yields following insights.

First, in decentralized decision making the socially responsible channel members of perfect competition lose consumer surplus even enough for channel coordination. Unlike the pure profit maximizing channel, the double marginalization of the socially responsible channel increases with decreasing retail price of the channel. Moreover, in the perfect welfare maximizing socially responsible centralized channel the retail price is equal to the marginal production cost. So, CSR is cost enhancing by inducing a higher output and lower price and the socially responsible channel behaves more competitively than a pure profit maximizing channel because it accepts less profit to act socially. Second, the contract-bargaining process resolves channel conflict and distributes surplus profit among the channel members. The wholesale prices of the contract-bargaining process are different when compared with the wholesale prices of a traditional supply chain.

The wholesale price of the manufacturer is less than its marginal production cost above a threshold of CSR practice. Even it is negative for the manufacturer's heavy CSR activity. As a result the manufacturer's pure profit may be negative, i.e., it pays the distributor to sell additional units that it produces for exhibiting CSR. The benefit of the distributor's wholesale price is not as that of the manufacturer but for the manufacturer's higher threshold of CSR. As a consequence, the distributor's profit and the retailer's profit are win-win. Third, the manufacturer's pure profit and its CSR are inversely proportional. Thus, the manufacturer cannot maximize the shareholder's value and the stakeholder's value simultaneously. In the contract-bargaining process, if the manufacturer's CSR is above 0.6805 then its shareholder's value destroys completely. Although the consumer surplus, which the manufacturer accrues, compensates the loss of pure profit, still the manufacturer should identify the intensity of CSR that balances shareholder's value and stakeholder's value.

Although the proposed model provides some insightful results, still it has some limits and may provide an interesting future research direction. First, for simplicity of analysis the demand is assumed as deterministic and linear in price. Models may be developed by considering some stochastic price dependent demand or some other well established deterministic demand. Secondly, in modeling and analysis it is assumed that as the leader of the channel the manufacturer practices CSR. Instead of this, it may be assumed that only the retailer or both the channel members are involved in CSR and perform it in a proportion. In the former case the CSR is the retailer's own and the manufacturer is independent of that. In the later case the consumer surplus accrued for CSR will be incorporated in the channel members' profits in the same proportion in which they practice CSR. Thirdly, in the contract-bargaining process the manufacturer and the distributor provide wholesale price discount. Instead of this some other transfer pricing policy may be used. Also, instead of using symmetric Nash bargaining product, asymmetric Nash bargaining product may be used to distribute surplus profit among the channel members. Although these extensions make the model robust and dynamic, still in future the basic result of the proposed model will remain unaltered.

Uncited references

Cachon and Lariviere (2005).

References


Please cite this article as: Panda, S., et al., Channel coordination and profit distribution in a socially responsible three-layer supply chain, International Journal of Production Economics (2015), http://dx.doi.org/10.1016/j.ijpe.2015.07.001.


Disposal cost sharing and bargaining for coordination and profit division in a three-echelon supply chain

S. Panda*, N. M. Modak* and M. Basu*

*Department of Mathematics, Bengal Institute of Technology, 1.no. Govt. Colony, Kolkata 700150 West Bengal, India; †Kalyani University, Mathematics, Kalyani, 741235, India

(Received 29 October 2013; final version received 10 March 2014)

This paper focuses on coordination and profit division in a manufacturer–distributor–retailer supply chain. The manufacturer supplies a perishable product to the retailer through the distributor in a single lot. The product deteriorates at a constant rate and it cannot be reworked. The retailer disposes of the product without any salvage value. The manufacturer and the distributor form a coalition that shares the retailer’s disposal cost aiming at channel coordination. Further, to divide the channel surplus first the retailer bargains with the coalition and then the manufacturer and the distributor bargain within the coalition. This simple side-payment contract resolves channel conflict and the channel members’ profits are win–win for the disposal cost share within a range. The proposed mechanisms for coordination and surplus division are illustrated by a numerical example.

Keywords: channel coordination; deterioration; disposal cost; profit sharing; bargaining

JEL Classification: C50; C71; P41

1. Introduction

The increasing trend of globalisation and fierce competition to hold a higher market share impel supply chain drivers to search for newer alternatives that integrate business activities beyond an organizations boundary. For these reasons, effective supply chain management has been receiving notable attention from the concerned communities and has become mainstream. The objective of effective supply chain management is to minimize the conflicting self-interested goals of the channel members, which often lead to suboptimal decisions. It has been well-established that the profit earned under decentralized self-interested decision making is always lower than the profit earned in a single central decision through cooperation. Coordination among channel members is therefore pressing for improving channel-wide performance. With supply chain coordination, a channel member(s) offer(s) a proper side-payment contract to one or more other channel members. The contract aligns all the channel members’ profit maximizing objectives with the best channel performance. A variety of side-payment contracts (e.g. quantity discounting, quantity flexibility, two-part tariff, revenue sharing, sales rebate, buy-back, credit options, and commitment to purchase quantity) have been used in supply chains as ways to cut out channel conflict. These contracts differ by contractual clauses among channel members and are primarily concerned with quantity, time, quality and price. The basic objective behind the design of a side-payment contract aimed at coordinating a supply chain is to transfer a portion of surplus from one chain member to another in order to ensure win–win outcomes.

Although the last two decades have witnessed a rapidly increasing number of publications on channel coordination in two-echelon supply chains, models addressing three-echelon supply chain coordination are notably fewer. In practice, it is more difficult to mitigate channel conflict in a three-echelon supply chain by applying a side-payment contract than in a two-echelon supply chain. When the number of channel members increases, the self-profit maximizing objectives of the channel increase. As a result, the dimension of the solution space increases and optimal solutions are now possible. The paper also discusses how surplus can be divided among the channel members through nested bargaining. First, the retailer and the coalition

bargain and then the manufacturer and the distributor bargain within the coalition for surplus division. Since surplus profit in the channel is generated through disposal cost sharing, it is the key decision variable in the bargaining process.

2. Literature review

Product perishability has enormous impact on decision making. Deterioration is the result of various effects on stock, such as damage, spoilage, obsolescence, decay, decreasing usefulness and many more. Ghare and Schrader (1963) were the first to treat deterioration in the inventory literature. They developed an Economic Order Quantity (EOQ) model for items with exponential decay and deterministic demand. Liu and Shi (1999) classified perishability and deteriorating inventory models into two major categories, namely decay models and finite lifetime models. Finite lifetime models assume a limited lifetime for each item. Blood cells, cans of fruit, foodstuffs, cosmetics and drugs are examples of items having fixed lifetimes. Decaying products are of two types. Products that deteriorate from the beginning and products that start to worsen after a certain time. For perishable products, disposal cost and salvage value have significant impact on order quantity. Some products, like fruit and vegetables, have low disposal cost. On the contrary, electronic goods and radioactive substances have high disposal cost. For some products (e.g., baked goods and books) there are salvage values because products can be used in alternative ways such as sale of the product for scrap, recycling or donation of the product for charity. On this subject, interested readers may note the papers of Panda (2011), Panda and Saha (2010) and Panda, Saha, and Basu (2007, 2008, 2013). However, research addressing contracts for supply chain coordination of perishable products is scarce. Jaggi and Verma (2009) developed a supplier–buyer chain for multiple items with time varying deterioration rate and discussed joint ordering policy aimed at coordinating the supply chain. Giri and Mati (2011) developed a vendor–buyer supply chain model for a deteriorating product with time-varying demand and production rate. Assuming the vendor follows a lot-for-lot policy, they determined the optimal production quantity. They showed numerically that the centralized production quantity is more favourable than the decentralized production quantity. Sadigh, Karimi, and Farahani (2011) used a game theoretic approach to coordinate a seller–buyer chain where the in-hand product of the seller deteriorates continuously and the buyer purchases the product continuously. On this subject, the work of Sana (2013) and Yang and Wee (2003) is worth mentioning. In all the models cited above, the disposal cost of deteriorated products is excluded. Also, either conventional specific side-payment contracts are used as a tool for coordination or integrated order quantities are used to improve supply chain performance. Further, the problems are analysed in a two-echelon rather than a three-echelon environment for simplicity of analysis.

Some researchers in recent years have considered a theoretical bargaining model to expand the view of negotiation and coordination in supply chains. Bargaining theory deals with resolving bargaining situations between two players. Bargaining problems can be defined in the simplest case as ‘two individuals who have the opportunity to collaborate for mutual benefit in more than one way’. There are two main streams of research and application in bargaining theory. First, the axiomatic approach, which requires that resulting solutions should satisfy a set of axioms. Second, the strategic approach, in which outcomes are predicted by using concept of subgame perfect equilibrium. There are few instances of the application of bargaining theory in supply chain coordination. An outline of it can be found in a review article by Nagarajan and Sosic (2008). Kohli and Park (1989) were the first to apply bargaining to supply chains, where the authors studied the negotiation of a quantity discount contract in an EOQ setting by applying the approach of Kalai and Smorodinsky (1975). The Kalai and Smorodinsky (1975) model suggests that both parties share the system surplus equally to achieve channel coordination. Considering logistics costs, Sucky (2004, 2005) proposed several bargaining models for Joint Economic Lot Scheduling (JELS) with and without lot streaming under both complete and incomplete information for uniform demand. Errogal and Wu (2001) considered a supply chain contracting for wholesale price and order quantity in a price sensitive electric market under a bargaining framework. They showed that, in subgame perfect equilibrium, the first best case is optimal for buyer and supplier. On this subject, the work of Susan, Li, and Ashley (1996), Reineers and Tapiero (1995), Sucky (2006) and Panda (2013a, 2013b) is worth mentioning.

Most of the existing literature addresses channel conflict problems in two-echelon supply chains, and not many papers have discussed double marginalization in three-echelon supply chains. Khouja (2003) and Munson and Rosenblatt (2001) have discussed the coordination problem in a three-echelon supply chain with quantity discount. Jaber, Osman, and Guiffrida (2006) considered price discount and profit sharing in a three-echelon supply chain. Jaber, Bonney, and Guiffrida (2006) considered price discount and profit sharing in a three-echelon supply chain with learning based continuous improvement. Sana (2011) developed a production inventory model with perfect and imperfect quality items in a three-layer supply chain, but did not discuss channel coordination. Saha, Panda, Modak, and Basu (2013) considered a three-echelon supply chain coordination problem in which demand is linear in price. They used pull promotion, mail-in-rebate, and push promotion, downward-direct-discount, as the coordination contracts and examined the superiority of one over the other. Seifert, Zequeira, and Liao (2012) studied coordination in a three-layer supply chain and examined the impact of sub-supply chain coordination by using price-only contracts. On this subject, the work of Huang and Huang (2010), Pal, Sana, and Chaudhuri (2012) and Roy, Sana, and Chaudhuri (2012) is worthy of attention.

In an EOQ environment, the manufacturer’s set-up cost is substantially higher than the ordering cost of the retailer (Sarmah, Acharya, & Goyal, 2007). Since the
manufacturer is dependent on the retailer’s EOQ (which is less than the manufacturer’s EOQ or the channel EOQ), the manufacturer must face higher loss of revenue than the retailer because of high set-up cost. To overcome this, the manufacturer induces the retailer to order larger quantities than the EOQ of the retailer. Although this tactic reduces the set-up cost of the manufacturer, the retailer overrules it for increments in inventory related costs. The problem further intensifies if (i) the number of channel members increases and (ii) the product deteriorates. To deal with short shelf life products in a manufacturer–retailer chain, where demand on the retailer’s side is stochastic, Pasternack (1985) examined several pricing and return policies. He showed that if the manufacturer offers full credit for all unsold goods or no return of unsold goods to the retailer, then solutions are suboptimal. He proved that if the manufacturer offers full credit for partial return of goods with the rest disposed of by the retailer for its salvage value, then channel conflict can be resolved. However, the returns policy for non-reworkable decaying products requires the manufacturer or the distributer to have a good disposal set-up in terms of workforce, money, preservation systems, infrastructure, etc. It is common practice for supermarket retail businesses that use common disposal for deteriorated inventories to be destroyed or recycled. Obviously, in such cases the retailer has less responsibility and fewer costs than the distributer or the manufacturer. That is why, as a side-payment contract, the upstream channel members of a multi-echelon vertical supply chain prefer compensation for disposal costs rather than returns policies.

The present study differs from previous work in the following ways. First, unlike Pasternack (1985), here it is assumed that, as an inducement tool, compensation for the disposal costs of deteriorated products is used. The compensation is provided by the manufacturer–distributer coalition. Second, instead of considering sub-supply chain coordination in a three-echelon supply chain as in Seifert et al. (2012), the paper discusses how channel conflict can be resolved by a side-payment contract. Third, the present paper explains a procedure to divide the surplus that is created through coordination by applying Nash bargaining to the products. The disposal cost compensation contract needs the upstream channel members to offer a higher compensation package for the disposal cost of deteriorated inventories. So, the retailer would have enough incentive to choose a higher target quantity, by its possibly benefiting all parties by increased profits compared with those in a decentralized setting. Since inventory decisions depend on sharing the disposal cost, it becomes a key contract parameter involved in such a supply chain coordination contract.

3. Model formulation and analysis

Notation
The following notation is used in developing the model:

\( q(t) \) Retailer’s instantaneous inventory level at time \( t \).
\( D \) Customer’s demand.
\( Q \) Common order quantity.
\( Q_0 \) Retailer’s order quantity in decentralized decision making.
\( h \) Holding cost per-unit product per-unit time.
\( c_d \) Disposal cost per-unit item.
\( T \) Cycle length for common order quantity.
\( T_0 \) Cycle length in decentralized decision making.
\( p_r \) Retailer’s set-up cost.
\( p_d \) Distributors’s set-up cost.
\( s_m \) Manufacturer’s set-up cost.
\( C \) Unit production cost of the manufacturer.
\( p_m \) Manufacturer’s per-unit product selling price.
\( p_d \) Distributors’s per-unit product selling price.
\( p_r \) Retailers per-unit product selling price.

3.1. Compensation for the disposal cost in channel coordination and the win–win outcome

Consider a three-echelon supply chain consisting of a manufacturer, a distributer and a retailer. All the channel members are risk neutral and seek to maximize profits. The manufacturer produces and supplies the product to the distributer in a single lot. The distributer supplies it to the retailer. The product deteriorates at a constant rate \( \theta \) and shortages are not allowed. Deteriorated product cannot be reworked or disposed of by the retailer without any salvage value. Under this model, the objective of each channel member is to maximize his own profit margin even if the resulting channel profit is suboptimal. In such a case, the channel profit and the profits of the channel members will depend on the retailers own profit maximizing order quantity, \( Q_{0_d} \). To overcome this problem, the model assumes that the channel members are willing to accept a common order quantity \( Q \) that is larger than \( Q_{0_d} \) provided all the channel members receive at least their decentralized profits. These restrictions are ensured by the commitment of the manufacturer–distributer coalition to share either a minimum or a maximum fraction of the retailer’s disposal cost jointly. As such, some surplus profit will be generated at either the retailer’s or the coalition’s end. This surplus will be shared by the channel members through bargaining. Since the surplus profit in the channel is generated through disposal cost sharing, the disposal cost sharing fraction is the key decision variable in the bargaining process.

In decentralized decision making at the beginning of the replenishment cycle, the retailer has \( Q_{0_d} \) units of inventory. As time progresses, the inventory level decreases due to uniform demand \( D \) and deterioration. The inventory level reaches to zero level after time \( T_{0_d} \). Then, the next replenishment cycle starts. If \( q(t) \) is the instantaneous inventory level of the retailer at time \( t \), then the governing differential equation is

\[
\frac{dq(t)}{dt} + \theta q(t) = -D
\]  

with the initial and terminal conditions \( q(0) = Q_{0_d} \) and \( q(T_{0_d}) = 0 \), respectively.
The total average profit of the retailer in the replenishment cycle is

\[ \pi'_d = \frac{1}{T_d} \left[ \left. \frac{\partial}{\partial Q_d} \left( \frac{Q_d}{T_d} \right) \right|_{Q_d=0} \right] \times e^{\alpha t_d} - 1 \]  

Using the terminal condition, the order quantity of the retailer is found to be

\[ Q_d = \frac{D}{\alpha} \left( e^{\alpha t_d} - 1 \right). \]  

The total average profit of the retailer in the replenishment cycle is

\[ \pi'_d = \frac{1}{T_d} \left[ \left. \frac{\partial}{\partial Q_d} \left( \frac{Q_d}{T_d} \right) \right|_{Q_d=0} \right] \times e^{\alpha t_d} - 1. \]  

Simplifying it is found that \( 2(2 + \theta T_d)/(2 - \theta T_d) \) and the necessary condition \( d\pi'_d/dT_d = 0 \) for the existence of an optimal solution, we get

\[ T'_d = \frac{2\sqrt{s_2}}{\sqrt{2D(h + \theta c_d + \theta p_d) + \theta \sqrt{s_2}}}. \]  

Substituting \( T'_d \) of (5) into (3), one can find the retailer’s optimal EOQ in decentralized decision making.

Furthermore,

\[ \frac{d^2 \pi'_d}{dT_d^2} = \frac{\left[ s_r + 4D(2D(h + \theta c_d + \theta p_d))/(2 - \theta T_d) \right]}{T_d}. \]  

The right-hand side of the above expression is negative if \( 2 - \theta T_d > 0 \). Now using \( T_d \) of (5) and simplifying it is found that \( 2 - \theta T_d = 2(2 + \theta T_d)/(2 - \theta T_d) \). Thus, \( d^2 \pi'_d/dT_d^2 < 0 \) and hence \( \pi'_d \) is concave.

The distributor’s and the manufacturer’s profit functions in decentralized decision making for the retailer’s EOQ are given by

\[ \pi'_d = \frac{1}{T_d} \left[ \left. \frac{\partial}{\partial Q_d} \left( \frac{Q_d}{T_d} \right) \right|_{Q_d=0} \right] \times e^{\alpha t_d} - 1 \]  

As pointed out, in a multi-echelon vertical manufacturing system, where the product is transferred to the customers downward through different echelons, the set-up cost of the manufacturer is substantially higher than the ordering costs of the distributor and the retailer. Thus, when the manufacturer opts for a lot-for-lot production policy, his objective is to produce as many units as possible. By doing so, the manufacturer reduces his set-up cost for per-unit product and makes more profit in a single production run. So the manufacturer will always try to receive orders of larger lot size than \( Q_{ds} \) from the distributor. Under the channel structure assumed in this paper, the distributor simply acts as an intermediary, because he receives the lot from the manufacturer and transfers it to the retailer, making a profit in the process. Thus, when the manufacturer produces larger lots, the distributor receives more profit. But the retailer has no reason to accept an order quantity \( Q \) larger than his decentralized EOQ because his system running cost in such cases will increase. As such, to compel the retailer to order a quantity \( Q \) larger than \( Q_{ds} \), the manufacturer and the distributor form a coalition that provides compensation for the disposal cost of the retailer. If all the channel members agree on a common order quantity \( Q \), then the retailer’s new average profit in a production cycle is found to be

\[ \pi' = \pi'_d - \pi'. \]  

The manufacturer’s and the distributor’s profit functions in decentralized decision making for the common order quantity \( Q \) are, respectively,

\[ \pi' = \frac{1}{T} \left[ (p_d - p_m)(Q - s_d) \right] \]  

\[ \pi'' = \frac{1}{T} \left[ (p_m - c)(Q - s_m) \right]. \]  

The manufacturer–distributor coalition compensates for the loss of the retailer’s profit corresponding to the common order quantity \( Q \). Thus, after providing compensation to the retailer, the profit function of the coalition for order quantity \( Q \) is

\[ Y = \pi'' - \pi' \times X. \]  

The first order condition \( dY/dT = 0 \) for the existence of an optimal solution, using the approximation \( e^{\alpha t_d} = (2 + \theta T_d)/(2 - \theta T_d) \), yields

\[ T' = \frac{2s_r + s_d + s_m}{\sqrt{2D(h + \theta c_d + \theta p_d) + \theta \sqrt{s_2}}}. \]  

Moreover,

\[ \frac{dY}{dT} = \frac{2s_r + s_d + s_m}{T'} \left[ \frac{4D(2D(h + \theta c_d + \theta p_d))}{(2 - \theta T')^2} \right]. \]
The expression on the right-hand side is negative if 
\[ 2 - \theta T > 0. \]
But
\[ 2 - \theta T = \frac{2\sqrt{(s_r + s_d + s_m)D(k + \theta k_d + \theta k)}}{2D(h + \theta k_d + \theta k)} - \theta^*(s_r + s_d + s_m) > 0. \]

Hence, the joint profit function of the coalition after providing the retailer’s loss to it is concave.

Proceeding in the same way as in the decentralized decision making, the optimal common order quantity of the channel is found to be

\[ Q^* = \frac{D}{\theta} \left( e^{\theta T} - 1 \right). \]  

(15)

The retailer will show interest in ordering \( Q \) rather than \( Q_d \) if he receives at least his decentralized profit.

The manufacturer–distributor coalition compensates for any loss of the retailer’s profit due to changed ordering policy through compensation for the disposal cost. If \( \lambda_{\text{min}} \) is the minimum percentage of the disposal cost that compensates for the retailer’s loss of profit due to the coordinated order quantity \( Q \), then using (10) the following expression can be found:

\[ X = \lambda_{\text{min}} \left[ c_d D \left( e^{\theta T} - 1 \right) \right]. \]

On simplification, the above expression yields

\[ \lambda_{\text{min}} = \frac{\pi_{d^*} - \pi^*}{c_d D \left( e^{\theta T} - 1 \right)}. \]  

(16)

Therefore, the retailer’s minimum profit for the coordinated order quantity is

\[ \pi_{\text{min}} = D \left( p_r + \frac{h}{\theta} + c_d \right) - \frac{s_r}{\theta} - D \left( p_d + \frac{h}{\theta} + c_d \right) \times \left( e^{\theta T} - 1 \right) + \lambda_{\text{min}} \left[ c_d D \left( e^{\theta T} - 1 \right) \right]. \]  

(17)

When the manufacturer–distributor coalition provides minimum compensation for the disposal cost, their profit corresponding to the coordinated order quantity will be maximum. As such, the basic question is in what proportions will the manufacturer and the distributor within the coalition share payment of compensation for the disposal cost. Without loss of generality, we assume the simplest case in which the distributor provides \( k_{\text{min}} \lambda_{\text{min}} \) and the manufacturer provides \((1 - k_{\text{min}}) \lambda_{\text{min}}\). Then the maximum profit of the distributor and the maximum profit of the manufacturer for the coordinated order quantity are, respectively,

\[ \pi_{d^*} = \frac{1}{\theta} \left[ (p_d - p_m)Q - s_d \right] - k_{\text{min}} \lambda_{\text{min}} \left[ c_d D \left( e^{\theta T} - 1 \right) \right]. \]  

(18)

\[ \pi_{m^*} = \frac{1}{\theta} \left[ (p_m - c)Q - s_m \right] - (1 - k_{\text{min}}) \lambda_{\text{min}} \left[ c_d D \left( e^{\theta T} - 1 \right) \right]. \]  

(19)

The value of the parameter \( k_{\text{min}} \) determines the maximum profits that the distributor and the manufacturer can gain. Thus, the parameter \( k_{\text{min}} \) is crucial to both parties and it depends on their negotiating powers. The member having greater negotiating power than the other will gain more profit for the coordinated order quantity.

The manufacturer–distributor coalition can provide compensation for the retailer’s disposal cost corresponding to the coordinated order quantity as long as their profit is not less than the decentralized profit. If \( \lambda_{\text{max}} \) is the maximum percentage of compensation on disposal cost that the manufacturer–distributor coalition provides, then it follows that

\[ \lambda_{\text{max}} = \frac{\pi_{d^*} + \pi_{m^*} - (\pi_{d^*} + \pi_{m^*})}{c_d D \left( e^{\theta T} - 1 \right)}. \]  

(20)

On simplification, the maximum percentage of compensation on disposal cost that the manufacturer–distributor coalition can provide to the retailer is found to be

\[ \lambda_{\text{max}} = \frac{\pi_{d^*} + \pi_{m^*} - (\pi_{d^*} + \pi_{m^*})}{\lambda_{\text{min}} \left[ c_d D \left( e^{\theta T} - 1 \right) \right]} - k_{\text{max}} \lambda_{\text{max}} \left[ c_d D \left( e^{\theta T} - 1 \right) \right]. \]  

(21)

\[ \pi_{\text{min}}^* = \frac{1}{\theta} \left[ (p_d - p_m)Q - s_d \right] - k_{\text{max}} \lambda_{\text{max}} \left[ c_d D \left( e^{\theta T} - 1 \right) \right]. \]  

(22)

Consequently, the retailer’s maximum profit is

\[ \pi_{\text{max}} = D \left( p_r + \frac{h}{\theta} + c_d \right) - \frac{s_r}{\theta} - D \left( p_d + \frac{h}{\theta} + c_d \right) \times \left( e^{\theta T} - 1 \right) + \lambda_{\text{max}} \left[ c_d D \left( e^{\theta T} - 1 \right) \right]. \]  

(23)

Note from Equations (18) and (21) that the value of \( k_i \), \( i = \{\text{min, max}\} \), is determined by negotiation between the manufacturer and the distributor. But (19) and (21) do not ensure that the distributor’s profit in both cases is greater than his decentralized profit, i.e. \( \pi_{d^*} \geq \pi_{d_{\text{d}}^*} \), \( i = \{\text{min, max}\} \). To ensure in both cases that the distributor receives at least his decentralized profit, by simplifying the inequality the upper limit of \( k_i \) can be found to be

\[ k_i \leq \frac{\pi_{d^*} - \pi_{d_{\text{d}}^*}}{\lambda_{\text{min}} \lambda_{\text{max}} \left[ c_d D \left( e^{\theta T} - 1 \right) \right]} \]  

\( i = \{\text{min, max}\} \).
Similarly, the range of \( k_i \), \( i \in \{\text{min, max}\} \), that ensures that the manufacturer’s profit corresponding to the coordinated order quantity after sharing compensation for the disposal cost with the distributor is at least equal to its decentralized amount, the order quantity can be determined from the inequality \( \pi_i = \pi_i^d \), \( i \in \{\text{min, max}\} \). Simplifying the inequality, the lower limit of \( k_i \) can be found to be

\[
k_i \geq 1 - \frac{\pi_i - \pi_i^s}{\lambda_i \left[ c_i D \left( \frac{\alpha_i}{\theta_i} - 1 \right) \right]} \\
\]

Thus, after providing compensation for the disposal cost to the retailer, the distributor and the manufacturer share the compensation payment in the proportions \( k_i \), \( i \in \{\text{min, max}\} \), between them such that it is always ensured that both parties receive at least their decentralized profits. The range of the compensation cost sharing fraction is given by

\[
1 - \frac{\pi_i - \pi_i^s}{\lambda_i \left[ c_i D \left( \frac{\alpha_i}{\theta_i} - 1 \right) \right]} \leq k_i \leq \frac{\pi_i^d - \pi_i^s}{\lambda_i \left[ c_i D \left( \frac{\alpha_i}{\theta_i} - 1 \right) \right]} \\
i = \{\text{min, max}\},
\]

\( \lambda_{\text{min}} \) and \( \lambda_{\text{max}} \) are the two bounds on compensation for the disposal cost of deteriorated products. For any \( \lambda \in (\lambda_{\text{min}}, \lambda_{\text{max}}) \), the profit of the manufacturer–distributor coalition and the profit of the retailer are win–win. Since \( k \) depends on any \( \lambda \in (\lambda_{\text{min}}, \lambda_{\text{max}}) \), the range of \( k \) for which the distributor and the manufacturer’s profit are win–win is found to be

\[
1 - \frac{\pi_i - \pi_i^s}{\lambda_i \left[ c_i D \left( \frac{\alpha_i}{\theta_i} - 1 \right) \right]} \leq k \leq \frac{\pi_i^d - \pi_i^s}{\lambda_i \left[ c_i D \left( \frac{\alpha_i}{\theta_i} - 1 \right) \right]} \\
\lambda \in (\lambda_{\text{min}}, \lambda_{\text{max}}). \tag{25}
\]

From (25), the following results can be realized. First, for \( \lambda = \lambda_{\text{max}} \), the range of \( k \) using (16) can be found to be

\[
1 - \frac{\pi_i - \pi_i^s}{\pi_i^d - \pi_i^s} \leq k_{\text{max}} = \frac{\pi_i - \pi_i^s}{\pi_i^d - \pi_i^s} \tag{26}
\]

Note that the lower bound of \( k_{\text{max}} \) may be negative when \( \pi_i - \pi_i^s > \pi_i^d - \pi_i^s \). The intuitive reason is as follows. When the manufacturer–distributor coalition provides minimum compensation for the disposal cost, the channel is coordinated and it receives the entire surplus profit of the channel. If the manufacturer and the distributor share the compensation within the coalition in such a way that the manufacturer’s share of surplus profit is greater than the retailer’s loss of profit due to the coordinated order quantity, then the lower bound of \( k_{\text{max}} \) is negative. Similarly, the upper bound of \( k_{\text{max}} \) may be greater than one when the portion of surplus profit received by the distributor is higher than the loss of profit of the retailer due to the coordinated order quantity. However, whether \( k_{\text{max}} \) is negative or greater than one depends on two factors, viz. the retailer’s profit loss corresponding to the coordinated order quantity and the surplus profit generated in the channel by coordination. Second, for \( \lambda = \lambda_{\text{max}} \), using (20), instead of a range \( k_{\text{max}} \) is found to be

\[
k_{\text{max}} = \frac{\pi_i^d - \pi_i^s}{\left( \pi_i^d - \pi_i^s \right) + \left( \pi_i^d - \pi_i^s \right)} \tag{27}
\]

The intuition is straightforward. When the manufacturer–distributor coalition provides maximum compensation for the disposal cost, their profit is the sum of the decentralized profits. Obviously, the retailer will take away the entire surplus profit. So, for a particular disposal cost compensation split, the coalition’s decentralized profit will be ensured. Figure 1 represents the range of \( k \) for \( \lambda \in (\lambda_{\text{min}}, \lambda_{\text{max}}) \). It indicates the same characteristics as those mentioned above.

If all channel members operate jointly, then under centralized decision making the channel profit is

\[
\pi = \frac{1}{T} \left[ (p_s - c)D - D \left( \frac{h}{D} + c_d \right) \times \left( \frac{\alpha D - 1}{\theta D} - 1 \right) \right] - (s_r + k_d + s_m)
\]

\( \alpha \), the optimal cycle length and the optimal order quantity can be found to be (14) and (15), respectively. That is, when the manufacturer–distributor coalition compensates the retailer for loss of profit through sharing the disposal cost corresponding to a changed order quantity, the system will be coordinated. Now, in what ratio the surplus profit will be shared by the channel members depends on their bargaining powers. Several bargaining models are available in the literature to divide the surplus among the channel members. Here we will consider the Nash bargaining product to discuss the issue.

### 3.2. Surplus sharing

As assumed throughout the paper, the manufacturer and the distributor form a coalition and the coalition as a single entity provides compensation for the disposal cost to the retailer. The bargaining process that is used in this paper for dividing the channel surplus is a nested bargaining process and is based on the Nash bargaining product, where bargaining takes place on the basis of the previous bargaining outcome. It consists of two steps. First, the

![Figure 1. Range of k with respect to A.](image)
manufacturer–distributor coalition bargains with the retailer. Second, once the coalition and the retailer finalize their share, the manufacturer and the distributor bargain within the coalition to divide the surplus that is received by their coalition from the first step between them. In the first step, any value of \( \lambda \in (\lambda_{\text{min}}, \lambda_{\text{max}}) \) eliminates channel conflict, the participating parties will bargain to determine an acceptable channel surplus, the manufacturer

receives its share of the channel surplus, the manufacturer–distributor coalition.

The necessary optimality condition with respect to compensation for the disposal cost yields

\[
\lambda^* = \alpha \lambda_{\text{max}} + (1 - \alpha) \lambda_{\text{min}}.
\]

Thus, \( X \) determines the division of the channel surplus between the retailer and the manufacturer–distributor coalition.

As soon as the manufacturer–distributor coalition receives its share of the channel surplus, the manufacturer and the distributor bargain between themselves to share it. Since the manufacturer and the distributor compensate the retailer for his disposal cost in the proportions \( k \) and \( (1 - k) \) within the range (25) of \( X \), the Nash bargaining product will be maximized with respect to \( k \). The Nash bargaining product is found to be

\[
\max_k \pi^N(k) = \pi^N_m(1 - k)\lambda
\]

where \( \beta \) and \( (1 - \beta) \), \( 0 \leq \beta \leq 1 \), are the negotiating powers of the manufacturer and the distributor, respectively, and

\[
\pi^m(k) = \frac{1}{\theta}[(p_d - p_o)Q - s_d] - k\lambda^* \left[ c_D \left( \frac{e^{rt}}{\theta t} - 1 \right) \right].
\]

The necessary condition for the existence of an optimal solution yields

\[
k = (1 - \beta)
\]

Therefore, the optimal profits of the retailer, the distributor and the manufacturer are, respectively,

\[
\begin{align*}
\pi^r &= \pi^d + (1 - \alpha)(\pi^m + \pi^d + \pi^r) - (\pi^m + \pi^d + \pi^r) \\
\pi^d &= \pi^m + \alpha(1 - \beta)\pi^r - (\pi^m + \pi^d + \pi^r) \\
\pi^m &= \pi^m + \beta(\pi^m + \pi^d + \pi^r) - (\pi^m + \pi^d + \pi^r) + \pi^r.
\end{align*}
\]

From the above bargaining process, three possible results are realized. First, the optimality criterion (30) interprets the fact that the manufacturer–distributor coalition provides the weighted average of the minimum and maximum possible compensation for the disposal cost. The weighted average is based on the negotiating powers. So, the player having more negotiating power than the other will acquire a larger surplus share than the other. Second, once the manufacturer and the distributor are within the coalition to share the channel surplus, which is received by their coalition, the negotiated outcome is the weighted average of their bargaining powers. Also it depends heavily on the previous bargaining outcome and the same conclusion can be drawn as in the previous case for the relation between negotiating power and channel surplus. Third, the coalition and the retailer have the same negotiating power, i.e. \( \alpha = \beta = 1/2 \), so \( X^* = (\lambda_{\text{min}} + \lambda_{\text{max}})/2 \).

As such, the coalition and the retailer will divide the channel surplus equitably. Moreover, if the manufacturer and the distributor have same negotiating power, then they share the surplus equitably between them. As a result, in addition to the decentralized profit, the retailer receives half of the channel surplus and the manufacturer and the distributor realize quarter of the channel surplus each, in addition to their decentralized profits.

4. Numerical illustration

Assume that annual demand is \( D = 10,000 \) units. The product deteriorates at a rate \( \theta = 0.4 \) per year. The set-up cost of the manufacturer is \( S_m = \$10,000 \) per production cycle. The ordering costs of the retailer and the distributor are, respectively, \( S_r = \$3000 \) and \( S_d = \$4000 \). The unit
production cost of the manufacturer is \( c = \$110 \). The selling prices of the manufacturer, the distributor and the retailer are \( p_m = \$130 \), \( p_d = \$140 \) and \( p_r = \$160 \), respectively. The holding cost and the disposal cost of the retailer are \( h = \$2 \) per unit per year and \( c_r = 0.8c \) per unit, respectively.

Under decentralized decision making, the replenishment cycle length and order quantity are \( T_{\text{in}} = 0.084 \), 51 yr and \( Q_{\text{in}} = 859.52 \) units. The optimal profit of the manufacturer, the distributor and the retailer are \( \pi_m = \$85085.9 \), \( \pi_d = \$54,376.2 \) and \( \pi_r = \$124,672 \), respectively. The total channel profit is \( \$264,134 \). When the channel members operate jointly through compensation for the disposal cost, the optimal cycle length is \( T^* = 0.196,58 \) yr and the optimal order quantity is \( Q^* = 2045.17 \) units. The minimum and maximum compensations for the disposal cost offered by the manufacturer–distributer coalition provides \( \lambda_{\text{min}} = 95.68\% \) and \( \lambda_{\text{max}} = 285.53\% \). For \( \lambda_{\text{min}} \), the optimal channel profit is \( \pi_m + \pi_d + \pi_r = \pi_m + \pi_d + \pi_r = \$331,575.9 \). For \( \lambda_{\text{max}} \), the optimal channel profit is \( \pi_m + \pi_d + \pi_r = \$331,575.9 \). The optimal profits for both cases are aligned with the optimal profit for centralized decision making. For the minimum compensation for the disposal cost, the manufacturer–distributer coalition receives the entire channel surplus of \( \$67,441.8 \), and for \( \lambda_{\text{max}} \) the retailer receives the same.

If the manufacturer–distributer coalition provides the fraction \( \lambda_{\text{max}} \) of the compensation for the disposal cost, then for any \( k \in (0.682, 0.8) \) the manufacturer’s profit and the distributor’s profit are greater than their decentralized profits. The manufacturer and the distributor will receive profits within the ranges \( \pi_m \in (\$85,085.9, \$152,528) \) and \( \pi_d \in (\$54,376.2, \$121,818) \) and the retailer will get his decentralized profit. However, if the manufacturer–distributer coalition provides \( \lambda_{\text{min}} \) compensation for the disposal cost, then \( k^* = 0.289 \) and the manufacturer and the distributor will receive their decentralized profits. When the retailer and the manufacturer–distributer coalition bargain with equal negotiating powers, i.e. \( a = 0.5 \), then the manufacturer and the distributor bargain within the coalition with equal negotiation powers, i.e. \( \beta = 0.5 \), so \( \lambda = 190.61\% \) and \( k = 149.21\% \). The profits of the manufacturer, the distributor and the retailer are \( \$101,946.35 \), \( \$71,236.65 \) and \( \$158,392.6 \), respectively. The retailer receives half of the channel surplus. The manufacturer and the distributor receive a quarter of it each. The profits of the manufacturer, the distributor and the retailer increase by \( 19.82, 31.00 \) and \( 27.04\% \), respectively, from decentralized decision making.

From the numerical example, the following managerial implications are realized. First, the range of the compensation fraction that the manufacturer–distributer coalition provides to the retailer indicates that the coalition may provide more than the total disposal cost to the retailer as a compensation package while still achieving a win–win outcome. This may happen if the surplus created for channel coordination is substantially higher than the total disposal cost of the retailer. The manufacturer–distributer coalition compensates for the loss of the retailer’s profit by providing a percentage \( \lambda_{\text{min}} \) of the disposal cost. For channel coordination, an amount of surplus \( (\lambda_{\text{max}} - \lambda_{\text{min}}) \) \times \( \text{disposal cost} \) is generated in the channel. Thus, to divide additional profit the coalition may provide \( \lambda_{\text{max}} \) plus a portion of \( \lambda_{\text{max}} - \lambda_{\text{min}} \) \^\text{disposal cost} \), which may be greater than one. Second, the profits of the manufacturer and the distributor are heavily influenced by the compensation fraction \( \lambda \). As \( \lambda \in (\lambda_{\text{min}}, \lambda_{\text{max}}) \) increases, the surplus profit that is received by the coalition decreases. As a result, the range of the cost sharing fraction \( k \) within the coalition is shortened. It ultimately provides a particular profit split when \( \lambda = \lambda_{\text{max}} \), i.e. the manufacturer and the distributor receive their decentralized profits (see Figure 1). Further, the lower bound of \( k \) may be negative and its upper bound may be greater than one. That is, instead of receiving a portion of the channel surplus, if the distributor provides a portion of its profit for the coordinated order quantity, then it will also receive at least its decentralized profit. The same conclusion can be drawn for the upper limit of \( k \) for the manufacturer also. Third, the model is highly sensitive to errors in estimating \( D \) and the set-up costs of the channel members, and moderately sensitive to the change in \( \theta \) (see also Figure 2). Low sensitivity is found for change in the parameter value \( h \). When \( D \) is increased with other parameter values fixed, it is found that the profits of the channel members increase. Reverse trends are found for the parameters \( \theta \) and \( h \).

5. Summary and concluding remarks

In this paper, we propose a manufacturer–distributer–retailer supply chain. The manufacturer produces a perishable product and supplies it to the retailer through the distributor in a single lot. Deteriorated product cannot be reworked and is disposed of by the retailer without any salvage value. For optimal channel profit, the manufacturer and the distributor form a coalition that provides a percentage of the disposal cost to the retailer as compensation. The channel members divide the surplus profit that is generated by channel coordination, through nested bargaining. First, the retailer and the manufacturer–distributer coalition bargain by using their negotiating powers. Based on the result of this bargaining, the manufacturer and the distributor bargain for surplus sharing using their bargaining powers within the coalition.

A number of interesting results are obtained, which are summarized below. First, the simple side-payment contract compensation for the disposal cost of deteriorated products cuts out channel conflict. Second, if the manufacturer–distributer coalition offers more than the full disposal cost as compensation to the retailer, then the channel is also coordinated and a win–win result for all channel members is ensured. Third, as the compensation fraction increases within its range, the retailer’s profit increases but the coalition’s profit decreases. If the coalition’s profit decreases, then it does not follow that the manufacturer’s profit as well as the distributor’s profit decrease. The profits of the manufacturer and the distributor depend on how they share the
retailer’s disposal cost within the coalition. If $k$ increases within its range, then the distributor’s profit decreases but the manufacturer’s profit increases. However, when $l$ attains its maximum value $l^*$, the coalition’s profit is least and the manufacturer and the distributor settle for their decentralized profits. Fourth, the profits of all the channel members depend heavily on the set-up cost of the manufacturer and the ordering costs of the retailer and the distributor, respectively. Thus, the compensation fraction’s range depends on these parameters. In fact, for a high retailer ordering cost, the range of the compensation fraction is low, whereas it is high for a high set-up cost and/or higher ordering cost of the manufacturer and/or the distributor.

In this paper, it is assumed that the manufacturer–distributor coalition provides compensation for the disposal cost corresponding to a coordinated order quantity. Instead, the coalition may provide compensation for the disposal cost of the retailer’s decentralized order quantity or only for the increased order quantity. These two cases can be analysed by replacing $Q^*$ by $Q^*_d$ and $Q^*_c$ by $(Q^*_c - Q^*_d)$ in Equations (16) and (20), respectively. Here it is assumed that the manufacturer and the distributor form a coalition and the coalition provides the compensation. Instead of this, either the manufacturer or the distributor may provide the compensation to the retailer. These two cases can be analysed under the present model setting by replacing $k = 0/1$.

The model presented in this paper may be extended in several ways. For example, the channel members may have some private cost information, the demand at the retailer’s end may be stochastic or price dependent, etc. To divide the channel surplus instead by considering the
system over a finite time horizon, a strategic bargaining process may be used to divide the channel surplus.

Acknowledgements
The authors thank the anonymous referees for their valuable comments and suggestions for the improvement of the paper.

References


Three-echelon supply chain coordination considering duopolistic retailers with perfect quality products

Nikunja Mohan Modak, Shibaji Panda, Shib Sankar Sana

Abstract

This paper explores channel coordination and profit division issues of a manufacturer-distributor-dupolistic retailers supply chain for a product, where the manufacturer supplies lotsize of the product that contains a random portion of imperfect quality item. In manufacturer-Stackelberg vertical game setting, the duopolistic retailers' three behaviours—Cournot, Collusion and Stackelberg are discussed. Besides analyzing the effect of imperfect quality product on optimal decisions, the paper depicts the hybrid contract mechanism so that all units quantity discount with franchise fee resolves channel conflict though unable to provide win-win outcome. For surplus profit division, the paper proposes two sequential bargaining processes—backward and forward, where outcome of a Nash bargaining is dependent on the previous. It is found that, for channel coordinated win-win profit, the manufacturer prefers Collusion, Stackelberg and Cournot, while the retailer prefers the reverse and both prefer backward sequential bargaining. But, the distributers preference depends on the target profit that it sets during the bargaining process. The proposed mechanisms are illustrated by a numerical example.

Keywords: Three-echelon supply chain, Imperfect quality product, Channel coordination, Bargaining

1. Introduction

One of the major issues in supply chain is to maintain goodwill with the customers, providing good quality products. Imperfect quality product found at the delivery time to the customers causes bad impression/impact of the seller that looses goodwill with customers to some extent. Advances in science and technology, improved mechanization and automation for large-scale production have made outstanding contributions to improve the quality of product. But, the percentage of good quality products in any manufacturing system could not reach at 100%. Consequently, it is necessary to separate imperfect quality products from the whole lot by screening process to sustain the existence of a business sector in a competitive market. Pricing is also an important decision factor which affects on the profit of the enterprise and it plays a significant role in demand (Kalton and Singh, 1992). Optimal pricing strategy (Cardenas-Barron, 2006, 2009a;b; Smith et al., 2009; Panda et al., 2008, 2013; Panda and Saha, 2010; Shah and Raykundaliya, 2010; Sarkar et al., 2013; Salvietti et al., 2014) is a major issue to attract the customers in any business organization in a given economy.

Coordination of pricing among the channel members under different channel structures has been studied in the marketing and operations management literature. Channel coordination not only decreases the overall channel costs, but increases the sales volume that results in more profit of the channel. Most studies to date have focused on coordination in traditional channel structure, mainly two-level supply chain consisting of a manufacturer and a retailer and few of them consider duopolistic retailers. Little attention has been given to the multi-level supply chain coordination.

In practice, there exist various forms of competition between the two retailers in the downstream supply chain. Cournot behaviour of duopolistic retailers is a common phenomenon in industry. Wal-Mart and Tesco, GOME and Suning, Carrefour and Auchan, etc., are the examples of the retailers who follow Cournot. Collusion is another realistic behavior of the duopolistic retailers where they do not compete with each other. In Collusion behavior, retailers perform secretly with a confidential agreement among them. Although, Collusion is illegal as the retailers play it tactically that is common in Chinese market. In another way, the duopolistic retailers may compete with each other following Stackelberg. In Stackelberg game, one retailer act as leader and the other as follower.

This paper considers a three-echelon supply chain with duopolistic retailers in the downstream and the retailers may play...
Cournot, Collusion and Stackelberg games. The manufacturer supplies a product in a single lot to the distributor that contains a random proportion of defective items. After receiving the lot, the distributor separates the imperfect items by screening process and sells in a secondary market with a discounted price. At the end of the screening process, the distributor satisfies the demand of two retailers with perfect quality products only. The purpose of this paper is to (i) examine the effect of imperfect quality product on the optimal decisions, (ii) verify whether the hybrid contract mechanism resolves the channel conflict or not and (iii) demonstrate how a nested bargaining process depicts win–win profits for the channel members after channel coordination. Also, the paper presents a preference analysis to highlight the channel members game and bargaining process preference.

2. Literature review

Thomas and Griffin (1996) mentioned that effective supply chain management requires planning and coordination among the various channel members including manufacturers, retailers and intermediaries, if any. Contracts with various coordination mechanisms have been discussed and implemented successfully in the supply chain co-ordination; for example, quantity discount (Panda et al., 2014b), two-part tariff (Ingene and Parry, 1995), revenue sharing (Panda, 2014a), buyback (Ding and Chen, 2008), mail in rebate (Saha et al., in press), disposal cost sharing (Panda et al., 2014a), profit sharing (Modak et al., 2015; Panda et al., 2015) etc. Ingene and Parry (1995) explored coordination of a channel consisting of a manufacturer and two competitive retailers where manufacturer acted as a Stackelberg leader. Yang and Zhou (2006) extended the work of Ingene and Parry (1995) by considering duopolistic retailers, who compete it in three different ways. However, application of coordination contract to resolve channel conflict in a supply chain that has multiple retailers in downstream is notably fewer. Wang et al. (2012) extended the work of Yang and Zhou (2006) and explored channel coordination by applying price discount contract. Xiao et al. (2007) investigated the coordination mechanism for a supply chain with one manufacturer and two competing retailers when the demands are disrupted. They showed that the supply chain is to be coordinated by either a linear quantity discount schedule or an all-unit quantity discount schedule. Lee and Rosenblatt (1985) investigated the effect of an imperfect production process on the optimal production cycle time. They assumed that defective items could be reworked instantly at a cost and they found that the presence of defective products motivated smaller lot sizes. Agnihotri and Kneett (1995) showed how the impact of defects on system performance was measured in an imperfect production process with 100% inspection followed by rework. Lee et al. (1997) developed a batch quantity model in a multi-stage production system, considering the proportions of defective items produced at each stage while ignoring rework. Ben-Daya et al. (2006) developed integrated inventory inspection models with and without replacement of nonconforming items found during inspection. Salameh and Jaber (2000) developed an EOQ (Economic Order Quantity) model for items received with imperfect quality under the assumption that defective items could be sold as a single batch at the end of 100% screening process. They determined the optimal decisions of the channel members in decentralized setting and performed preference analysis by considering the equal demands of the duopolistic retailers. However, they did not discuss about centralized decision and channel coordination issues. Unlike Yang and Zhou, this paper assumes three-echelon supply chain with duopolistic retailers. The manufacturer supplies the product in a single lot that contains random proportion of defective items and the distributor differentiates perfect and imperfect quality products. Assumption of random proportion of imperfect items in a lot is well-established and well-studied (Salameh and Jaber, 2000; Sana, 2011; Khan et al., 2011) in inventory literature. The paper discusses the effect of imperfect quality items on optimal decisions. Secondly, all unit quantity discounts with franchise fee is used as the coordination mechanism to cut out channel conflict in all the three games, which the retailers play. Although the contract resolves channel conflict, it does not ensure win–win profits for the channel members. Thirdly,
to determine win–win profits of all the channel members, the paper applies a sequential bargaining process that consists of two Nash bargaining and determination of two limits of franchise fee. Entire sequential bargaining process is nested, i.e., outcome of one limit of franchise fee and bargaining have direct influences on the other two. It is assumed that in the multi-member supply chain, only two channel members at a time participate in the process. As such, the process may flow either from the manufacturer to the retailers, i.e., forward or from the retailers to the manufacturer, i.e., backward. The paper discusses these two cases separately and presents a comparative study for acceptability of the process by the channel members. Fourthly, for win–win coordinated profits the paper depicts which channel member prefers what game of the retailers and bargaining process.

3. Notation

The following notations are used to develop the proposed model.

- \( \pi_i \): the profit function of the \( i \)-th retailer, \( i = 1, 2 \)
- \( \alpha \): the profit function of the distributor
- \( w_0 \): the wholesale price per unit determined by the manufacturer
- \( w_d \): the wholesale price per unit of perfect product determined by the distributor
- \( w_p \): the sale price per unit of imperfect product determined by the distributor
- \( p_i \): the sale price per unit charged to customer by the retailer \( i \)
- \( c_i \): marginal cost per unit of the \( i \)-th retailer, \( i = 1, 2 \)
- \( c_d \): marginal cost per unit of the distributor
- \( s \): cost of screening per unit product
- \( r \): proportional probability of imperfect items, a random variable
- \( f(r) \): the probability density function of the random variable \( r \)

4. The model

Assume that a manufacturer produces a single type product and delivers in a single lot to the distributor. The lot contains a random proportion \( r \) (\( 0 \leq r \leq 1 \)) imperfect quality product that follows a probability distribution. After receiving the lot, the distributor performs screening process to separate perfect and imperfect quality products. Distributor supplies perfect quality product to two retailers to satisfy their demand. Demand at retailer \( i, (i = 1, 2) \) is downward sloping and in the form \( D_i = a_i - b_i p_i + /p_{i-1} \) (\( i = 1, 2 \)) where \( a_i > 0 \) is the market potential of the retailer \( i \). \( a_i \) denotes the measure of sensitivity of retailer-\( i \)’s sales to changes of the retailer-\( i \)’s price. \( b_i \) is the degree of substitutability between retailers, which reflects the impact of the marketing mix decision of the retailers on customer demand. Distributor sells imperfect quality product at a lower price in a secondary market in a single lot. Two retailers compete on price and set their prices independently. Assume that the manufacturer as the stackelberg leader and the distributor is its immediate follower. The distributor supplies the perfect quality product to two retailers. Under these assumption we first find the decentralized decisions.

4.1. Decentralized system

In decentralized channel the expected profit functions of the manufacturer, the distributor and the retailers can be represented as follows:

- \[ \pi_m = (1 + E(r))(D_1 + D_2)(w_0 - c_m) \] (1)
- \[ \pi_d = (w_d + E(r)w_p - (1 + E(r))(s + c_d + w_0))(D_1 + D_2) \] (2)
- \[ \pi_i = (p_i - w_0 - c_i)(a_i - a_ip_i + /p_{i-1}) \] (\( i = 1, 2 \)) (3)

where \( E(.) \) is the expectation corresponding to the pdf \( E(.) \) of the imperfect quality product.

In the manufacturer Stackelberg setting, this is a three-stage game. In first stage, the manufacturer announces wholesale price of the product. Based on this, in the second stage game, the distributor determines wholesale price of perfect quality products. In third stage, following the distributor’s wholesale price, two retailers make there further sales decisions. Two retailers in the channel may behave in three ways, (i) Cournot i.e., the retailers compete, (ii) Collusion, i.e., the retailers jointly take decision and (iii) Stackelberg, i.e., one the retailers takes decision and based on that the other retailers makes decision. In next sub-sections, we shall discuss how the members of the decentralized three-echelon supply make their decisions under these three situations.

4.1.1. Retailers play the Cournot game

Suppose duopolistic retailers pursue the Cournot solution, i.e., each retailer independently sets the retail price by assuming its rival’s selling price as a parameter. To find the subgame perfect equilibrium of this three-stage game we use backward induction. First the retailers simultaneously determine the optimal selling prices given \( w_d \). Based on these two prices the distributor determines the optimal selling price of the perfect quality product. Finally the manufacturer determines the optimal wholesale price based on the distributor’s wholesale price for perfect quality product.

For any given \( w_d \), the optimal retail prices of retailers 1 and 2 are given by (solving \( d\pi_i/dp_i = 0 \) and \( d\pi_i/dp_2 = 0 \))

- \[ p_1 = \frac{2a_1 + d_1(w_d + c_1) + \beta(b_1 + d_2(w_p + c_2))}{4d_1^2 - \beta^2} \] (4)
- \[ p_2 = \frac{2a_2 + d_2(w_d + c_2) + \beta(b_2 + d_1(w_p + c_1))}{4d_2^2 - \beta^2} \] (5)

Now, total demand at the distributor’s end is \( D_1 + D_2 = D - w_d \), where \( A = a_1 + a_2 + \beta(b_1 + d_1(w_p + c_1) - 2a_1 - d_1^2 - b_2 + d_2(w_p + c_2)) \) and \( B = (2a_2 - \beta^2)w_d + a_2 - 2a_2b_2)/4d_2^2 - \beta^2) \). Substituting \( p_1 \) and \( p_2 \) in \( \pi_m \) and solving \( d\pi_m/dw_d = 0 \), we get

- \[ w_d = \frac{A + 1}{B} = (1 + E(r))(s + c_d - w_0 - B)] \] (6)

Substituting \( w_d \) in \( \pi_m \), we have

- \[ \pi_m = \frac{1}{2}(1 + E(r))(A - B(1 + E(r))(w_m - c_m) - B(1 + E(r))(s + c_d) - w_dE(r))] \] (7)

and

- \[ d\pi_m/dw_d = -(1 + E(r))^2B \]

Reorganizing the terms of \( B \), we have \( B = (a_1 - \beta^2b_2)w_d + a_2 - \beta^2)/4d_2^2 - \beta^2) \). Since \( 0 < \beta < a_1 \), it is clear that \( B \) is positive. It means \( \pi_m \) is a concave function of \( w_m \). Therefore, the solution of \( d\pi_m/dw_m = 0 \) is an optimal wholesale price that is set by manufacturer and is presented in Table 1.
Please cite this article as: Modak, N.M., et al., Three-echelon supply chain coordination considering duopolistic retailers with perfect quality products. International Journal of Production Economics (2015), http://dx.doi.org/10.1016/j.ijpe.2015.05.021

4
N.M. Modak et al. / Int. J. Production Economics ∎∎∎∎ (∎∎∎∎∎∎∎∎) ∎∎∎–∎∎∎

Table 1
The optimal solutions for the centralized decision and three games of decentralized decision.

<table>
<thead>
<tr>
<th>Optimal</th>
<th>Decentralized decision</th>
</tr>
</thead>
<tbody>
<tr>
<td>Collusion (i = cm)</td>
<td>Stackelberg (j = u)</td>
</tr>
<tr>
<td>ω_{im}</td>
<td>ω_{im} = (A_{i} - X_{i1})/41 = X_{i1}</td>
</tr>
<tr>
<td>ω_{jm}</td>
<td>ω_{jm} = (A_{j} - X_{j1})/41 = X_{j1}</td>
</tr>
<tr>
<td>p_{im}</td>
<td>p_{im} = (a_{i} + a_{j} - 2)/2 (α + a_{i} + a_{j} - 2)</td>
</tr>
<tr>
<td>p_{jm}</td>
<td>p_{jm} = (a_{i} + a_{j} - 2)/2 (α + a_{i} + a_{j} - 2)</td>
</tr>
</tbody>
</table>

Centralized decision

Where X_{i1} = (1 + E)[i + c + c_{i} - E](w_{im} - c_{i} + c_{j} - E w_{jm} - c_{j}), a_{i} = a_{i} + a_{j} - 2(α + c_{i} + c_{j} - E w_{im} - c_{i} + c_{j} - E w_{jm} - c_{j}), p_{im} = (a_{i} + a_{j} - 2)/2 (α + a_{i} + a_{j} - 2), p_{jm} = (a_{i} + a_{j} - 2)/2 (α + a_{i} + a_{j} - 2), X_{j1} = (1 + E)[i + c + c_{i} - E](w_{jm} - c_{i} + c_{j} - E w_{im} - c_{i} + c_{j} - E w_{jm} - c_{j}).

4.1.3. Retailers play Stackelberg game

Under manufacturer-Stackelberg game setting at the final level assume that one of the two retailers, say, retailer-1 acts as the Stackelberg leader and the other i.e., retailer-2 acts as the Stackelberg follower. That is, to find the subgame perfect equilibrium of the entire three-stage game, using backward induction one first need to find the Stackelberg equilibrium between the retailers. Thus, for given w_{im} and p_{im}, retailer-2 maximizes its profit. Based on it retailer-1 finds optimal p_{i1} given w_{im}. The rest of the game can be approached same as the previous two cases. The optimal values of the decision variables (see appendix) are presented in Table 1. Consequently, using these values the optimal profits of the channel members are calculated and are also presented in Table 1.

4.1.4. Comparison of optimal decisions

Comparing the wholesale price of the manufacturer and the distributor under three scenarios we have the following proposition.

Proposition 1. Wholesale price of the manufacturer and the distributor have similar properties and are as follows:

(i) w_{im} > w_{jm} if \( p > 0 \) and (2a_{i} - 2a_{j} + a_{j} - 2p_{im}) < 0.

(ii) w_{im} > w_{jm} if \( p < 0 \) and (2a_{i} - 2a_{j} + a_{j} - 2p_{jm}) > 0 and \( a_{j} < a_{i} \).

(iii) w_{im} > w_{jm} if \( p > 0 \) and \( (2a_{i} - 2a_{j} + a_{j} - 2p_{im}) > 0 \) and \( a_{j} > a_{i} \).

(iv) w_{im} > w_{jm} if \( p > 0 \) and (2a_{i} - 2a_{j} + a_{j} - 2p_{jm}) < 0 and \( a_{j} > a_{i} \).

(v) w_{im} > w_{jm} if \( p < 0 \) and (2a_{i} - 2a_{j} + a_{j} - 2p_{jm}) > 0 and \( a_{i} < a_{j} \).

(vi) w_{im} > w_{jm} if \( p > 0 \) and (2a_{i} - 2a_{j} + a_{j} - 2p_{jm}) < 0 and \( a_{j} < a_{i} \).

is optimal Table 1. Using the optimal values in (1)-(3) the optimal profits of the channel members are calculated and are presented in Table 1.

4.2. Retailers play the collusion game

Assume that the duopolistic retailers recognize their interdependence and agree to act in union in order to maximize the total profit in the downstream retail market. The total expected profit of the downstream retail market in such case is

\[
\sigma_{e} = \sum_{r=1}^{2} \sum_{i=1}^{2} (p_{i} - w_{im} - c_{i} - c_{j} + \alpha) (X_{i1} - \alpha p_{i} - \beta) \tag{8}
\]

The expected profit functions of the manufacturer and the distributor are same as in (1) and (2). Using backward induction as in the previous case the subgame perfect equilibrium of this manufacturer-Stackelberg game can be found (see appendix) and

optimal values of the other decision variables are calculated and using these values the optimal profits of the channel members are calculated and are also presented in Table 1.
Fig. 1

0.5

0.2

Fig. 3

0.3

0.6

Table 1

omitted.

Proof. See Appendix.

Wholesale prices of the distributor for the retailers three behaviours are same as the manufacturer and hence the proof is omitted. Proposition 1 indicates that based on $\Psi$ and the retailers price sensitivity factors superiority of wholesale prices of the manufacturer and distributor can be identified. Note that, when $a_1 = a_2$, $a_1 = c_2$ and $c_1 = c_2 = 0$ then $\Psi = 0$ and it gives same outcome of special case of Yang and Zhou (2006).

Generally, $a_1^m > a_2^m > a_3^m$ but when $\beta = 0$ then $a_4^m = a_5^m$ and graphical representation of the manufacturer profits are given in Fig. 1. From Fig. 1 one can observe that the differences of the optimal profit of the manufacturer among the three cases increase with increasing $\beta$.

From Table 1 one may note that profits of the distributor in all cases are half of the manufacturer’s profits. So, profits of the distributor posses same behavior as of profits of the manufacturer. Graphical representation of comparison of the retailers profits is presented in Fig. 2 while Fig. 3 represents the comparison of retail prices.

Figs. 2 and 3 depict that the differences of the prices and profits of the retailers in three games decrease as $\beta$ decreases. When $\beta=0$, the outcomes of Cournot, Collusion and Stackelberg games are invariant. Thus, cross price effect of the downstream retailers has significant impact on the channel members profits. Also, note from Figs. 2 and 3 that the retailers profits are maximum when they play Collusion. This is quite obvious because at the downstream the retailers cooperate. As a result the upstream channel members profits are minimum in that case. That is why, Collusion game is downstream market is illegal in some countries. In the next subsection we have developed decisions of centralized channel.

4.2. Centralized decision

Suppose that all the channel members of the decentralized three-echelon supply chain are willing to form a vertically integrated system. Then, the decision maker in this system sets prices $p_1$ and $p_2$ at its two retail outlets. Let $\pi_t$ denotes the expected profit of the integrated system, then

$$\pi_t = \sum_{i=1}^{2} [p_i - c_i - X_i | \theta_i]$$

Taking the second-order partial derivatives of $\pi_t$ with respect to $p_1$ and $p_2$, we obtain $\partial^2 \pi_t / \partial p_1 \partial p_2 = -2a_1$, $\partial^2 \pi_t / \partial p_1 \partial p_2 = -2a_2$ and $\partial^2 \pi_t / \partial p_1 \partial p_2 = \partial^2 \pi_t / \partial p_2 \partial p_1 = 2b$. Let $\Delta_1$ and $\Delta_2$ denote respectively the first and second-order principal minors of Hessiam matrix of the total channel profit, $\pi_t$. Then, we have $\Delta_1 = -2a_1 < 0$ and $\Delta_2 = 4a_1a_2 - 4b^2 > 0$. Hence, the Hessiam matrix of the total channel profit, is negative definite. So, the profit function of centralized channel is a concave function of $p_1$ and $p_2$. Optimal selling prices of the duopolistic retailers can be obtained by solving the necessary conditions $\partial \pi_t / \partial p_1 = 0$ and $\partial \pi_t / \partial p_2 = 0$ and are displayed in Table 1. Centralized demand in two retail outlets and centralized channel profit are also given in Table 1.

Please cite this article as: Modak, N.M., et al., Three-echelon supply chain coordination considering duopolistic retailers with perfect quality products. International Journal of Production Economics (2015), http://dx.doi.org/10.31016/j.ijpe.2015.05.021
5. Effect of imperfect quality product

Generally speaking, it is common in marketing that selling price of imperfect quality product is lower than its production cost and so we assume that \( s + c_t + c_m < w_d \). Differentiation of wholesale prices, selling prices, profits of the channel members is given in the appendix. From the appendix observe that wholesale price of the manufacturer in all three cases decreases with increasing expected rate of imperfect quality product while wholesale price of the distributor and the selling price of the retailer increases. Fig. 4 represents the behaviour of wholesale prices of the channel members. Profits of all the channel members in all three cases decrease with increasing expected rate of imperfect quality product. Thus we have the following proposition (Fig. 5).

**Proposition 2.** When selling price of imperfect quality product is lower than its production cost then wholesale price of the manufacturer, profit function of all channel members decrease but wholesale price of the distributor increase with increasing expected rate of imperfect quality product in decentralized channel for all three behaviors of the duopolistic retailers.

The conclusion of Proposition 2 will be opposite if the distributor can sell the imperfect quality product more than its production cost i.e., if \( s + c_t + c_m > w_d \). Effect of expected rate of imperfect quality product in centralized decision can be found by differentiating optimal selling prices, demand and centralized profit with respect to \( E(t) \) as

\[
\frac{dp_i}{dt} = \frac{1}{2} \frac{d^2 \pi_i}{dx^2} \quad i = 1, 2
\]

\[
\frac{d\pi_c}{dt} = -\left( a_i - \beta c_t + c_m - w_d \right) \quad j = 1, 2
\]

That is, selling price of the product in centralized scenario increases while demand and profit decreases with increasing expected rate of imperfect quality product. Thus we have the following proposition.

**Proposition 3.** Selling price of the product in centralized scenario increases while demand and profit decreases with increasing expected rate of imperfect quality product if \( s + c_t + c_m > w_d \) otherwise the effect will be reversed.

6. Channel coordination using all unit quantity discount with franchise fee

In the three-echelon supply chain for all the competitive behaviors of the retailers, the channel is not coordinated. For channel coordination, assume that an upstream channel member provides all unit quantity discount to its immediate downstream channel member and charge a franchise fee. All unit quantity discount has been well studied and well applied to resolve channel conflict in supply chain literature. When all unit quantity discount is applied, a channel member provides discounts on the wholesale price to the other member anticipating centralized quantity to be ordered. Franchise fee is another contract that is used for cutting out channel conflict, where the upstream channel member supplies the product to the downstream channel member at its own marginal cost and charge a franchise fee for profit enhancement. We apply these two coordination contracts jointly for channel coordination in Cournot, Collusion and Stackelberg games of the retailers separately.

6.1. Retailers play Cournot game

Assuming that by providing discount, the manufacturer charges wholesale price \( p_{mU_d} \) to distributor and claims franchise fee \( f_{fl} \) from the distributor. The distributor supplies the product to the ith retailer \( (i = 1, 2) \) at a wholesale price \( p_{rU_d} \) and charges \( f_i \) franchise fee from the ith retailer. Then the expected profit function of retailers, distributor and manufacturer are as

\[
\pi_{rU_d} = (p_i - p_{rU_d} - c_m)(q_i - a_i + \beta p_i) \quad (i = 1, 2) \tag{13}
\]

\[
\pi_{dU_d} = \sum_{i=1}^{2} \left( (p_i - p_{rU_d} - c_m)(q_i - a_i + \beta p_i) + f_i \right) \tag{14}
\]

Under Cournot behavior of the retailers the necessary conditions \( \frac{d\pi_{rU_d}}{dp_i} = 0 \) \( (i = 1, 2) \) yield

\[
\rho_i = \frac{2a_i + \beta c_t}{2c_{t1} + 2a_2 + \beta c_{t2}} \quad (i = 1, 2) \tag{15}
\]

On the other hand, based on the retailers decision the distributor will maximize its profit function by determining the discount factors, which are different for different retailer. This assumption is quite reasonable and common in marketing practice. Generally, an upstream channel member will provide how much discount to its
different downstream channel members that depends on how much it receives from those members. That is, the amount of quantity discount is proportional to the order quantity and hence profit. As such, the necessary conditions for the existence of the optimal solution yields
\[ \rho_{\text{opt}} = \frac{a_{0} + a_{1} + a_{2} + \beta(a_{0} + a_{1} + a_{2} - \beta^{2})}{2w_{d}^{\alpha}(a_{0} + a_{1} + a_{2} - \beta^{2})} \]  
\( i = 1, 2 \)

(18)

where \( w_{d} = (1 + \epsilon)(1 - \theta)W_{d} \).

Obviously for channel coordination \( \rho_{\text{opt}} = \rho_{\text{opt}}^{\prime} \), (i = 1, 2), which on simplification suggests

\[ \rho_{\text{opt}}^{\prime} = \frac{a_{0} + (G_{2} + a_{1})(\theta - 1) + (1 + \epsilon)(1 - \theta)w_{d}^{\alpha}}{2w_{d}^{\alpha}(a_{0} + a_{1} - \beta^{2})} \]

\( i = 1, 2 \)

(19)

The result of (19) is quite interesting. Firstly, \( \rho_{\text{opt}}^{\prime} \) (i = 1, 2) may be negative i.e., initially the manufacturer has to provide some money to the distributor for channel coordination but finally it makes up its loss and gains through franchise fee. This scenario shows that for channel coordination the intermediary plays a major role, even if it initially demands money from upstream channel members to implement integrated decision. Second, it suggests that the manufacturer provides different discounts to different channel members under the same distributor. But, in the decision making context it is not possible as well as feasible. Thus, we assume that the manufacturer sets the discount factor for the distributor as the weighted mean of the discount factors of its corresponding retailers. Then, the discount factor of the manufacturer for the distributor is

\[ \rho_{\text{h}}^{\text{opt}} = \frac{\sum_{i=1}^{n} \rho_{\text{opt}}^{\prime} p_{i}}{\sum_{i=1}^{n} p_{i}} \]  

(20)

Thus, the manufacturer sets discount factor \( \rho_{\text{h}}^{\text{opt}} \) on its wholesale price. In response the distributor provides \( \rho_{\text{h}}^{\text{opt}} \) (i = 1, 2) discount on \( q_{d} \) of the retailers under the all unit quantity discount with franchise fee is

\[ \rho_{\text{h}}^{\text{opt}} = \frac{\sum_{i=1}^{n} \left( p_{i} - \rho_{\text{opt}}^{\prime} w_{d}^{\alpha} - c_{i} \right) p_{i}}{\sum_{i=1}^{n} p_{i}} \]  

(21)

Solving the necessary conditions for maximizing \( \rho_{\text{h}}^{\text{opt}} \) i.e.,

\[ \rho_{\text{h}}^{\text{opt}} = \frac{a_{1} + (G_{2} + a_{1})(\theta - 1) + (1 + \epsilon)(1 - \theta)w_{d}^{\alpha}}{2w_{d}^{\alpha}(a_{0} + a_{1} - \beta^{2})} \]

\( i = 1, 2 \)

(22)

Proceeding similar way as in Subsection 6.1, the amount quantity discount which will coordinate the channel are as follows:

\[ \rho_{\text{opt}}^{\prime} = \frac{X_{d}}{q_{d}} \]  

( i = 1, 2 \)

and

\[ \rho_{\text{h}}^{\text{opt}} = \frac{\rho_{\text{opt}}^{\prime} D_{i}^{1} + \rho_{\text{opt}}^{\prime} D_{i}^{2}}{D_{i}^{1} + D_{i}^{2}} \]  

(24)

Please cite this article as: Modak, N.M., et al., Three-echelon supply chain coordination considering duopolistic retailers with perfect quality products. International Journal of Production Economics (2015), http://dx.doi.org/10.1016/j.ijpe.2015.05.021

6.3. Retailers play Stackelberg game

Similar to decentralized scenario, the retailer-2 (follower) first optimizes its profit then the retailer-1 (leader) optimizes its profit function. The optimal quantity discount for coordination when the retailers play Stackelberg game is found as

\[ \rho_{\text{opt}}^{\prime} = \frac{X_{1} - c_{1} + a_{0}(1 + \epsilon)(1 - \theta)w_{d}^{\alpha}}{2w_{d}^{\alpha}(a_{0} + a_{1} - \beta^{2})} \]  

\( i = 1, 2 \)

(26)

\[ \rho_{\text{opt}}^{\prime} = \frac{a_{1} + (G_{2} + a_{1})(\theta - 1) + (1 + \epsilon)(1 - \theta)w_{d}^{\alpha}}{2w_{d}^{\alpha}(a_{0} + a_{1} - \beta^{2})} \]  

\( i = 1, 2 \)

(27)

and

\[ \rho_{\text{opt}}^{\prime} = \frac{2a_{1} + a_{2} + \beta(a_{0} + a_{1} - \beta^{2})}{2w_{d}^{\alpha}(a_{0} + a_{1} - \beta^{2})} \]  

\( i = 1, 2 \)

(28)

where

\[ \rho_{\text{opt}}^{\prime} = \frac{2a_{1} + a_{2} + \beta(a_{0} + a_{1} - \beta^{2})}{2w_{d}^{\alpha}(a_{0} + a_{1} - \beta^{2})} \]  

\( i = 1, 2 \)

(29)

Now, the expected profits of the channel members for jth \( j = \text{ct, cw, sg} \) game are respectively as

\[ \sigma_{i}^{\text{ct}} = \frac{a_{i} + (G_{2} + a_{1})(\theta - 1) + (1 + \epsilon)(1 - \theta)w_{d}^{\alpha}}{2w_{d}^{\alpha}(a_{0} + a_{1} - \beta^{2})} \]  

\( i = 1, 2; j = \text{ct, cw, sg} \)

(31)

\[ \sigma_{i}^{\text{cw}} = \frac{a_{i} + (G_{2} + a_{1})(\theta - 1) + (1 + \epsilon)(1 - \theta)w_{d}^{\alpha}}{2w_{d}^{\alpha}(a_{0} + a_{1} - \beta^{2})} \]  

\( i = 1, 2; j = \text{ct, cg, sg} \)

(32)

\[ \sigma_{i}^{\text{sg}} = \frac{a_{i} + (G_{2} + a_{1})(\theta - 1) + (1 + \epsilon)(1 - \theta)w_{d}^{\alpha}}{2w_{d}^{\alpha}(a_{0} + a_{1} - \beta^{2})} \]  

\( i = 1, 2; j = \text{ct, cg, sg} \)

(33)

Observe that \( \sigma_{i}^{\text{ct}} = \sigma_{i}^{\text{cw}} = \sigma_{i}^{\text{sg}} \), (i = \text{ct, cw, sg}) i.e., the channel is coordinated and we have the following proposition.

Proposition 4. All unit quantity discount with franchise fee coordinates the three-echelon supply chain under the duopolistic retailers Cournot, Collusion and Stackelberg behaviors.

Now, considering the situation without franchise fee i.e., before settlement of franchise fee we have

\[ \sigma_{i}^{\text{ct}} - \rho_{\text{opt}}^{\prime} w_{d}^{\alpha} = \sigma_{i}^{\text{cw}} \]  

(34)

\[ \sigma_{i}^{\text{ct}} - \rho_{\text{opt}}^{\prime} w_{d}^{\alpha} = \sigma_{i}^{\text{sg}} \]  

(35)

That is, before the settlement of franchise fee both the retailers get more money in Collusion game than Cournot game. Again,
The retailers get maximum wealth when they play collusion game and minimum in Cournot game. Since total profit of the channel members is equal to the centralized profit, it is obvious that first preference of the manufacturer will be Cournot and last is Collusion behavior of the duopolistic retailers. Thus we have the following proposition.

**Proposition 5.** Under the all unit quantity discount before the settlement of franchise for retailers preference sequence are (i) Collusion (ii) Stackelberg and (iii) Cournot, while the manufacturer prefers oppositely.

### 6.4. Some limits of franchise fee

Although the proposed all unit quantity discount with franchise fee resolves channel conflict, it is acceptable to the channel members only when they receive win-win outcomes. Panda et al. (2014b) have mentioned that in a pure profit maximizing supply chain, surplus profit due to channel coordination is arbitrarily distributed among the channel members when prices are endogenous.

The win-win outcomes of the channel members are ensured if they receive at least equal to their respective decentralized profits i.e., $s_{ij}^{D} = s_{ij}^{D}$, $(i = 1, 2, j = ct, cn, sg)$ and $s_{ij}^{D} = s_{ij}^{D}$, $(j = ct, cn, sg)$ and so on, which on simplification yields

\[
\begin{align*}
&f_{ij}^{D} \geq (p_{i}^{D} - \rho_{i}^{D}w_{a} - c_{0}D_{j}^{D} - f_{j}^{D}) \\
&f_{ij}^{D} \geq (1 + E(r))D_{j}^{D} + D_{j}^{D} \rho_{ij}^{D}w_{m} - c_{m}^{D} = f_{ij}^{D} \\
&f_{ij}^{D} \geq f_{ij}^{D} + \sum_{i} \left[ s_{ij}^{D} \rho_{ij}^{D}w_{a} + E(r)w_{m} - (1 + E(r))c_{i} - c_{j} \\
&+ \rho_{ij}^{D}w_{m} (D_{j}^{D}) \right] (j = ct, cn, sg)
\end{align*}
\]

Now to determine the other limits of franchise fee and particular surplus profit split for the channel members we consider the following section.

### 6.5. Surplus profit division

Although the inequalities indicate the upper limits of the franchise fees that the retailers provide to the distributor and lower limit of franchise fee that the distributor provides the manufacturer, still it does not ensure win-win profits for the channel members. Thus, how the surplus profit generated through coordination will be distributed among the channel members given all will achieve win-win profits. To determine this one first need to determine the range of franchise fees, that provide win-win outcomes. The best possible approach in this direction is to apply Nash bargaining product. In this context note that the distributors act as the mediator, who receives the lot from the manufacturer and distributes among the retailers. By doing so it makes profit. Interestingly, channel coordination the distributor plays central role because it actually maintains the lot streaming between the manufacturer and the retailers. Thus, when the coordination contracts resolve channel conflict, there may arise two questions: (i) What is the minimum amount of franchise fee that the distributor accept? (ii) What is the maximum percent of franchise fee that the distributor can provide to it’s manufacturer? These two questions are interrelated and one definitely get different results when approaches from the manufacturer to the retailers and from the retailers to the manufacturer i.e., backward sequential bargaining and forward sequential bargaining. The intuitive reason behind this is because of considering multi player Nash bargaining is quite reasonable and common in marketing practice. In marketing deal on implementation of contract issues and then division of surplus generated through contract is done between two players. But how one can use the bargaining to divide the surplus if the amount of surplus due to coordination, i.e., the ranges of franchise fees are not explicitly identified. Thus, we apply the process that first identifies the range of franchise fee and then determine the profit split. It is done in two steps by considering two immediate next or immediate previous channel members. Also, it can be done either by approaching from forward or from backward. In the next subsections, we apply these two processes and analyze the results.

### 6.5.1. Backward-sequential-bargaining for surplus profit division

In this process the optimal decisions for the ranges of franchise fees and surplus profit division are made in one-to-one basis. First the distributor and the retailers interact and then the distributor and the manufacturer find the decisions. The objectives of all channel members are to determine the shares of surplus profit that is generated through channel coordination. The sequence of events in this process are as follows:

1. Each retailer interacts with the distributor separately and determines the lower limit of franchise fee.
2. The distributor bargains with each retailer independently to determine the franchise fee within its range. Decentralized profit plus the surplus share is the optimal profit of the ith retailer. The distributor’s intermediate profit is the decentralized profit plus cumulative surplus shares, which it acquired from all of it’s retailers.
3. Based on the intermediate profit the distributor and the manufacturer identify the upper limit of franchise fee. This limit can be determined by assuming that the distributor accept the franchise fee as long as it’s decentralized profit is reserved.
4. The manufacturer and the distributor bargain for surplus share. The distributor’s optimal profit is decentralized profit plus accumulated surplus from all of it’s retailers minus surplus share to the manufacturer. The manufacturer’s profit is decentralized profit plus surplus share that it receives from the distributor.

The distributor deals with each of it’s retailers independently because it has different reservations for different retailers. How
how much franchise fee it will give the manufacturer. Since the players operate under symmetric information, the distributor takes a risk by supplying the information to the retailers that it has to pay $f_i^{ct}$ to the manufacturer and settles the franchise fees that it will take from the retailers. Later when the distributor determines the franchise fee, that it will give to the manufacturer through negotiation, may be higher or lower than $f_i^{ct}$. That is, the risk which the distributor takes on franchise fee during the entire negotiation process is sometimes beneficial and sometimes not. This feature is quite common in marketing, where the players have the opportunity to take decision freely. Thus, minimum amount of franchise fee that the distributor will accept from $i$th ($i=1,2$) retailer is

$$f_i^{ct} = \min \left\{ f_i^{ct} \mid D_I + f_i^{ct} \right\}, \quad i = 1, 2, j = ct, cn, sg$$

(37)

The distributor's decentralized and centralized profits corresponding to its $i$th ($i=1,2$) retailer are respectively shown as

$$\pi_i^{ct} = \pi_i^{ct} = \left[ \pi_i^{ct} \mid \pi_i^{ct} + (1 + \varepsilon_t)(s + c_z + \rho_i^{ct} \pi_i^{ct}) \right]$$

(38)

where $D_I$ denotes demand of the $i$th ($i=1,2$) retailer in decentralized Cournot behavioral scenario. The minimum amount of franchise fee that the distributor will accept from $i$th ($i=1,2$) retailer if $\pi_i^{ct} \geq \pi_i^{ct} = \pi_i^{ct}$, $j = ct, cn, sg$, i.e.,

$$\frac{f_i^{ct}}{D_I} = \pi_i^{ct} = \left[ \pi_i^{ct} \mid \pi_i^{ct} + (1 + \varepsilon_t)(s + c_z + \rho_i^{ct} \pi_i^{ct}) \right]$$

(39)

Clearly, $f_i^{ct}$ cannot be determined uniquely until $f_i^{ct}$ is determined. But least amount of franchise fee is acceptable to the distributor from the $i$th retailer only when it pays minimum amount of franchise fee $f_i^{ct}$ to the manufacturer, i.e.,

$$f_i^{ct} = \frac{f_i^{ct}}{D_I} = \frac{\pi_i^{ct}}{D_I} = \left[ \pi_i^{ct} \mid \pi_i^{ct} + (1 + \varepsilon_t)(s + c_z + \rho_i^{ct} \pi_i^{ct}) \right]$$

(40)

Interestingly profit of the manufacturer is equal to its decentralized profit when the distributor gives $f_i^{ct}$ amount of franchise fee. But it is reasonable to assume that the distributor has to share a reasonable portion of surplus profit with the manufacturer, i.e., $f_i^{ct} > f_i^{ct}$ otherwise the manufacturer has no reason to accept the contract. Suppose the distributor predetermines a target that it will pay $f_i^{ct} / f_i^{ct} > f_i^{ct}$ amount of franchise fee to the manufacturer. The intuitive realization is straightforward. As a mediator, the distributor first settles the franchise fee without knowing about
channel members in backward bargaining process for Cournot behavior of the retailers in Fig. 6.

Now, if the distributor bargains with the manufacturer for $f_m^t (f_m^t, f_m^t)$ that will maximize their profit then the Nash bargaining product can be found as

$$\max_{f_m^t} \left[ d_n^t + f_m^t + f_m^t f_m^t \right] = \text{Nash bargaining product}$$

where

$$d_n^t = \left[ \sum_{i=1}^{n} (\pi_i D_i') \theta_i + c_{mt} \right]$$

and

$$d_n^t = \left[ 1 + (\pi_i D_i') \theta_i + c_{mt} \right]$$

Optimal value of $f_m^t$ is

$$f_m^t = \frac{d_n^t + f_m^t + f_m^t f_m^t}{2}$$

Therefore, optimal profits of the retailers, the distributor and the manufacturer after the sequential-bargaining process are

respectively given as

$$\mu_{i,j}^t = \frac{d_n^t + f_m^t + f_m^t f_m^t}{2} \quad i = 1, 2; \quad j = \text{ct, cn, sg}$$

$$\mu_{i,j}^t = \frac{2d_n^t + f_m^t + f_m^t f_m^t}{4} \quad j = \text{ct, cn, sg}$$

$$\mu_{i,j}^t = \frac{2d_n^t + f_m^t + f_m^t f_m^t}{4} \quad j = \text{ct, cn, sg}$$

Note that, $d_n^t / D_i^t < 1$ and $d_n^t / D_i^t > 0$ for all channel members. Therefore, optimal profits of the channel members are $d_n^t$ and when it increases retailers' profits decrease but profits of the manufacturer and the distributor are both $d_n^t$. This represents this characteristics.

From the above bargaining process results can be realized. Firstly, unlike the centralized decision, the ranges of the contract parameters ensure win-win opportunities for all the channel members. Moreover, the channel is coordinated and maximum benefit is distributed among the channel members. Secondly, retailers share the surplus profit equitably with the distributor. Thirdly, the distributor shares the surplus profit for channel coordination with the manufacturer equitably. From the above discussion we have the following proposition.

**Proposition 6.** All unit quantity discount with franchise fee coordinates the channel and the channel members get win-win profit through backward-sequential-bargaining for $f_m^t (f_m^t, f_m^t)$ (i = 1, 2; $j = \text{ct, cn, sg}$) and $f_m^t (f_m^t, f_m^t)$ and agreed on franchise fee $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_m^t$, $f_
that ensures the its decentralized profit then
\[
\frac{\Delta f_m}{\Delta f_m} = -\Delta f_m + \sum_{i=1}^{n} \left( \frac{\partial f_m}{\partial t} \right)_{ct, cn, sg}
\]
\[
\Delta f_m' = j = ct, cn, sg
\]
(51)

Any \( f_m \) in \( (\Delta f_m, \Delta f_m') \) ensures win–win profits for the manufacturer and the distributor. Note that first stage of forward-sequential-bargaining process will be possible only when \( \Delta f_m > \Delta f_m' \), i.e., if \( \Delta f_m' > \Delta f_m \). The forward-sequential-bargaining process depends on distributor’s managing capability and decision making power. When the distributor assures that the cumulative amount of franchise fee from retailers will exceed the amount \( \Delta f_m' - \Delta f_m \) then it will participate in bargaining with the manufacturer for a \( f_m = \left( f_m', f_m'' \right) \) that will maximize their profits. In such case the Nash Bargaining product can be found as
\[
\max \left( x_i + y_i \right)_{f_m', f_m''} = j = ct, cn, sg
\]
(52)

Optimal value of \( f_m \) is
\[
f_m = \frac{a_f + \sqrt{a_f^2 + 2 \Delta f_m'a_f}}{2}, j = ct, cn, sg
\]
(53)

After bargaining the intermediate profit is
\[
\Delta f_m = -\Delta f_m' + \sum_{i=1}^{n} \left( \frac{\partial f_m}{\partial t} \right)_{ct, cn, sg}
\]
\[
\Delta f_m' = j = ct, cn, sg
\]
(54)

As soon as the distributor finds its intermediate profit, it will demand the franchise fee and bargain with the retailers independently because it has different reservations for different retailers. Applying similar approach as in the backward sequential bargaining, we have the minimum amount of franchise fee that the distributor will accept from ith \( (i = 1, 2) \) retailer can be found as
\[
f_m = \frac{a_f + \sqrt{a_f^2 + 2 \Delta f_m'a_f}}{2}, \Delta f_m', \Delta f_m'' \quad i = 1, 2, j = ct, cn, sg
\]
(55)

The region of franchise fee for the retailers is presented in Fig. 8. Since, any \( f_m = \left( f_m', f_m'' \right) \) 1 = 1, 2, 2 solves channel conflict, the distributor and the ith retailer bargain to determine an acceptable \( f_m = \left( f_m', f_m'' \right) \) that will maximize the Nash bargaining product. The Nash Bargaining product can be found as
\[
\max \left( x_i + y_i \right)_{f_m', f_m''} = j = ct, cn, sg
\]
(56)

Optimal value of \( f_m \) is
\[
f_m = \Delta f_m' - \Delta f_m'' + \left( \frac{\Delta f_m''}{\Delta f_m'} \right)_{ct, cn, sg}
\]
(57)

Thus, the ith retailer’s optimal profit after bargaining can be found from (31) by replacing \( f_m \) by \( f_m' \) of (59). The feasible region of win–win outcome of all channel members in forward bargaining process for Cournot behavior of the retailers displayed in Fig. 7.

Therefore, optimal profits of the retailers, the distributor and the manufacturer after the sequential-bargaining process are respectively given as
\[
\frac{\Delta x_m}{\Delta x_m} = \frac{\Delta y_m}{\Delta y_m} = \frac{\Delta z_m}{\Delta z_m} = \frac{\Delta f_m}{\Delta f_m} = \frac{\Delta f_m'}{\Delta f_m'} = \frac{\Delta f_m''}{\Delta f_m''} \quad i = 1, 2, j = ct, cn, sg
\]
(58)

\[
\Delta x_m' = \Delta y_m' = \Delta z_m' = \frac{\Delta x_m + \Delta y_m + \Delta z_m}{2}, \Delta f_m' = \Delta f_m'' \quad j = ct, cn, sg
\]
(59)

Note that, \( \Delta x_m' = \Delta y_m' = \Delta z_m' = \Delta f_m' = \Delta f_m'' \), that is the bargaining profit of the retailers as well as the distributor decrease but the profit of the manufacturer increases with increasing \( f_m' \) and it is indicated in Fig. 9. From the above discussion and Fig. 9 we have the following proposition.

**Proposition 7.** All unit quantity discount with franchise fee coordinates the channel and the channel members get win–win profit through forward-sequential-bargaining when \( f_m = \left( f_m', f_m'' \right) \) (i = 1, 2, j = ct, cn, sg) and \( f_m = \left( f_m', f_m'' \right) \) and agreed on franchise fee \( f_m' \) (i = 1, 2, j = ct, cn, sg).

6.5.3. Comparison of two bargaining processes

Comparing optimal profits of the retailers, the distributor and the manufacturer obtained in two sequential-bargaining processes we get
\[
\frac{\Delta x_m}{\Delta x_m} = \frac{\Delta y_m}{\Delta y_m} = \frac{\Delta z_m}{\Delta z_m} = \frac{\Delta f_m}{\Delta f_m} = \frac{\Delta f_m'}{\Delta f_m'} = \frac{\Delta f_m''}{\Delta f_m''} \quad j = ct, cn, sg
\]
(60)

Table 2

<table>
<thead>
<tr>
<th>Systems</th>
<th>( w_m )</th>
<th>( w_m' )</th>
<th>( f_m' )</th>
<th>( f_m'' )</th>
<th>( f_m'' )</th>
<th>( f_m'' )</th>
<th>( f_m'' )</th>
<th>( f_m'' )</th>
<th>Channel profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cournot</td>
<td>315.30</td>
<td>268.18</td>
<td>307.19</td>
<td>296.08</td>
<td>10,086.92</td>
<td>5040.81</td>
<td>1199.79</td>
<td>696.91</td>
<td>17,022.11</td>
</tr>
<tr>
<td>Collusion</td>
<td>315.30</td>
<td>268.18</td>
<td>307.19</td>
<td>296.08</td>
<td>2008.12</td>
<td>1199.79</td>
<td>696.91</td>
<td>3,022.11</td>
<td>17,022.11</td>
</tr>
<tr>
<td>Stackelberg</td>
<td>315.30</td>
<td>268.18</td>
<td>307.19</td>
<td>296.08</td>
<td>10,086.92</td>
<td>5040.81</td>
<td>1199.79</td>
<td>696.91</td>
<td>17,022.11</td>
</tr>
<tr>
<td>Centralized</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Please cite this article as: Modak, N.M., et al., Three-echelon supply chain coordination considering duopolistic retailers with perfect quality products. International Journal of Production Economics (2015), http://dx.doi.org/10.3106/j.appe.2015.05.021
7. Numerical example

In this section, a numerical study is carried out to demonstrate the behavior of the proposed model and to gain some insights of the problem which are being studied. Assume that the potential market demand of the retailer-1 and the retailer-2 is \( q_d = 1000 \) units respectively and these are found by adding \( \alpha_1 = 0.2 \) and \( \alpha_2 = 0.3 \) respectively. Other parameter values are \( \alpha_1 = 0.9, \alpha_2 = 1.0, \rho = 0.4, c_w = 10, c_y = 2, c_1 = 1.5, c_2 = 1.5, w_g = 5, s = 0.15. \) Suppose the random amount of imperfect quality items, \( r, \) is uniformly distributed between 0.02 and 0.20. Hence, \( E(r) = (0.02 + 0.20)/2 = 0.11. \) Optimal prices and optimal individual expected profits and total channel profit in Cournot, Collusion, Stackelberg bargaining processes are presented in Table 2.

From Table 2, one can easily verify that retail price of the product is maximum in Collusion game of the duopolistic retailers. The retailers also get maximum profit in Collusion game, e.g., retailer-1 gets 8.36% more profit in Collusion game compared to Cournot game. The manufacturer and the distributor get maximum profits in the duopolistic retailers’ Cournot behavior. Total channel profit is maximum in Cournot and minimum in Collusion behavior of the duopolistic retailers. Thus, numerical results justify the analytical results for the channel members preference of game. The centralized channel profit is 87% more than total channel profit of the Cournot game. So, channel coordination is essential to achieve improved channel performance.

Table 3 illustrates ranges and bargaining outcomes of franchise fees in backward and forward sequential bargaining processes for coordination and win–win outcome. Tables 2 and 3 indicate that before the settlement of franchise fee the retailer-1 gets 9280, 10186, 10013, while the retailer-2 gets 9153, 9153 and 15857 respectively in Cournot, Stackelberg and Collusion games respectively and these are found by adding \( \alpha_i \) and \( \beta_i \) \( (i = 1, 2, j = ct, sg, cn). \) That is, the retailers mostly prefer Collusion game instead of the other two. For the numerical example of two bargaining processes, we assume that the distributor sets the target franchise fee that it will pay to the manufacturer is \( f_{ri} = f_{ri}^C + (\alpha_i - \alpha_2 - \alpha_1 - \alpha_3 - 2\alpha_i + \alpha_2 + \alpha_3) \) in backward ‘sequential bargaining. On the other hand, in forward sequential bargaining the distributor assumes that the target franchise fee that it can collect from the \( i \)th \( (i = 1, 2) \) retailers is \( f_{ri} = f_{ri}^C + (\alpha_i - \alpha_2 - \alpha_1 - \alpha_3 - 2\alpha_i + \alpha_2 + \alpha_3) \) for all \( i \) and \( j = ct, sg, cn). \) Based on these assumptions, \( f_{ri}^C = f_{ri}^C + (\alpha_i - \alpha_2 - \alpha_1 - \alpha_3 - 2\alpha_i + \alpha_2 + \alpha_3) \) and \( f_{ri} = f_{ri}^C + (\alpha_i - \alpha_2 - \alpha_1 - \alpha_3 - 2\alpha_i + \alpha_2 + \alpha_3) \) are calculated and are presented in Table 3. For example, for backward sequential bargaining \( f_{11} = 55864.6 \) and for backward sequential bargaining \( f_{11} = 7019.8 \) and \( f_{21} = 8040.7. \) Note that, the ranges of franchise fees of the retailer-1 under Cournot behaviour for backward and forward sequential bargaining processes are (5550, 8080) and (4461, 8080) respectively. That is, the range of franchise fee of the retailer-1 is more in backward sequential bargaining than forward sequential bargaining and so the retailer-1 prefers forward sequential bargaining instead of backward sequential bargaining. Same result can be found for the retailer-2. Similarly, the manufacturer also prefers backward sequential bargaining because upper limit of franchise fee that it gets from the distributor is less when compared with forward sequential bargaining. This scenario may change because the preference of bargaining process depends on the decisions of the distributor on target franchise fee and its outcome is given in Proposition 8.

Table 4 shows the amount of quantity discount that the upstream channel members provide to its downstream members. Interestingly, \( \alpha_{b, c} < 0, \) i.e., for channel coordination the manufacturer initially has to pay to the distributor but through settlement franchise fee it makes win–win profit. Also all the channel members optimal bargaining profits are win–win, i.e., higher than their decentralized profits and hence acceptable to all the channel members.

8. Summary and concluding remarks

The paper considers a three-echeleon supply chain, where in the downstream two retailers may play Cournot, Stackelberg or Collusion game. Also, the paper assumes that the manufacturer supplies the product to the distributor in a single lot that contains random proportion of defective items. It is found that expected imperfect quality product has considerable impact on pricing. In fact, when the distributor sells the product below its marginal cost, the manufacturer reduces its wholesale price whereas the retailer increases its wholesale price. The intuitive reason is straightforward. The manufacturer helps the distributor to reduce its loss and the distributor does the same by increasing wholesale price. Reverse trend may be observed when the selling price of the defective items is higher. All unit quantity discount with franchise fee coordinates the channel through enough to provide win–win profits for all the channel members. It is found that the manufacturer prefers the retailers games is in the order Cournot, Stackelberg or Collusion, whereas the retailers prefer the reverse order because in Collusion game the retailers cooperate and control the market by setting prices. For surplus profit division the paper proposes two sequential bargaining processes, which are quite new in the literature. The bargaining processes are used to determine the limits of the franchise fees and to split the surplus profit trough negotiation between two consecutive stages.

Table 3

<table>
<thead>
<tr>
<th>Bargaining process</th>
<th>Game</th>
<th>( f_{ri}^C )</th>
<th>( f_{ri}^C )</th>
<th>( f_{ri} )</th>
<th>( f_{ri} )</th>
<th>( f_{ri} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Backward-</td>
<td>Cournot</td>
<td>(5550, 8080)</td>
<td>6815</td>
<td>(5053, 8456)</td>
<td>6808</td>
<td>(46,949, 58,885)</td>
</tr>
<tr>
<td></td>
<td>Collusion</td>
<td>(11,208, 14,096)</td>
<td>12,952</td>
<td>(10,002, 15,126)</td>
<td>13,014</td>
<td>(71,694, 85,731)</td>
</tr>
<tr>
<td></td>
<td>Stackelberg</td>
<td>(8037, 8979)</td>
<td>7568</td>
<td>(5106, 8433)</td>
<td>6814</td>
<td>(48,587, 60,376)</td>
</tr>
<tr>
<td></td>
<td>Forward-</td>
<td>(4461, 8808)</td>
<td>6271</td>
<td>(3913, 8456)</td>
<td>6238</td>
<td>(46,949, 60,322)</td>
</tr>
<tr>
<td></td>
<td>Cournot</td>
<td>(9964, 14,096)</td>
<td>12,330</td>
<td>(9600, 15,126)</td>
<td>12,363</td>
<td>(71,694, 86,965)</td>
</tr>
<tr>
<td></td>
<td>Stackelberg</td>
<td>(4950, 8979)</td>
<td>6959</td>
<td>(4058, 8433)</td>
<td>6245</td>
<td>(48,587, 61,015)</td>
</tr>
</tbody>
</table>

Please cite this article as: Modak, N.M., et al., Three-echeleon supply chain coordination considering duopolistic retailers with perfect quality products. International Journal of Production Economics (2015), http://dx.doi.org/10.1016/j.ijpe.2015.05.021

\[ a^{(3)} - a^{(1)} = \frac{a_{1i} - (a_{1i} + a_{2j}) + f_{ri} + (f_{ri} + f_{ri}^C)}{4} \]
\[ a^{(3)} - a^{(1)} = \frac{B(f_{ri} - a_{1i}) + f_{ri} + f_{ri}^C - 2f_{ri}^C}{4(f_{ri} + f_{ri}^C)} \]
\[ j = ct, cn, sg \]
of the supply chain. As such, the mediator distributor plays the central role by setting a target profit in each of the two processes at the first stage of bargaining by assuming that it can make up the target in the next stage of bargaining. This is quite common in marketing practice that for the best output of the channel, the mediator makes a commitment in the first stage of negotiation and tries to overcome the loss due to the commitment in the next stage of negotiation. The paper also presents a preference analysis of the bargaining processes for the channel members. It indicates that the manufacturer and the distributor prefer backward bargaining, whereas the retailers prefer the other.

Although the paper provides some interesting managerial insights, still it has some limitations. The paper considers linear price dependent demand. Instead of this some other well established demand functions may be considered. Then, it is essential to verify whether the managerial insights can be generalized or not.

The paper uses sequential bargaining that has two stages. More investigation is needed to verify the applicability of the process in general case. That is, whether the process can be applied in an echelons supply chain or not. Then only it can be concluded that the proposed process is dynamic for surplus profit division among a supply chain members.

Appendix A

A.1. Decentralized Collusion solution

The necessary conditions to obtain the optimal total expected profit of the downstream retail market yield (i.e., $\alpha_{m}/\alpha_{p} = 0$ and $\delta_{m}/\delta_{p} = 0$) $\beta_{m} = \frac{1}{2}$

$$p_{1} = \frac{1}{2} \left( \frac{\alpha_{m} + \beta_{m}}{\alpha_{p}} \right)$$

and

$$p_{2} = \frac{1}{2} \left( \frac{\alpha_{m} + \beta_{m}}{\alpha_{p}} \right)$$

Taking the second-order partial derivatives of $\pi$, with respect to $p_{1}$ and $p_{2}$, we obtain $\delta_{m}/\alpha_{p} = -2\alpha_{1}$ and $\delta_{m}/\alpha_{p} = -2\alpha_{2}$ and $\delta_{m}/\alpha_{p} = -2\alpha_{1}$ and $\delta_{m}/\alpha_{p} = -2\alpha_{2}$.

Let $\alpha_{1}$ and $\alpha_{2}$ denote respectively the first and second-order principal minors of Gaussian matrix of the total channel profit, $\alpha_{i}$. Then, we have $\alpha_{1} = -2\alpha_{1} < 0$ and $\delta_{m}/\alpha_{p} = -2\alpha_{2} - 4\alpha_{i} > 0$.

Hence, the Gaussian matrix of the total profit of the retailers is negative definite. Thus, for any given $\omega_{m}$, the total expected profit for the downstream retail market, $\pi$, is a concave function of $p_{1}$ and $p_{2}$.

Total demand of the downstream retail market is

$$D_{1} + D_{2} = \frac{1}{2} \left( \alpha_{1} + \alpha_{2} - 2\beta \right)$$

where $\alpha_{1} = \alpha_{1} + \alpha_{2} - (\alpha_{1} - \beta_{1})(\alpha_{2} - \beta_{2}) + C_{1}/2$ and $\alpha_{2} = (\alpha_{1} - \beta_{1})(\alpha_{2} - \beta_{2}) + C_{2}/2$. The distributor's expected profit function is

$$E(\pi) = \left[ A_{1} - 4\alpha_{1} \alpha_{2} - 2\beta \omega_{m} \right] + \frac{E(\pi)}{\delta_{m} \delta_{p} - (1 + E(\pi))}$$

(A.3)

Now, $\frac{E(\pi)}{\delta_{m} \delta_{p}} = (\alpha_{1} + \alpha_{2} - 2\beta) < 0$ (due to $\beta < \alpha_{i}$). Using (24) and (22) in (5), we have the expected profit function of the manufacturer $\pi_{m}$, $\pi_{m}$ is a concave function of $\omega_{m}$. The distributor's optimal wholesale price is (solving $\frac{E(\pi)}{\delta_{m} \delta_{p}} = 0$)

$$\omega_{m}^{*} = \frac{1}{2} \left( 1 + \frac{E(\pi)}{\delta_{m} \delta_{p}} \right)$$

(A.4)

Using (24) and (22) in (5), we have the expected profit function of the manufacturer $\pi_{m}$, $\pi_{m}$ is a concave function of $\omega_{m}$ (since, $\frac{E(\pi)}{\delta_{m} \delta_{p}} < (1 + E(\pi)) \alpha_{1} + \alpha_{2} - 2\beta < 0$ due to $\beta < \alpha_{i}$). The necessary condition, i.e., $\delta_{m} \delta_{p} = 0$, to obtain the maximum profit of the manufacturer yields

$$\omega_{m}^{*} = \frac{1}{2} \left( 1 + \frac{E(\pi)}{\delta_{m} \delta_{p}} \right)$$

(A.5)

Substituting (A.5) in (A.4), we have the distributor's optimal wholesale price of the perfect quality products as follows:

$$\omega_{m}^{*} = \frac{1}{2} \left( 1 + \frac{E(\pi)}{\delta_{m} \delta_{p}} \right)$$

(A.6)

Using (A.6), the optimal selling prices of the perfect quality product at the retailer’s end are

$$p_{1}^{*} = \frac{1}{2} \left( \frac{\alpha_{m} + \beta_{m}}{\alpha_{p}} \right)$$

and

$$p_{2}^{*} = \frac{1}{2} \left( \frac{\alpha_{m} + \beta_{m}}{\alpha_{p}} \right)$$

(A.7)

Using (A.6)−(A.8), the expected profits of the manufacturer, the distributor and the ith retailer are displayed in Table 1.

A.2. Decentralized Stackelberg solution

For any given $\omega_{m}$ and $\pi_{1}$, the maximizing-profit of retailer-2 responds with the retail price of $p_{2} = (\alpha_{2} + \alpha_{2} + \alpha_{2} + \alpha_{2} + \alpha_{2})/(2\alpha_{2})$ (solving $\omega_{m}^{*} = 0$). Substituting retailer-2’s reaction function in retailer-1’s profit function, we have

$$\pi_{1} = (\pi_{1} - \delta_{m}) / \delta_{m}$$

(A.9)

Retai-l’s profit is a function of $p_{1}$ alone and a concave function of $p_{1}$, $(\pi_{1} - \delta_{m}) / \delta_{m} > 0$. Thus, for any $\omega_{m}$ set by the distributor, retailer-1 who acts as the leader can obtain his optimal retail price below, setting $\omega_{m} = 0$.

$$p_{1} = F_{1} + F_{1} \omega_{m}$$

(A.10)

Where

$$F_{1} = [2(\alpha_{2} + \alpha_{2} + \alpha_{2} + \alpha_{2} + \alpha_{2})] / \delta_{m}$$

and

$$F_{1} = [2(\alpha_{2} + \alpha_{2} + \alpha_{2} + \alpha_{2} + \alpha_{2})] / \delta_{m}$$

This optimal retail

Table 4

<table>
<thead>
<tr>
<th>Bargaining process</th>
<th>Game</th>
<th>$\alpha_{m}$</th>
<th>$\beta_{m}$</th>
<th>$\alpha_{p}$</th>
<th>$\beta_{p}$</th>
<th>$\omega_{m}$</th>
<th>$\omega_{p}$</th>
<th>$\pi_{1}$</th>
<th>$\pi_{2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Backward-sequential bargaining</td>
<td>Cournot</td>
<td>0.323</td>
<td>0.310</td>
<td>-1.026</td>
<td>1.6051</td>
<td>11.09</td>
<td>2465</td>
<td>2345</td>
<td></td>
</tr>
<tr>
<td>Backward-sequential bargaining</td>
<td>Collision</td>
<td>0.048</td>
<td>0.048</td>
<td>-1.820</td>
<td>14.971</td>
<td>10.095</td>
<td>2684</td>
<td>2238</td>
<td></td>
</tr>
<tr>
<td>Forward-sequential bargaining</td>
<td>Stackelberg</td>
<td>0.280</td>
<td>0.310</td>
<td>-1.026</td>
<td>16.770</td>
<td>9.176</td>
<td>3099</td>
<td>2915</td>
<td></td>
</tr>
<tr>
<td>Forward-sequential bargaining</td>
<td>Cournot</td>
<td>0.280</td>
<td>0.310</td>
<td>-1.026</td>
<td>16.585</td>
<td>9.157</td>
<td>3221</td>
<td>2907</td>
<td></td>
</tr>
</tbody>
</table>

Please cite this article as: modak et. al., three-echelon supply chain coordination considering duopolistic retailers with perfect quality products. International Journal of Production Economics (2015), http://dx.doi.org/10.1016/j.ijpe.2015.05.021
price will give the maximum profit of the retailer-1 as

\[
s^*_w = \left(2a_2a_1 + \beta(\beta_1 - \beta)(\beta_1 - \beta)\frac{\mu_0}{(\beta_2 - \beta_1)^2}\right) \left(\frac{\beta_2 - \beta_1}{(\beta_2 - \beta_1)^2}\right).
\]

(A.11)

Retailer-2’s optimal sales price and maximum profit from follower-ship can be determined by substituting retailer-1’s optimal leadership retail price in retailer-2’s reaction function. Then, the relevant expressions are

\[
p^*_w = F_2 + \frac{\partial \pi}{\partial p_2} \left(\frac{\partial \pi}{\partial \xi_2}\right)^{-1}, \quad \xi_2 = \left(\frac{\partial \pi}{\partial \xi_2}\right)^{-1}.
\]

(A.12)

where \(F_2 = \left(2a_2a_1 + \alpha_2(\alpha_1 - \alpha_2) - \beta(\beta_2 - \beta)(\beta_2 - \beta)\frac{\mu_0}{(\beta_2 - \beta_1)^2}\right)\left(\frac{\beta_2 - \beta_1}{(\beta_2 - \beta_1)^2}\right)\) and \(\xi_2 = \left(\frac{\partial \pi}{\partial \xi_2}\right)^{-1}\). Thus the maximum profit of the retailer-2 is

\[
s^*_w = \left(2a_2a_1 + \alpha_2(\alpha_1 - \alpha_2) - \beta(\beta_2 - \beta)(\beta_2 - \beta)\right)\left(\frac{\beta_2 - \beta_1}{(\beta_2 - \beta_1)^2}\right) - 3\beta_2\left(\mu_0 + \xi_2\right)^2\left(\frac{\beta_2 - \beta_1}{(\beta_2 - \beta_1)^2}\right) - \left(15a_2a_1 - \beta_2\right)^2.
\]

(A.13)

The distributor follows the retailers’ reaction functions \(p^*_w\) and \(p^*_w\) for any \(w_0\), value she sets for perfect quality products. The profit function of the distributor is

\[
s_d = (D_1 + D_2)w_0 + \frac{\mu_0}{2}(1 + \epsilon)(D_1 + D_2)(c + c_w + w_0).
\]

(A.14)

The downward demand is thus \(D_1 + D_2 = F_1 - F_2w_0\), where \(F_1 = a_0(a_0 - a_1)(\beta_2 - \beta_1)(\beta_2 - \beta_1)F_2\) and \(F_2 = a_0(a_0 - a_1)(\beta_2 - \beta_1)(\beta_2 - \beta_1)F_4\).

\[
F_4 = \frac{\pi_0}{\beta_2} - 2\xi_2 < 0 \quad \text{(due to} \quad \beta < \alpha, \quad \text{and} \quad F_1 > 0, \quad F_2 > 0).\]

Therefore, when the duopolistic retailers play Stackelberg game in the downward demand market, the manufacturer will set an optimal wholesale price as (solving \(\frac{\partial \pi}{\partial w_0} = 0\))

\[
w_0^* = \frac{1}{2(1 + \epsilon)} \left(\frac{F_1}{\beta_2} \left(1 + \epsilon\right)\left(c + c_w + w_0\right) + \frac{\mu_0}{2}(1 + \epsilon)w_0\right).
\]

(A.17)

As the manufacturer knows the distributor’s reaction function \(w_0^*\) for any \(w_0\) value he/she can set to maximize the profit:

\[
s_m = (1 + \epsilon)(D_1 + D_2)(c + c_w + w_0).
\]

(A.16)

The manufacturer’s expected profit, \(E_{\pi_m}\), is also a concave function of \(w_0\) as \(E_{\pi_m} / w_0 - (1 + \epsilon)\left(F_1 - F_2w_0\right) < 0 \quad \text{(due to} \quad \beta < \alpha, \quad \text{and} \quad F_1 > 0, \quad F_2 > 0)\).

\[
\frac{\partial E_{\pi_m}}{\partial w_0} = \left(\frac{F_1}{\beta_2} \left(1 + \epsilon\right)\left(c + c_w + w_0\right) + \frac{\mu_0}{2}(1 + \epsilon)w_0\right).
\]

(A.18)

\[
p^*_m = \frac{1}{2(1 + \epsilon)} \left(\frac{F_1}{\beta_2} \left(1 + \epsilon\right)\left(c + c_w + w_0\right) + \frac{\mu_0}{2}(1 + \epsilon)w_0\right).
\]

(A.19)

\[
\frac{\partial E_{\pi_m}}{\partial w_0} = \left(\frac{F_1}{\beta_2} \left(1 + \epsilon\right)\left(c + c_w + w_0\right) + \frac{\mu_0}{2}(1 + \epsilon)w_0\right).
\]

(A.20)

The expected profits are given in Table 1.

A.3. Comparison of the wholesale prices

Comparison of the wholesale price of the manufacturer and the distributor under three scenario are as follows:

\[
w_0^* - w_0^* = \frac{2\beta(1 + \epsilon)(\beta_1 - \beta_2)(\beta_1 - \beta_2)\mu_0}{(\beta_2 - \beta_1)^2}.
\]

(A.21)

where \(\mu_0 = \frac{\partial \pi}{\partial \xi_2}\) and \(\alpha_2 = \frac{\pi_0}{\beta_2} - 2\xi_2 < 0\). Clearly, \(w_0^* = w_0^*\) when \(\beta = 0\) or \(a_2 = a_1\) or \(\beta = 0\). But \(w_0^* > w_0^*\) if \((a_2 - a_1)\beta > 0\) i.e., either \(a_2 > a_1\) and \(\beta > 0\) or \(a_2 < a_1\) and \(\beta < 0\). Otherwise \(w_0^* < w_0^*\).
\[
\begin{align*}
\frac{d^2 y}{dx^2} &= \frac{F(s + c_2 + c_m - w_0)}{4} \frac{F}{X_i} (A.34) \\
\frac{d^2 y}{dx^2} &= \frac{B(s + c_2 + c_m - w_0)}{16} \frac{A}{X_i} (A.35) \\
\frac{d^2 y}{dx^2} &= \frac{(d_1 + d_2 - 2s(s + c_2 + c_m - w_0))}{2d_1} \frac{2d_1}{(d_1 + d_2 - 2s)} (A.36) \\
\frac{d^2 y}{dx^2} &= \frac{F(s + c_2 + c_m - w_0)}{8} \frac{F}{X_i} (A.37) \\
\frac{d^2 y}{dx^2} &= \frac{\alpha_2(d_1, a_1, a_2) - \beta_1 \rho_1 - \beta_1 \rho_2 s}{2d_4 d_1 a_1 a_2} \frac{X_1}{C_0} (A.38) \\
\frac{d^2 y}{dx^2} &= \frac{s + c_2 + c_m - w_0}{8} \frac{X_1}{C_0} (A.39) \\
\frac{d^2 y}{dx^2} &= \frac{s + c_2 + c_m - w_0}{4} (A.40)
\end{align*}
\]

References


Please cite this article as: Modak, N.M., et al., Three-echelon supply chain coordination considering duopolistic retailers with perfect quality products. International Journal of Production Economics, http://dx.doi.org/10.1016/j.ijpe.2015.05.021.
Pricing and replenishment policies in dual-channel supply chain under continuous unit cost decrease

S. Panda,⇑, N.M. Modak, S.S. Sana, M. Basu

Department of Mathematics, Bengal Institute of Technology, 1. no. Govt. Colony, Kolkata 700150, India
Department of Mathematics, University of Kalyani, Kalyani 741235, India
Department of Mathematics, Bhangar Mahavidyalaya, Bhangar 743502, India

Abstract

This paper explores pricing and replenishment policies for a high-tech product in a dual-channel supply chain that consists of a brick-and-mortar channel and an internet channel. The unit cost of the product decreases over its short life cycle. Assuming the manufacturer as the Stackelberg leader, the optimal pricing and replenishment policy is analysed mathematically. It is found that there is a severe price competition between the retail and online channel, and product compatibility has a significant impact on the pricing policy. In particular (i) customers’ higher retail channel preference above a threshold leads to non-coexistence of dual-channel, (ii) the dual-channel is non-profitable for product compatibility outside an interval and (iii) higher or lower retail price in comparison to online price is dependent on product compatibility. Also, the retailer’s higher setup cost may lead to non-existence of online channel. Finally, a profit sharing mechanism through wholesale price adjustment resolves channel conflict. A numerical example is illustrated to justify our proposed model.

1. Introduction

The rapid growth of internet based electronic commerce has attracted the manufacturers of several companies such as, IBM, HP, Sony, Kodak, Panasonic, Cisco, etc to introduce direct online channels to their existing brick-and-mortar retail networks. Reduced cost for searching, increasing contact with the customers and detail specification and information of the products through the internet enable the manufacturer to enhance it’s market coverage. The growth of US online marketing is forecasted at 8% in 2010 and is set to reach 14% by 2012. Two third (2/3) of the marketers believe that online business must be complemented by traditional marketing activities [1]. As a result, manufacturers redesign their traditional channel structures by engaging in direct sales to reach different customer’s segments that cannot be reached by the traditional retail channel. This channel structure births to the dual channel. In fact, manufacturers who sell only through retailers are now considering the option of selling directly to end customers. Since, in dual-channel of same/substitutable product is sold through retail store as well as online channel. Consequently, the customers have alternatives to choose the channel that is better suited for their needs [2].

Decreasing property of price component and diminishing of demand over time due to introduction of upgraded versions of components are now important characteristics of high-tech industrial market. In high-tech industries such as communication...
and computers, some component cost is decreasing around one percent per week [3]. Thus, production and sale in one week earlier or later leads to about 1% loss or gain. As a matter of fact, in decreasing unit cost environment, the decision maker always remains in searching the appropriate selling price because it has a considerable impact on demand as well as on optimal ordering policy. Thus, optimal pricing strategy [4–12] is a major issue to attract the customers in any business organization in a global economy.

Various aspects of dual-channel supply chain, such as advantages and disadvantages of online channel in addition to brick-and-mortar channel, when to open an online channel, pricing policies, replenishment policies, price competition, retail services, sales effort in retail channel, return policies, etc. have been explored extensively in supply chain literature. Interestingly, there is no research till date that has discussed pricing and replenishment policies for the hi-tech products, whose unit cost decreases continuously in their short life span. Hi-tech products have high online compatibility and tech savvy customers that generally considers the specifications of the products through online channels and compare the retail prices with the products in online manufacturers’ suggested retail prices. In such situation there is a need for the manufacturer to identify online price and replenishment/purchase policy of a product that reduces total channel cost effectively that increases channel profit.

In inventory literature, there are a few models concerning continuous cost changing. Both Buzacott [13] and Erel [14] have proposed two inventory models where the unit cost of the product increases under time. Erel [14] has developed the model under the assumption that the unit cost of the product increases in compound nature. Whereas, Buzacott [13] has assumed compound increments of both unit cost and setup cost. Goyal et al. [15] have developed inventory models under decreasing feature of unit cost. Khouja and Goyal [16] have suggested that the model of Buzacott [13] can be used for continuous unit cost change, if the rate of change of unit cost is same as the ordering cost. Erel’s [14] model is also applicable in this purpose. But, if the rate of inflation is less than 10%, then the models provide wrong approximation. Teng and Yang [17] and Teng et al. [18] have developed inventory models under partial backlogging where demand and cost fluctuate over time. In both the models, optimal replenishment policy and optimal purchasing policy have been determined to minimize system running cost. They have claimed that this policy fits for today’s high-tech market. Khouja and Goyal’s [16] model may be considered as a special case of Teng and Yang [17] and Teng et al. [18] with constant demand and unit cost dependent holding cost. In unit cost decrement inventory literature, interested readers may consult the paper of Khouja and Park [19] which provides interesting review of the literature associating it with the existing industrial scenario. Khouja and Park [19] have developed an inventory model to determine optimal operating policy in which the unit cost of the product decreases continuously by a constant percentage. Under the restriction of equal cycle lengths for finite time horizon, they have derived an approximate close form value for the optimal cycle length to minimize system operating cost. Panda [20] has determined the optimal pricing and replenishment policy in a decreasing demand with time and price sensitive market where the unit cost of the product decreases linearly with time. Cárdenas-Barrón et al. [21] have suggested a heuristic algorithm to solve the vendor management inventory system with multi-product and multi-constraint based $EOQ$ model with backorders, considering two classical backorders costs: linear and fixed. Sarkar and Majumder [22] have investigated an integrated vendor-buyer supply chain model to reduce total cost of the channel by considering the setup cost reduction of the vendor.

As indicated above, in addition to traditional brick-and-mortar channel, a new channel provided to the customers directly through internet is prevailing in practice because of its intuitive advantages. As a result, dual-channel supply chain has got enormous attention and become in main stream. Extensive researches have been done addressing variety of problems in dual-channel supply chain. For example, Levary and Mathieu [23] have examined the profits of retail store, online store and dual-channel, and have concluded that the dual-channel provides maximum profit. Ahn et al. [24] have discussed about the pricing decisions of a dual-channel supply chain, where the retail channel and the online channel operate in spatially separated markets. Huang et al. [25] have determined the optimal pricing strategies in a retail-e-tail supply chain by considering price dependent demand, a degree of substitution across the channel and the overall market potential. Yan [26] has developed a dual-channel supply chain incorporating differentiated branding strategy. He has concluded that it does not resolve full channel conflict although differentiated branding strategy alleviates channel competition and conflict. Dan et al. [27] have determined optimal retail service and prices in centralized and de-centralized dual-channel supply chain. Chen et al. [28] have proposed the manufacturer’s pricing strategies in a dual-channel supply chain. They have also showed that the channel conflict can be resolved by applying two-part tariff or a price sharing agreement. In this direction, the works of Saha [29], Hua et al. [29], Qi et al. [30], Xing et al. [31], Ma et al. [32] are worth mentioning.

Coordination among the channel members has potentiality to realize the benefits of the members of the chain. To coordinate the members of a supply chain, contracts are designed effectively among the decentralized decision makers such that the difference between outcome of a centralized decision and decentralized decisions can be neutralized. The basic objective behind designing a coordination contract is to incentivize decentralized channel members to act coherently with one another. Variety of side-payment contracts like as quantity discount [30,31], quantity flexibility [31,34], two-part tariff [35], revenue sharing [36,37], sales rebate [38], buy back [39], credit option [40], mail-in-rebate [41], disposal cost sharing [42,43] etc., have been used in supply chains as the ways of cutting out of channel conflict. These contracts differ by contractual clauses among channel members and are primarily concerned with quantity, time, quality and price. For detailed discussion on channel coordination, the survey papers of Cachon [44] and Sarmah et al. [45] are referred to the readers.

Although supply chain literature has rich content on two-echelon supply chain coordination, there are few papers which are dealt with resolving channel conflict in a dual-channel supply chain. Cai [46] has showed that hybrid revenue
sharing and linear price relationship between the retail channel price and direct channel price coordinate a dual channel
supply chain. Boyaci [47] has proposed that revenue sharing, wholesale price, buy back contracts can’t resolve channel
contlict though a penalty contract coordinates the channel. Cai et al. [46] have proposed that it is possible to achieve
win–win profits in a dual channel supply chain applying price discounting, though they have not discussed channel
coordination issues. Yao and Liu [48] have compared the proﬁt gains under Bertrand and Stackelberg equilibrium pricing
strategies but they have not discussed about double marginalization of the channel. In a manufacturer-Stackelberg
dual-channel supply chain, Chen et al. [27] have investigated that a contract with a wholesale price and a direct channel
price offered by the manufacturer can resolve the channel conﬂict. They have also suggested that complementary agree-
ments such as two-part-tariff, negotiated proﬁt sharing in some specific ranges coordinate the channel and the channel
members’ proﬁts are win–win.

The purpose of the paper is to determine optimal pricing and replenishment policy in a supply chain while the
manufacturer operates an online channel besides retail channel. The manufacturer sales the product through retail and
online channels simultaneously. As the product has a short life time and it’s cost decreases continuously with respect to time,
demand of the products in both the channels is sensitive with price that results in different selling prices per unit. In
conventional inventory models with pricing strategy, the number of price changes is pre-determined, i.e., the times of price
changes are known earlier over a ﬁnite time horizon. Relaxing this assumption, in this paper, we assume that several
replenishments of equal cycle lengths may be done over a ﬁnite time horizon and the decision maker has the opportunity
to adjust the unit selling prices in each of the replenishment cycle to maximize the proﬁt. Generally, in decentralized
decision making, the retailer decides the number of replenishment cycle that maximizes it’s proﬁt, whereas, in centralized
channel, the manufacturer proposes the replenishment number. Moreover, we apply negotiation of proﬁt sharing
mechanism to determine the optimal wholesale price of the manufacturer that resolves channel conﬂict. The research
reported in this paper differs from the prior works is the following aspects. First, we consider a product whose unit cost
decreases continuously with time and becomes obsolete after it’s ﬁnite life time. These features have not considered in
earlier studies. Second, we consider the system over the ﬁnite lifetime of the product instead of a single replenishment/production
cycle. As a result, in addition to selling prices in the channels, the optimal number of replenishment cycles in decentralized
and centralized channel are different. We use proﬁt sharing mechanism to align the replenishment cycles of both the channel
structure. Third, the paper addresses the effect of product compatibility on the optimal selling prices and optimal order
quantity of the channel.

2. Notation

Following notations are used to develop the proposed model.

\( L \) the time horizon under consideration

\( T \) the cycle time during the planning horizon

\( n \) the total number of replenishments over \([0, L]\) (a decision variable)

\( D_i^t \) demand rate of the product in retail channel of ith replenishment

\( h_i \) inventory holding cost per unit per unit time of the retailer

\( h_m \) inventory holding cost per unit per unit time of the manufacturer

\( s_r \) the ordering or/and set-up cost of the retailer

\( s_m \) the ordering or/and set-up cost of the manufacturer

For ith replenishment (\( i = 1, 2, \ldots, n \)):

\( c(i) \) unit production cost of the manufacturer

\( w \) wholesale price of the manufacturer to the retailer

\( p_r \) unit selling price of the retailer

\( p_i \) unit selling price of the manufacturer in direct channel.

3. Model formulation

Assume that a manufacturer sales a hi-tech product through an online channel in parallel with the retail channel. The
channel is operated over the ﬁnite time horizon \( L \) in which the retailer replenishes \( n \) times after every time interval \( T \) such
that \( nT = L \). As in Khouja [16], Khouja and Goyal [13], Panda [17], the unit cost of the product decreases continuously with
respect to time at a rate \( c(t) = z - blt \), \( t \in (0, \infty) \). The \( z \) is the introductory unit cost of the product and \( b \) is the unit cost’s
time sensitive parameter. It is quite reasonable to assume that \( z/b > L \), i.e., the unit cost of the product is positive over the
planning horizon \( L \). In the ith (\( i = 1, 2, \ldots, n \)) replenishment cycle the demands in the retail and direct channels are linear in
unit selling prices (Van [26], Yue and Liu [49]) and are of the forms

\( D_i^t = (c_0 - bt_i) + r_i(p_r - p_i), \quad i = 1, 2, \ldots, n \)
and
\[ D_i' = (1 - \theta) \alpha - b_i p_i^r + r_i (p_i^o - p_i^e), \quad i = 1, 2, \ldots, n, \]
where, \( \alpha > 0 \) is the market potential. The parameter \( \theta \), \((0 < \theta < 1)\) is the compatibility of the product with the retail channel. Product compatibility is how the consumer perceives the product into the customer’s lifestyle choices. When the product closely matches the individual’s needs, wants, beliefs, values, and consumptions patterns of the customers, it can be considered highly compatible with the consumers’ choice. The percentage of the primary demand \( \alpha \) that goes to the retail channel is \( \theta \) and when the value of \( \theta \) is greater, the product’s compatibility with the retail channel is larger and more consumers would purchase the product from the retail channel. Computer-related products, books, information, magazines, and digital products have more compatibility with the direct channel than the products like water, rice, gasoline, and milk. Here, \( b_i > 0 \) and \( b_r > 0 \) are the price sensitivity factors in retail channel and online channel respectively. The \( r_i > 0 \) and \( r_r > 0 \) are the cross-price effects which reflect the degree of price competition between the channels. According to Yan [26], we assume that the price sensitivity factors and cross-price effect in the retail channel and in the online channel are equal, i.e., \( b_i = b_r = b \) and \( r_i = r_r = r \). It is quite reasonable that \( b > r \), i.e., the effect of price sensitivity of a channel is greater than cross-price effect. Thus, in the ith \((i = 1, 2, \ldots, n)\) replenishment cycle, the demands of the product in the retail channel and in the online channel are respectively as follows.
\[ D_i^r = \alpha - b_i p_i^r + r_i p_i^e, \quad i = 1, 2, \ldots, n \]
and
\[ D_i^o = (1 - \theta) \alpha - b_i p_i^r + r_i (p_i^o - p_i^e), \quad i = 1, 2, \ldots, n \]
Quite often in the ith \((i = 1, 2, \ldots, n)\) replenishment cycle, the selling price in the online channel is higher than the manufacturer’s wholesale price, i.e., \( p_i^o > w_i \), \( i = 1, 2, \ldots, n \). Otherwise, the retailer purchases the product through the online channel rather than from the manufacturer. Also, for profitability of the retailer, in the ith \((i = 1, 2, \ldots, n)\) replenishment cycle, the selling price of the retailer is higher than the manufacturer’s wholesale price, i.e., \( p_i^o > w_i \), \( i = 1, 2, \ldots, n \). Under this model setting, our objective is to find out optimal decisions in decentralized as well as centralized systems.

3.1. Decentralized dual-channel-supply chain

In decentralized decision making, the manufacturer and the retailer are interested to achieve maximum individual profit. Interactions between the manufacturer and the retailer are considered as a Stackelberg game. The manufacturer acts as the Stackelberg leader of the channel and the retailer is its follower. In Stackelberg game, leader makes first move and follower then reacts by consistent playing the best move with available information. The objective of the leader is to design own move in such a way that his own profit is maximum, after considering all rational moves follower can devise [50]. In this way, the manufacturer first announces the wholesale price and selling price of the product in the online channel. Based on the manufacturer’s decision, the retailer determines the retail price and orders ‘n’ replenishments to the manufacturer each of length \( T \) over \( L \) so that \( n T^* = L \). In the ith \((i = 1, 2, \ldots, n)\) replenishment cycle, profit function of the retailer is
\[ \pi_i^o(p_i^o) = D_i^o p_i^o - D_i^o w_i - s_i - D_i^o h_i^o T, \quad (i = 1, 2, \ldots, n) \]
In Eq. (3), 1st, 2nd, 3rd and 4th terms represent sales revenue, purchase cost, set up cost and holding cost of the product respectively. The profit function of the retailer over the planning horizon is
\[ \pi_o(n^r p_i^o) = \sum_{i=1}^{n} \sum_{r=1}^{T_i} \pi_i^o(p_i^o) = \sum_{i=1}^{n} \sum_{r=1}^{T_i} [D_i^o (p_i^o - w_i) - h_i^o T] = n^r s_o. \]
In the decentralized decision making, the profit function of the manufacturer is the sum of two profit functions. One is for the quantities which is sold through the retail channel and the other is for the online channel. The profit function of the manufacturer, in the ith replenishment cycle, is
\[ \pi_i^{m}(w_i, p_i^{o,h}) = D_i^o w_i - D_i^o c (i-1) T_r + D_i^o (p_i^{o,h} - c) (i-1) \frac{T_i^*}{T} - D_i^o s_i^r - D_i^o h_i^o T (i=1, 2, \ldots, n) \]
In Eq. (5), first and second terms represent the manufacturer’s net profit from the retail channel and the remaining terms represent the manufacturer’s profit from the online channel. Total profit of the manufacturer over the planning horizon is
\[ \pi^{m}(w_i, p_i^{o,h}) = \sum_{i=1}^{n} \sum_{r=1}^{T_i} \pi_i^{m}(w_i, p_i^{o,h}) = \sum_{i=1}^{n} \sum_{r=1}^{T_i} [D_i^o (w_i - c (i-1) \frac{T_i^*}{T}) + D_i^o (p_i^{o,h} - c) (i-1) \frac{T_i^*}{T} h_i^o T] = n^r s_o. \]
Total profit of the retailer in the planning horizon is a function of \( p_i^{o,h} \), \((i = 1, 2, \ldots, n)\) and \( n^r \), where \( n^r \) is a discrete variable. On the other hand, the profit function of the manufacturer is a function of \( w_i \) and \( p_i^{o,h} \), \((i = 1, 2, \ldots, n)\). Since the manufacturer is the Stackelberg leader of the channel, it first determines the wholesale price and selling price of the product.
through the online channel where it assumes that the retailer will consider multiple replenishment cycles. As a follower, the retailer then sets the retail price and number of replenishment cycles in the time horizon. Now, for the concavities of the profit functions we have the following proposition.

Proposition 1.

(i) For given \( n' \), the manufacturer’s profit function over \( L \) is a concave function of \((w_i, p_i^{(a)})\), \(i = 1, 2, \ldots, n'\) and optimal wholesale price and retail price in the online channel are

\[
w_i = \frac{a[r + (b)]}{2[|b + r| - r^2]} \left\{ \frac{bh_i}{4r^2} \right\} \frac{1}{2} \left\{ (i - 1) \frac{L}{n'} \right\} (i = 1, 2, \ldots, n').
\]

(ii) For given \( n' \) and the manufacturer’s optimal \((w, p^{(a)})\), \(i = 1, 2, \ldots, n'\) pair, the retailer’s profit function is concave over \( L \) and it’s optimal retail price is

\[
p^{(a)} = \frac{a[r + (1 - b)]}{2[|b + r| - r^2]} \left\{ \frac{bh_i}{4r^2} \right\} \frac{1}{2} \left\{ (i - 1) \frac{L}{n'} \right\} (i = 1, 2, \ldots, n').
\]

Proof. See Appendix A.

Using the optimal selling prices of the retail channel and online channel, optimal order quantities, optimal profits in the ith replenishment cycle over the entire time horizon can be found which are presented in Table 1.

From (9), we have \( \frac{dp^{(a)}(w, p^{(a)})}{dw} = \frac{r}{2[|b + r| - r^2]} > 0 \), i.e., in the ith replenishment cycle, the optimal selling price of the retailer decreases when the selling price of the online channel decreases. Further, \( \frac{dp^{(a)}(w, p^{(a)})}{dw} - \frac{1}{2} > 0 \), i.e., the optimal retail price is also decreasing with decreasing optimal wholesale optimal of the manufacturer. As the selling price of the retailer decreases with decreasing values of \( p^{(a)} \) and \( w \) in the ith replenishment cycle, the manufacturer has the option to control the retailer’s selling price by introducing an online channel, where he/she sets the wholesale price \( w \) and unit selling price \( p^{(a)} \). This result is quite similar to Chiang et al. [51]. It is also observed that

\[
\frac{\partial p^{(a)}}{\partial p^{(a)}} = \frac{L}{n'} \sum_{i=1}^{n'} \frac{D_i^{(a)}}{|b + r|} + \left( p^{(a)} - w \right) \frac{bh_i}{2r^2} > 0.
\]

That means, the optimal total profit of the retailer over \( L \) increases with increasing optimal online selling price. The intuitive reason is straightforward. When the online selling price increases, it forces some demands of the online channel to switch to the traditional retail channel. As a result, the retailer’s profit increases. On the other hand,

\[
\frac{\partial p^{(a)}}{\partial w} = \frac{L}{n'} \sum_{i=1}^{n'} \frac{D_i^{(a)}}{|b + r|} + \left( p^{(a)} - w \right) \frac{bh_i}{2r^2} < 0.
\]

Here, the optimal total profit of the retailer decreases with increasing wholesale price of the manufacturer which is quite obvious. However, the optimal selling prices and wholesale prices satisfy the following proposition. \( \square \)

Proposition 2. In the planning horizon \( L \), for given \( n' \), (i) \( p_1^{(a)} > p_2^{(a)} > \ldots > p_{n'}^{(a)} \), (ii) \( p_1^{(a)} > p_2^{(a)} > \ldots > p_{n'}^{(a)} \) and (iii) \( w_1 > w_2 > \ldots > w_{n'} \) hold.

Proof. See Appendix A.

Proposition 2 indicates that the optimal selling prices in the channels and optimal wholesale price of the manufacturer decrease with the increasing replenishment number. That means, the optimal prices in the ith cycle are higher than those of \((i + 1)\)th, \(i = 1, 2, \ldots, n' - 1\) cycle. The reasonable explanation is as follow. As the product has limited lifetime, it’s unit cost
Table 1
Optimal solutions in decentralized and centralized decision making (where $A = \{r_h = \beta \in C \}$ and $B = \{2 \alpha r_h \in C \}$).

<table>
<thead>
<tr>
<th>Decentralized scenario</th>
<th>Manufacturer</th>
<th>Retailer</th>
</tr>
</thead>
<tbody>
<tr>
<td>4th Replenishment $i = 1, 2, \ldots, n'$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wholesale price</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$p$</td>
<td>$\frac{1}{p}(c - 1) \frac{1}{\bar{p}}$</td>
<td>$\frac{1}{p}(c - 1) \frac{1}{\bar{p}}$</td>
</tr>
<tr>
<td>Selling price</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\tilde{p}(c - 1) \frac{1}{\bar{p}}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Quantity sold</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\tilde{d} \left[ 2 \tilde{b} (c - 1) \frac{1}{\bar{p}} \right] + \tilde{d} \left[ 2 \tilde{b} (c - 1) \frac{1}{\bar{p}} \right]$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Manufacturing quantity</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\tilde{b} \left[ 2 \tilde{b} (c - 1) \frac{1}{\bar{p}} \right] + \tilde{b} \left[ 2 \tilde{b} (c - 1) \frac{1}{\bar{p}} \right]$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Profit</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\tilde{d} \left[ 2 \tilde{b} (c - 1) \frac{1}{\bar{p}} \right] + \tilde{d} \left[ 2 \tilde{b} (c - 1) \frac{1}{\bar{p}} \right]$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Over the sales season L</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Order quantity</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\tilde{d} \left[ 2 \tilde{b} (c - 1) \frac{1}{\bar{p}} \right] + \tilde{d} \left[ 2 \tilde{b} (c - 1) \frac{1}{\bar{p}} \right]$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Generalized scenario</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4th Replenishment $i = 1, 2, \ldots, n'$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Selling price</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\tilde{x} \left[ 2 \tilde{b} (c - 1) \frac{1}{\bar{p}} \right] + \tilde{x} \left[ 2 \tilde{b} (c - 1) \frac{1}{\bar{p}} \right]$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Demand</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\tilde{x} \left[ 2 \tilde{b} (c - 1) \frac{1}{\bar{p}} \right] + \tilde{x} \left[ 2 \tilde{b} (c - 1) \frac{1}{\bar{p}} \right]$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Channel profit</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\tilde{d} \left[ 2 \tilde{b} (c - 1) \frac{1}{\bar{p}} \right] + \tilde{d} \left[ 2 \tilde{b} (c - 1) \frac{1}{\bar{p}} \right]$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Over the sales season L</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Order quantity</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\tilde{d} \left[ 2 \tilde{b} (c - 1) \frac{1}{\bar{p}} \right] + \tilde{d} \left[ 2 \tilde{b} (c - 1) \frac{1}{\bar{p}} \right]$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Channel profit</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\tilde{d} \left[ 2 \tilde{b} (c - 1) \frac{1}{\bar{p}} \right] + \tilde{d} \left[ 2 \tilde{b} (c - 1) \frac{1}{\bar{p}} \right]$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

S. Peels et al. (2005) Mathematical and Computer Modelling Volume 42 Pages 913-929
decreases continuously with time. Now, the wholesale price of the product decreases because it depends on the unit cost of the product. As a result the selling prices in the channels also decrease because the channel members are willing to sell more product by lowering the selling prices before the products obsolesce.

Now, the optimal pricing strategy provided in Table 1 is acceptable to the channel members only when \( p_i^{\text{wh}} > w_i \) and \( p_i^{\text{wh}} > w_i \) for \( i = 1, 2, \ldots, n' \). Therefore, \( p_i^{\text{wh}} > w_i \) holds if

\[
\theta > \frac{b}{d} \left( \left( \frac{1}{2} \right)^{i-1} \frac{L}{n} \right) \frac{L(r_{\text{wh}} + 3(b + r)h_i)}{2d'n} 
\]

(11)

Also, \( p_i^{\text{wh}} > w_i \) holds if

\[
\theta < 0.5 \left( \frac{L(b + 2r)(h_i + h_m)}{4d'n} \right) + \varphi_{\text{wh}}, \quad \text{(say)}
\]

(12)

The right hand side of (11) depends on \( n' \) and \( i \), whereas right hand side of (12) is dependent on \( n' \) but independent of \( i \). Since, the system operates over the planning horizon \( L \), where \( n' \) replenishment cycles are made, the maximum value of the right hand side of (11) is at \( i = 1 \) is

\[
\frac{b}{d} \left( \frac{L(r_{\text{wh}} + 3(b + r)h_i)}{2d'n} \right) + \varphi_{\text{wh}}, \quad \text{(say)}
\]

and we have the following Lemma.

**Lemma 1.** For given \( n' \), the manufacturer will participate in the dual-channel for a product that’s unit cost decreases continuously over \( L \) if \( \theta < \theta^{\text{wh}} \).

**Lemma 1** indicates that the customer’s channel preference is one of the determining factors for operating an online channel besides the traditional retail channel. When \( \theta < \theta^{\text{wh}} \), the retailer can’t do business because its selling price is less than the manufacturer’s wholesale price. On the other hand, for \( \theta > \theta^{\text{wh}} \), the manufacturer can’t set the optimal selling price as the online price is there. Note that, we consider the maximum of lower threshold of product’s compatibility with the retail channel. As there are multiple replenishment cycles over \( L \), the retailer’s optimal selling prices are profitable in these cycles only when the lower limit of \( \theta \) is maximum among all of these values. As online selling price is higher than the wholesale price, it does not ensure that the retailer will participate in the profit making retail-e-tail channel for a product. This experiences continuous unit cost decrease over \( L \) because of its setup cost. In the \( i \)th \((i = 1, 2, \ldots, n') \) replenishment cycle, the retailer will participate in the dual-channel only when it’s profit is positive, i.e., \( \theta^{\text{wh}i} > 0 \), i.e., if

\[
\theta > \frac{b}{d} \left( \left( \frac{1}{2} \right)^{i-1} \frac{L}{n} \right) \frac{L(r_{\text{wh}} + 3(b + r)h_i)}{2d'n} 
\]

(13)

On the other hand, the manufacturer will operate the online channel until the demand in the online channel is positive, i.e., \( D^{\text{on}} > 0 \), i.e., if,

\[
\theta < \frac{2(b + r)}{2d' + r} \left( \frac{2(b + r)}{2d' + r} \right)^2 \frac{L(r_{\text{wh}} + 3(b + r)h_i/L)}{2d'/(2d' + r)} \frac{b(2b + 3r)}{(2b + r)} \left( \frac{1}{2} \right)^{i-1} \frac{L}{n} \right) = \varphi_{\text{on}{i}}, \quad (i = 1, 2, \ldots, n'),
\]

(14)

where \( \varphi_{\text{on}{i}} \) also depends on \( n' \) and \( i \) and attains its minimum value when \( i = 1 \), i.e.,

\[
\theta < \frac{2(b + r)}{2d' + r} \left( \frac{2(b + r)}{2d' + r} \right)^2 \frac{L(r_{\text{wh}} + 3(b + r)h_i/L)}{2d'/(2d' + r)} \frac{b(2b + 3r)}{(2b + r)} \left( \frac{1}{2} \right)^{i-1} \frac{L}{n} \right) = \varphi_{\text{on}{i}}, \quad (i = 1, 2, \ldots, n'),
\]

Unlike the retailer’s case, for positive profit of the manufacturer we consider here the positive online demand because the setup cost for operating the online channel is included in the system from where the manufacturer supplies the product. It is assumed that the setup cost of the manufacturer consists of setup cost for manufacturing the product and setup cost for operating the online channel because we concentrate only on overall profitability (profit from the retail channel and profit from the online channel) of the manufacturer.

When the customers’ retail channel preference lies in between \( \theta \) and \( \varphi \), Eqs. (13) and (14) suggest that the manufacturer can successfully operate a profitable dual-channel. Now,

\[
\varphi - \varphi^{\text{on}} = \frac{b}{d} \left( \left( \frac{1}{2} \right)^{i-1} \frac{L}{n} \right) \frac{L(r_{\text{wh}} + 3(b + r)h_i/L)}{2d'n} > 0
\]
and
\[ \hat{\beta} = \beta^{\text{opt}} = \frac{(2b + 3\beta) - 2bx^2}{2(2b + r)} + \frac{(2b + 3\beta + 4\beta^2) - (2b + 3\beta + r)\beta}{4\alpha(2b + r)} > 0 \]

hold. This implies that \( \max(\hat{\beta}, \beta^{\text{opt}}) = \hat{\beta} \) and \( \min(\hat{\beta}, \beta^{\text{opt}}) = \beta^{\text{opt}} \), i.e., \( \hat{\beta}, \beta^{\text{opt}} \) are nested in \( (\beta^{\text{opt}}, \hat{\beta}) \). This result is quite obvious because the manufacturer will operate the online channel only when both the channels are profitable. The retail channel of the manufacturer will be profitable not only for the retail price is equitable to the wholesale price but also reasonably higher because it must over compensate the costs related entire retail channel running cost. It is possible only when the customers’ retail channel preference is higher than \( \hat{\beta} \) but lower than \( \beta^{\text{opt}} \). Similarly, the retailer will participate in the dual-channel and will make profit if \( \theta > \hat{\beta} \) holds. Thus, the proposition is as follows:

**Proposition 3.** Over the planning horizon \( L \), for given \( n' \), the manufacturer can operate a profitable retail-online channel when the customers’ retail channel preferences \( \in (\beta^{\text{opt}}, \hat{\beta}) \).

However, as far as the competition between the retail channel and online channel is concerned, we have the following proposition.

**Proposition 4.** For given \( n' \), in the \( i \)-th replenishment cycle the optimal retail price is higher than online selling price if \( \theta \in (\theta^i, \theta^{\text{opt}}) \), while reverse may be noted for \( \theta \in (\hat{\theta}, \theta^i) \).

**Proof.** See Appendix A.

**Proposition 4** suggests that the customer’s channel preference intensifies the channel competition. To attract more customers, the retailer sets the retail price less than online selling price when the customers’ preference for the retail channel is below a threshold of the products channel compatibility. On the other hand, the retailer sets the retail price greater than the online selling price to earn higher profit margin when \( \theta \) is above the threshold \( \theta^i \). Thus, before making any pricing decision, the channel members must consider the customers’ channel preference as a decision factor. This result can be further justified as follows.

\[ \frac{d\theta^i}{d\theta} = \frac{ab}{2(b + r)^2 - r^2} < 0 \]

\[ \frac{d\theta^{\text{opt}}}{d\theta} = \frac{a(3b + 4\beta^2)}{4(b + r)(b + 2\beta^2)} > 0, \]

i.e., selling price in the retail channel increases and online selling price decreases when the customers’ preference for the retail channel increases. As more profit gains are the objectives of the channel members, the channel members decide about the selling prices based on the product compatibility. Also, note that

\[ \frac{d\omega_i}{d\theta} = \frac{ab}{2(b + r)^2 - r^2} > 0, \]

i.e., the wholesale price of the manufacturer increases with the increment of the customers’ retail channel preference. This is quite reasonable. Since the retailer sets higher selling price because of customers’ higher preference for the retail channel, the manufacturer increases also it’s wholesale price in order to acquire some margins from the retailer’s profit. However, the channel members can apply such strategy and counter strategy until the product compatibility with the retail channel lies in \( (\theta^i, \theta^{\text{opt}}) \). It is very interesting to note that \( \frac{d\omega_i}{d\theta} = 2(\theta^i) \sqrt{(b + r)m_i}/\lambda_k > 0 \), i.e., the lower threshold of profitable product compatibility with the retail channel is directly proportional to the retailer’s set up cost.

As such in the \( i \)-th replenishment cycle, \( \theta^i < \theta \) holds if

\[ s_i > \frac{L}{n'(b + r)} \frac{\omega^i}{\frac{4}{b^2}} \left( \frac{\omega_i}{L(b + r)} - \frac{\omega^i}{6\beta^2} \right) \frac{b}{\frac{4}{4}} \]

(15)

In such case, \( \theta^* < \theta < \theta^{\text{opt}} \) holds, i.e., the lower limit of customers retail channel preference is higher than the upper limit of product compatibility in between which the manufacturer can set higher online price in comparison to the retail price. Obviously in this case the manufacturer can not run the online channel because it cannot set the profitable online selling price. Thus the manufacturer can operate the profitable online only when the retailer’s set up cost is reasonably low. Then, we have the following proposition.

**Proposition 5.** The manufacturer cannot operate the online channel if

\[ s_i > \frac{L}{n'(b + r)} \frac{\omega^i}{\frac{4}{b^2}} \left( \frac{\omega_i}{L(b + r)} - \frac{\omega^i}{6\beta^2} \right) \frac{b}{\frac{4}{4}} \]

holds.
Appendix A

Proposition 6 Over the selling season \( L \) the retailer's profit is maximum for \( n_0^* \) number of replenishments, where \( n_0^* \) is given by

\[
n_0^* = \begin{cases} 
|n_0| & \text{if } n_0^* > n_0 + 1 \\
|n_0| + 1 & \text{otherwise} 
\end{cases}
\]  

(16)

and

\[
n_0 = (-d + \sqrt{d^2 + b^2})^{1/4} + (-d - \sqrt{d^2 + b^2})^{1/4},
\]

where \( b = L^2(A - h|b|)2\omega - b|2|2\omega - \beta|5/96(b + r)s, \) and \( d = L^3[3A(A - 2b|b| + 2b^2|b|)/192(b + r)s]; \) \( |n_0| \) denotes largest integer not greater than \( n_0. \)

Proof. See Appendix A.

Proposition 6 suggests that, based on the manufacturer's wholesale price and online selling price, the retailer chooses \( n_0^* \) number of replenishment over \( L \) that maximizes its profit. As the manufacturer is dependent on the retail channel, he/she will follow the retailer's replenishment policy and, based on it, the manufacturer will apply the online pricing schedule. \( \square \)

3.2. Centralized policy

The traditional centralized policy views the system as single entity where there is one central planner who makes all decisions so as to maximize the profit of the whole chain. The centralized policy determines suitable selling prices of the product for both retail and direct channel as well as production cycle so as to maximize the total system profit. The relevant costs
considered for the retailer and the manufacturer in this policy are similar to those in the decentralized replenishment policy. 

Profit function of the integrated channel in the ith replenishment cycle is

\[ \pi_i = D_i^m \theta_i \rho_i - D_i^m \frac{h_i^m}{2} - D_i^c \psi_i [i-1] |T| + D_i^c \theta_i \rho_c^m - D_i^c \frac{h_c^m}{2} - D_i^c \psi_c [i-1] |T| - s_i - s_m. \]  \hspace{1cm} (17)

The optimal solution of (17) is presented in the following proposition.

**Proposition 7.** (i) For given \( n' \), the profit function of the integrated channel over \( L \) is a concave function of \((p_i^r, p_i^d)\), \( i = 1, 2, \ldots, n' \) and the optimal selling price of the product for retail and direct channel are respectively

\[ p_i^r = \frac{a(i+b)}{2a} - \frac{h_i^r}{4a^2} \left[ (i-1) - \frac{L}{n'} \right] \]  \hspace{1cm} (18)

and

\[ p_i^d = \frac{a(i+b)}{2a} - \frac{h_i^d}{4a^2} \left[ (i-1) - \frac{L}{n'} \right] \]  \hspace{1cm} (19)

**Proof.** See Appendix A.

Using the optimal values of \( p_i^r \) and \( p_i^d \), we obtain the demand of the product in retail and direct channel in the ith replenishment cycle and multiplying these with cycle length, we get the amount of quantity sold through retail and online channel respectively which are displayed in Table 1. Also, the order quantity which are produced by the manufacturer is equal to the product of total demand and cycle length, the profit of the integrated channel in ith replenishment cycle as well as over the planning horizon are displayed in Table 1. Now, the optimal values of the centralized decision satisfy the following properties. \( \square \)

**Proposition 8.** In the centralized channel over the planning horizon \( L \), for given \( n' \), (i) \( p_i^r > p_{i+1}^r > \ldots > p_{n'}^r \), (ii) \( p_i^d > p_{i+1}^d > \ldots > p_{n'}^d \).

**Proof.** See Appendix A.

This result is quite similar to the decentralized decision making process. As the unit cost of the product decreases continuously, higher number of replenishment always leads to decrement of selling price in both the channels. Now, the manufacturer would be interested opening the online channel only when online channel demand in the ith replenishment cycle is positive, i.e., \( D_i^o > 0 \), which, after simplification, yields

\[ \theta < 1 - \frac{|b + i| h_i^o - h_o^i |T|}{2an'} \]  \hspace{1cm} (20)

Here, \( \varphi_{ios} \) depends on \( n' \) and i and it attends minimum value for \( i = 1 \), i.e.,

\[ \theta < 1 - \frac{|b + i| h_i^o - h_o^i |T|}{2an'} - \frac{bx}{a} - \varphi \]  \hspace{1cm} (21)

Although, in centralized channel, the manufacturer and the retailer co-operate and take decision jointly, the product compatibility has an impact on the manufacturer’s decision for opening the online channel. In the ith replenishment cycle, if the customers’ retail channel preference is higher than threshold \( \varphi \), then the manufacturer’s decision for opening an online channel is not profitable because its online demand is negative in such cases. On the other hand, there must be a competition between the retail channel and the online channel in the centralized process though the channel members co-operate. The channel members take decision jointly but the market potential remains same. When the manufacturer operates an online channel, some customers switches to the online channel. As a result the retailer’s demand decreases and it earns less profit. Besides the selling prices of the retail and online channel, the customers’ channel preference determines the divisions of potential market demand. So, like the decentralized decision making process, here also the selling prices of the retail channel may be higher than the online channel, i.e., \( p_i^r > p_i^d \), \( i = 1, 2, \ldots, n' \), i.e., if

\[ \theta > 0.5 - \frac{(b + 2i) h_i^r - h_o^i |T|}{4an'} \]  \hspace{1cm} (22)

Also, note that for any \( ii(i = 1, 2, \ldots, n') \),

\[ \varphi - \varphi = \frac{bx}{a} - \frac{(b + 4i) h_i^r - (3b + 4i) h_o^i}{4an'} > 0 \]  \hspace{1cm} (23)

Thus from (25), (26), (27), we have the following proposition. \( \square \)
Proposition 9. For given \( \theta \), the manufacturer will operate an online channel if \( \theta \in (0, \bar{\theta}) \). The online selling price is higher than retail price for any \( \theta \in (0, \bar{\theta}) \) and the retail price is higher than the online price for any \( \theta \in (\bar{\theta}, 1) \).

Proposition 9 demonstrates that, when the channel members cooperate and take decision jointly, the manufacturer’s decision is to open an online channel that is profitable only when the customers’ retail channel preference is below the threshold number \( \theta \). Interestingly below this threshold number of the product compatibility, there exists a price competition between the retail channel and the online channel. If the customers’ retail channel preference is within \( (0, \bar{\theta}) \), then the online price will be higher than the retail price and the reverse is set for \( \theta \in (\bar{\theta}, 1) \). Thus, for a profitable centralized retail-online channel, the channel will set the selling prices according to the customers’ channel preference.

Proposition 10. Over the selling season \( L \), the number of replenishments for which system’s profit is maximum is given by

\[
N^*_0 = \begin{cases} 
\lceil \sqrt{n_0} \rceil, & \text{if } p^{\alpha \beta}(n_0) > p^{\alpha \beta}(\lceil n_0 \rceil + 1) \\
\lceil n_0 \rceil + 1, & \text{otherwise}
\end{cases}
\]

and

\[
n^*_0 = \left( d_s - bL + b^2 + bL \right) ^{+} + \left( d_s - bL + b^2 \right) ^{+} \tag{23}
\]

where

\[
b_L = - \left[ 2aL/(b + \theta h_s + (1 - \theta)h_n) - bL/(2x - bL/2(b + h_s + h_n)) \right]/24(h_s + h_n),
\]

\[
d_L = L/3(b + r)(h_s + h_n) - 2rth(h_s + h_n) + 4bh^2 + 3bh(h_s + h_n)/48(h_s + h_n) \text{ and } \lceil n_0 \rceil \text{ denotes the largest integer which is not greater than } n_0.
\]

Proof. Same as Proposition 6.

Now, \( p^{\alpha \beta} > p^{\alpha \beta} + \pi^{\alpha \beta} \) holds, i.e., the channel is not coordinated. This is quite obvious as indicated in supply chain literature that cooperative integrated decision is always more profitable than decentralized system. In the next section, we demonstrate a profit sharing mechanism assuming that the manufacturer and the retailer jointly take the centralized decision, which is the channel best decision and share the total channel profit in a portion that ensures win-win profit. \( \square \)

3.3. Profit sharing for channel coordination

As the manufacturer and the retailer are separate and independent economic entities, a key issue is to develop mechanisms that can align their objectives and coordinate their activities so as to optimize system performance. To obtain centralized channel profit, there is a need to devise coordination mechanisms that are not only able to coordinate the activities but also able to align the objectives of independent supply chain members. The difficulties for coordination in this model are due to different optimal cycle length and hence different pricing policy for centralized and decentralized scenarios. For accepting the centralized cycle length, which is less than the decentralized cycle length, the retailer’s cost will increase and there is no reason that the retailer will adopt centralized policy unless proper incentive. As an incentive, manufacturer can offer the retailer to share the surplus profit if the retailer adopt centralized decisions. Under profit sharing mechanisms, the system performance is first optimized and the resultant benefit is then shared between the manufacturer and the retailer. This solution can be considered as a cooperative solution. Its implementation, however, depends on the development of a profit sharing scheme that is acceptable to both parties.

The manufacturer provides incentive to the retailer for accepting centralized selling price, \( p^c \) and cycle length, \( L/n_0 \) by offering him/her to the surplus profit proportionally according to their decentralized profit. To obtain centralized channel profit manufacturer has also sell the product at a price \( p^c \) through the online channel. Surplus profit for accepting centralized policy over the planning horizon \( L \) is \( \pi_{\alpha \beta} = \pi^{\alpha \beta} - (\pi^{\alpha \beta} + \pi^{\alpha \beta}) \). The manufacturer and the retailer get additional profits \( \pi^{\alpha \beta} (\pi^{\alpha \beta} + \pi^{\alpha \beta} \pi_{\alpha \beta}) \pi_{\alpha \beta} \) and \( \pi^{\alpha \beta} (\pi^{\alpha \beta} + \pi^{\alpha \beta} \pi_{\alpha \beta}) \pi_{\alpha \beta} \) respectively over the planning horizon \( L \). Now, the question is how they implement the profit sharing policy in different cycle. For this purpose, we propose that the profit surplus can be shared between them by just adjusting wholesale price properly. Thus, for a particular cycle, the manufacturer and the retailer get additional profit \( \pi^{\alpha \beta} (\pi^{\alpha \beta} + \pi^{\alpha \beta} \pi_{\alpha \beta}) \pi_{\alpha \beta} \) and \( \pi^{\alpha \beta} (\pi^{\alpha \beta} + \pi^{\alpha \beta} \pi_{\alpha \beta}) \pi_{\alpha \beta} \) respectively.

In the \( i \)th \( i = 1, 2, \ldots, n_0 \) replenishment cycle, profit of the retailer is given by

\[ \frac{dW_s}{d\theta} = -\left( \frac{p_\theta - p_0}{\beta} - \frac{\beta}{\alpha} \right) + \left( \frac{\beta^2}{\alpha} \right) \frac{p_\theta}{s} \]

After simplification, the wholesale price of the product in the ith \((i = 1, 2, \ldots, n_c)\) replenishment cycle can be found as

\[ w_i = \frac{p_i}{\alpha} \left( \frac{\beta^2}{\alpha} \right) \frac{p_i}{s} + s_i \]

Thus, through proper choice of wholesale price, profit sharing mechanism can be implemented and the decentralized channel can achieve profit equal to centralized profit which also assure win-win outcomes for all the channel members.

4. Numerical illustration

Assume that a manufacturer sells a hi-tech product and decides the sales season \(L\) as 120 days. At the beginning of the sales season the unit cost of the product is \(c = 500\) and the cost decreases at a rate \(\beta = 0.25\) per day. Other parameter values are \(a = 100, b = 0.4, r = 0.1, h = 0.15, h_0 = 0.12\) per unit per day, \(s_0 = 5000, s_1 = 10000\) per replenishment cycle. Also, assume that the customer’s retail channel preference is \(\alpha = 0.5\). The optimal values are presented in Table 2.

In decentralized setting the retailer replenishes thrice over the planning horizon whereas in centralized channel total number of replenishment is 4. As indicated, the wholesale price and the selling prices in retail and online channel decrease in every next replenishment cycle. From Table 2, one can easily observe that the manufacturer’s and the retailer’s profit increase 12.8% and 15.4% respectively from the decentralized channel when they adopt centralized policy through profit sharing mechanism. Both in centralized and decentralized channel, optimal order quantities over the planning horizon increase in each and every next replenishment. Optimal online channel price and retail price are higher than the manufacturer’s wholesale price for \(\theta \in (0.1526, 0.5162)\). Also, \(\theta = 0.8564\) and \(\theta = 0.354\). Thus for profitable retail-online channel customers’ retail channel preference must be \((0.354, 0.5162)\). Interestingly as indicated, customers’ retail channel preference intensifies price competition between the channel. In the present model, selling in decentralized setting for \(\theta > 0.4295, 0.4203, 0.4111\) in the 1st, 2nd and 3rd replenishment cycles, the retail prices are higher than the online selling prices (see Fig. 1). In centralized channel there is also price competition that is presented in Fig. 2. However, as in the decentralized case, the manufacturer can operate a profitable integrated channel when the customers’ retail channel preference \(\theta \in (0.4982, 0.791)\). Also, the retail price is higher than the online price for customers’ retail channel preference in \((0.4982, 0.791)\). Notice that the product compatibility has lesser impact on centralized channel when compared with decentralized channel.

<table>
<thead>
<tr>
<th>System</th>
<th>(n)</th>
<th>(w)</th>
<th>Retail price</th>
<th>Online price</th>
<th>Order quantity</th>
<th>(p^{n-1})</th>
<th>(p^{n+1})</th>
<th>Total profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Decentralized</td>
<td>1</td>
<td>85</td>
<td>105.1</td>
<td>91.1</td>
<td>2582</td>
<td>9112</td>
<td>81819</td>
<td>91111</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>82.2</td>
<td>103.8</td>
<td>90.3</td>
<td>3019</td>
<td>12652</td>
<td>100758</td>
<td>122410</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>86.4</td>
<td>100.4, 97.4</td>
<td>88.7, 81.7</td>
<td>3165</td>
<td>13239</td>
<td>116685</td>
<td>129924</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>86.4</td>
<td>82.6, 79.9</td>
<td>88.4, 84.6</td>
<td>3237</td>
<td>15085</td>
<td>117826</td>
<td>122084</td>
</tr>
<tr>
<td>Centralized</td>
<td>1</td>
<td>–</td>
<td>92</td>
<td>91.1</td>
<td>3211</td>
<td>–</td>
<td>–</td>
<td>101444</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>–</td>
<td>89.1, 82.2</td>
<td>89.8</td>
<td>3765</td>
<td>–</td>
<td>–</td>
<td>137663</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>–</td>
<td>89.84, 79</td>
<td>88.7, 83.7</td>
<td>3952</td>
<td>–</td>
<td>–</td>
<td>146162</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>–</td>
<td>86.4, 84.9, 81.1, 77.4</td>
<td>88.4, 84.6, 80.9, 77.2</td>
<td>4043</td>
<td>–</td>
<td>147942</td>
<td></td>
</tr>
<tr>
<td>Profit sharing</td>
<td>4</td>
<td>75.8</td>
<td>66.5, 61.6</td>
<td>86.8, 84.9, 81.1, 77.4</td>
<td>88.4, 84.6, 80.9, 77.2</td>
<td>4043</td>
<td>–</td>
<td>147942</td>
</tr>
</tbody>
</table>

Notice that the product compatibility has lesser impact on centralized channel when compared with decentralized channel.

Fig. 1. Price competition with respect to product compatibility in decentralized system.
The setup cost of the retailer has an impact on the manufacturer’s decision for operating the retail-online channel. As the lower threshold of product compatibility is directly proportional to the retailer’s setup cost, $\theta$ may be higher than $\theta_{dc}$. In such cases, the manufacturer can not set the online selling price and hence can not operate the retail-online channel. In the present model setting, if $s_r > 2361$ holds then the manufacturer can not operate the online channel (see Fig. 3). Thus, for a profitable retail-online channel, apart from the customers’ retail channel preference, the setup cost of the retailer should be reasonably low. From Table 2, it is noticed that profit sharing mechanism coordinates the channel and divides the surplus profit between the channel members. Also the average wholesale price of the manufacturer in profit sharing mechanism is 68.8 and it is lower than the average wholesale price, 83 of the decentralized decision making. This is quite obvious.

5. Summary and concluding remarks

In this paper we consider a retail–etail channel supply chain for a product that experiences continuous unit cost decrease over the planning horizon. Existing literature in this direction discusses the replenishment and pricing policy over the planning horizon for a single business entity like retailer or manufacturer. But as indicated before, in the current business scenario, the overall channel performance optimization is prevailing in practice and coexistence of retail and online channels are quite common. In such scenario, the present model proposes pricing and replenishment policy and analyzes how the customers’ channel preference affects the individual and integrated decisions of the channel members. The paper also discusses, in decentralized decision making, how the channel performance can be maximized through a transfer pricing policy. The model proposes the following managerial insights.

First, the retail price is directly proportional to online price and wholesale price. The manufacturer has the control, to some extent, on the retail price because it sets the wholesale price and online selling price. Second, the optimal selling prices and wholesale price of the product decrease continuously over the planning horizon because the products’ unit cost decreases continuously. In this scenario, the manufacturer must determine a planning horizon shorter than the lifetime of the product. Third, product compatibility has significant impact on the successful operation of profitable retail-online channel. As indicated earlier, if the customers retail channel preference lies in an interval then the manufacturer operates an online channel besides the traditional brick-and-mortar channel for a product that experiences continuous cost decrease. It is interesting that there is a price competition between the retail and online channel and customers’ channel preference intensifies.
the price competition. When the customers’ retail channel preference is below a certain threshold, the manufacturer sets online price higher than the retail price otherwise the retail price is higher. In integrated channel, there is also a price competition though the channel members cooperate. Thus, in centralized channel, there is an interval of customers’ retail channel preference for profit making retail–online channel. Fourth, the setup cost of the retailer influences the manufacturer’s decision for opening the online channel. The manufacturer cannot operate the online channel when setup cost of the retailer is above a threshold number because the manufacturer cannot set the online price in that case. Fifth, the profit sharing mechanism coordinates the channel and divides the surplus profit.

The model presented in this paper has some limitations. First, we have considered linear price dependent demand function. Although price dependent demand functions are used extensively in economics and marketing, still it is necessary to examine whether the managerial implications found in this paper can be generalized to other demand function or not. Second, we have assumed that the channel members know all information though it is not common in practice. Thus, model may be developed by considering that the channel members have private information. Third, we have used simple profit sharing mechanism to coordinate the channel. Instead of this mechanism, some other well established coordination contracts may be used to study the proposed model elaborately. Another interesting direction of future research is to be considered for multiple retailers while there is retail price competition. Also, the manufacturer wants to sell the product through an online channel. Although members of different replenishment cycles and price competition among the retailers make the model more complex in comparison to the present one, still it will be more dynamic and will be close to real business, in practice.

Appendix A

Proof of Proposition 1. For given \( n' \)

\[
\frac{\partial^2 \pi_{1}^{\text{ch}}(p_{1}^{\text{ch}})}{\partial p_{1}^{\text{ch}}^2} = -\frac{L}{p_{1}^{\text{ch}}} \left[ a \theta + (b + r) p_{1}^{\text{ch}} r p_{1}^{\text{ch}} - (b + r) \left( p_{1}^{\text{ch}} - w_{1} \right) \frac{Lh_{1}}{2p_{1}^{\text{ch}}} \right] \tag{A.1}
\]

\[
\frac{\partial^2 \pi_{1}^{\text{ch}}(p_{1}^{\text{ch}})}{\partial p_{1}^{\text{ch}}^2} = -\frac{2L}{p_{1}^{\text{ch}}} (b + r) < 0
\]

From (A.1), equating right hand side to zero and simplifying the stationary point, we have

\[
p_{1}^{\text{ch}}(w_{1}, p_{1}^{\text{ch}}) = \frac{w_{1}}{2} \left[ \frac{r}{2(b + r)} p_{1}^{\text{ch}} + \frac{a \theta}{2(b + r)} \right] + \frac{Lh_{1}}{4p_{1}^{\text{ch}}} (i = 1, 2, \ldots, n').
\]

Again,

\[
\frac{\partial^2 \pi_{1}^{\text{ch}}(w_{1}, p_{1}^{\text{ch}})}{\partial w_{1} \partial p_{1}^{\text{ch}}} = \frac{L}{p_{1}^{\text{ch}}} \left[ (1 - \theta) a + r \frac{a \theta + (b + r) \frac{L}{p_{1}^{\text{ch}}}}{2(b + r)} \right] - \frac{r}{2} \left( (i - 1) \frac{L}{p_{1}^{\text{ch}}} \right) - \frac{Lh_{1}}{2p_{1}^{\text{ch}}} \left[ \frac{r^{2}}{2(b + r)} - (b + r) \right] + rw_{1}
\]

\[
\frac{\partial^2 \pi_{1}^{\text{ch}}(w_{1}, p_{1}^{\text{ch}})}{\partial w_{1} \partial p_{1}^{\text{ch}}} = \frac{r^{2}}{2(b + r)} (b + r)
\]

\[
\frac{\partial^2 \pi_{1}^{\text{ch}}(w_{1}, p_{1}^{\text{ch}})}{\partial w_{1} \partial p_{1}^{\text{ch}}} = \frac{L}{p_{1}^{\text{ch}}} \left[ \left( a \theta + (b + r) \frac{L}{p_{1}^{\text{ch}}} \right) \frac{r}{2} \left( (i - 1) \frac{L}{p_{1}^{\text{ch}}} \right) + \frac{Lh_{1}}{2p_{1}^{\text{ch}}} \left[ \frac{r^{2}}{2(b + r)} - (b + r) \right] + rw_{1} \right]
\]

\[
\frac{\partial^2 \pi_{1}^{\text{ch}}(w_{1}, p_{1}^{\text{ch}})}{\partial w_{1} \partial p_{1}^{\text{ch}}} = \frac{r^{2}}{2(b + r)} (b + r)
\]

\[
\frac{\partial^2 \pi_{1}^{\text{ch}}(w_{1}, p_{1}^{\text{ch}})}{\partial w_{1} \partial p_{1}^{\text{ch}}} = -\left( b + r \right)
\]

\[
\frac{\partial^2 \pi_{1}^{\text{ch}}(w_{1}, p_{1}^{\text{ch}})}{\partial w_{1} \partial p_{1}^{\text{ch}}} = \frac{\partial^2 \pi_{1}^{\text{ch}}(w_{1}, p_{1}^{\text{ch}})}{\partial w_{1} \partial p_{1}^{\text{ch}}} - \frac{\partial^2 \pi_{1}^{\text{ch}}(w_{1}, p_{1}^{\text{ch}})}{\partial w_{1} \partial p_{1}^{\text{ch}}}
\]

\[
\frac{\partial^2 \pi_{1}^{\text{ch}}(w_{1}, p_{1}^{\text{ch}})}{\partial w_{1} \partial p_{1}^{\text{ch}}} = \left( \frac{\partial^2 \pi_{1}^{\text{ch}}(w_{1}, p_{1}^{\text{ch}})}{\partial w_{1} \partial p_{1}^{\text{ch}}} \right)^2 - \frac{2(b + r)^2 - r}{r} > 0.
\]
Equating (A.2) and (A.3) to zero and solving for $p_i^{(th)}$ and $w_i$ the result can be realized. Substituting the optimal values of $p_i^{(th)}$ and $w_i$ in Eq. (9), we get the retailer’s optimal selling price in $i$th ($i = 1, 2, ..., n'$) replenishment cycle displayed in Eq. (10). Substituting the optimal values of $p_i^{(th)}$ and $p_i^{(th)}$ in Eqs. (1) and (2), we get the demand of the product in retail and direct channel per unit time for $i$th ($i = 1, 2, ..., n'$) replenishment cycle and multiplying these with cycle length, we get the amount of quantity sold through retail and online channel respectively which are displayed in Table 1. Also, the order quantity that the manufacturer faces is equal to the product of total demand and cycle length, the profit functions of the manufacturer and the retailer in $i$th ($i = 1, 2, ..., n'$) replenishment cycle are displayed in Table 1.

**Proof of Proposition 2.** For given $n'$ we have, $c(t) = x - bt$. Thus for $i = 1$,
\[
p_1^{(th)} - p_1^{(h)} = \frac{1}{2}(c(0) - c(L/n')) - \frac{bl}{2n'} > 0
\]
that is, $p_1^{(th)} > p_1^{(h)}$.

For $i = 2$,
\[
p_2^{(th)} - p_2^{(h)} = \frac{1}{2}(c(L/n') - c(2L/n')) - \frac{bl}{2n'} > 0
\]
that is, $p_2^{(th)} > p_2^{(h)}$.

For $i = m$,
\[
p_m^{(th)} - p_m^{(h)} = \frac{1}{2}(c((m - 1)L/n') - c((m)L/n')) - \frac{bl}{2n'} > 0
\]
that is, $p_m^{(th)} > p_m^{(h)}$.

Hence, we can say $p_i^{(th)} > p_i^{(h)}$ for all $i = 1, 2, ..., n'$ i.e., $p_1^{(th)} > p_1^{(h)} > \ldots > p_n^{(th)}$.

Other results can be obtained in similar way and hence omitted.

**Proof of Proposition 4.** Comparing the selling prices of the product in retail channel and direct channel in $i$th ($i = 1, 2, ..., n'$) replenishment cycle, we get $p_i^{(th)} > p_i^{(th)}$ if
\[
\theta > \frac{2(b + r)}{3b + 6r} \left\{ \frac{2(b + r)}{3b + 6r} \right\} \left\{ (i - 1) \frac{L}{n'} \right\} = \theta_{th}^{0}
\]
Again, comparing $\theta_{th}^{0}$ with $\theta_{th}^{0}$, we get
\[
\theta_{th}^{0} - \theta_{th}^{0} = \frac{(b + 2r)(d - 2bc)}{2(3b + 6r)a} \left\{ \frac{b + 2r}{4(3b + 6r)} \right\} \left\{ (7b + 8r)h_a + (b + 4r)h_a \right\} L = 0
\]
and comparing $\theta_{th}^{0}$ with $\theta_{th}^{0}$, we get
\[
\theta_{th}^{0} - \theta_{th}^{0} = \frac{(b + 2r)(d - 2bc)}{2(3b + 6r)a} \left\{ \frac{b + 2r}{4(3b + 6r)} \right\} \left\{ (7b + 8r)h_a + (b + 4r)h_a \right\} L > 0
\]
Hence, $\theta_{th}^{0} < \theta_{th}^{0}$, Thus one can easily realize that $p_i^{(th)} > p_i^{(th)} > w_i$ if $\theta \in (\theta_{th}^{0}, \theta_{th}^{0})$ and $p_i^{(th)} > p_i^{(th)} > w_i$ if $\theta \in (\theta_{th}^{0}, \theta_{th}^{0})$.

**Proof of Proposition 6.** $\frac{a^2}{l^2} = 0$ gives
\[
16(b + r)sL^3 = \left( \frac{a^2}{l^2} \right) L^3 \left( 9a^2 - b \left( x - \frac{bl}{2} \right) \right)^2 \left( \frac{3A(A - 2b^2)}{2} \right) = 0
\]
Using Cardano’s method for solving the cubic equation, we get
\[
n_0 = \left( -a + \sqrt{a^2 + b^2} \right)^{\frac{1}{3}} + \left( -a - \sqrt{a^2 + b^2} \right)^{\frac{1}{3}}
\]
where $b = L^3(A - h_a)2\omega - b(2s - \beta L)\omega b + rjs$, and $d = L^3(3A(A - 2b^2)) + 2b^2 p_i^2 / 192(b + r)s$.

The analytical solution for finding the number of replenishment over the selling season $L$ gives the optimal profit of the retailer which can be an integer or can not be an integer. But the number of replenishment must be integer. It is very simple to find the integer solution of replenishment for the retailer. Suppose $[n_0]$ denotes the largest integer not greater than $n_0$.

Then the retailer will accept $[n_0]$ if $n_i^{(th)}([n_0]) > n_i^{(th)}([n_0]) + 1$ otherwise $[n_0] + 1$ is the better solution for the retailer. Hence optimal number of replenishment for the retailer is given by
Proof of Proposition 7. From Eq. \( \frac{\partial^2 n_i}{\partial x^2} \) gives

\[
2p_i'q_i' - 2(b + r)p_i' + 2\theta a + \frac{(b + r)h_k}{2} - \frac{rh_k}{2} + bc(i - 1) = 0
\]

and \( \frac{\partial^2 n_i}{\partial x^2} \) gives

\[
-2(b + r)p_i' + 2p_i' + (1 - \theta)a = \frac{rh_k}{2} + \frac{(b + r)h_k}{2} + bc(i - 1) = 0
\]

Solving we have the selling price of the product for retail and direct channel in ith, \( i = 1, 2, \ldots, n \) replenishment cycle. Again,

\[
\frac{\partial^2 n_i}{\partial x^2} = -2(b + r)
\]

\[
\frac{\partial^2 n_i}{\partial x^2} = -2(b + r)
\]

\[
\frac{\partial^2 m_i}{\partial p^2} = 2r
\]

\[
\frac{\partial^2 m_i}{\partial p^2} = 2r
\]

\[
\frac{\partial^2 m_i}{\partial p^2} = 2r - \frac{\partial^2 m_i}{\partial p^2} \left( \frac{\partial^2 m_i}{\partial p^2} \right)^2 - 4(b + r)^2 > 0
\]

That is, \( n_i \) is a concave function of \( p_i \) and \( p_i' \).

Proof of Proposition 8. We have \( c(t) = 2 - \beta t \). Thus for \( i = 1, \)

\[
p_i'' - p_i' = \frac{1}{2} \left( c(t) - c(L/n') \right) = \frac{\beta L}{2n'} > 0
\]

that is, \( p_i'' > p_i' \).

For \( i = 2, \)

\[
p_i'' - p_i' = \frac{1}{2} \left( c(L/n') - c(2L/n') \right) = \frac{\beta L}{2n} > 0
\]

that is, \( p_i'' > p_i' \).

For \( i = 3, \)

\[
p_i'' - p_i' = \frac{1}{2} \left( c((m - 1)L/n') - c((m)L/n') \right) = \frac{\beta L}{2n} > 0
\]

that is, \( p_i'' > p_i' \).

Hence, we can say \( p_i'' > p_i' \) for all \( i = 1, 2, \ldots, n \) i.e., \( p_i'' > p_i' > \ldots > p_n'' \). Other results can be obtained in similar way and hence omitted.

References

Corporate social responsibility, coordination and profit distribution in a dual-channel supply chain

Abstract

The dual-channel supply chain model has become increasingly popular in the industry and describes a scenario in which a firm, in addition to selling through the traditional supply chain of manufacturer and retailer, opens a direct channel to the customer through Internet sales. However, in the current global business environment, corporate social responsibility (CSR) is a determining factor of choices of the customers. Based on the above important factors, this article introduces a corporate social responsibility in two-echelon dual-channel supply chain. In addition to operating an online channel, the manufacturer intends to increase stakeholders’ welfare by exhibiting CSR. The pricing decisions for both the cases of the decentralized and centralized scenarios are studied analytically as well as numerically. The paper also examines the effect of the degree of concern of the manufacturer regarding CSR on product compatibility and discusses feasibility of the successful operation of a dual-channel supply chain. Finally, channel coordination through all unit quantity discounts with the agreement of a franchise fee and surplus profit division through bargaining is discussed analytically.

Keywords: Dual-channel; CSR; Product compatibility; Coordination

Introduction

Currently, an impressive growth of e-commerce in the highly competitive global market drives manufacturers to introduce online channels. A manufacturer captures the markets situated in geographically diverse locations through an online channel via the Internet that reduces the time consumption for purchasing, and hence it can also increase market share. In Western Europe, e-commerce spending hit 128 billion Euros in 2013, up 14.3% from that in 2012. In 2010, the Czech Republic earned 24% of the country’s total turnover through online channels. Global e-commerce sales topped 1 trillion for the first time in 2012 [23]. It is forecasted that e-commerce spending in 2017 will
reach 191 billion Euros in Western Union, a compound
annual growth rate of approximately 11% [24]. E-
commerce sales in the US were increased by 15.8% in
2013 compared to the year 2011. It is observed that
consumers prefer alternatives and choose the one that
is better suited to their needs, which compels manu-
facturers to restructure their traditional brick-and-
mortar channels by engaging in direct sales through
Internet channels [48].

Corporate Social Responsibility (CSR) can be
defined as a doctrine that promotes the expansion of
social stewardship by businesses and organizations.
The CSR approach is holistic and integrated with the
core of the business strategy by addressing social and
environmental impacts of the businesses. CSR suggests
that corporations embrace responsibilities toward a
broader group of stakeholders, such as customers, employees, etc., besides their regular financial obliga-
tions to stockholders [20]. In the current global busi-
ness environment, CSR is now a determining factor in
consumers’ and clients’ choices that cannot be ignored
by companies. The companies that fail to maximize
their adoption of a CSR strategy will be left behind.
Recently, empirical evidence has shown that customers
are willing to pay a higher price for products with CSR
attributes and that CSR programmes influence 70 per-
cent of all consumers’ buying decisions [4,10]. As a
result, many leading international brands, such as
Walmart, Nike, Adidas and Gap, have been compelled
to incorporate CSR in their complex supply chains via
a code of conduct [3].

Several research works have been conducted in the
area of supply chain coordination. Many of them
concentrate on the direct collaboration between two
individual members, whereas the models dealing with
resolving channel conflict in dual-channel supply
chains are notably fewer. The purpose of this present
article is to incorporate CSR in a dual-channel supply
chain comprising a manufacturer and a retailer. In
addition to traditional retail channels, the manufacturer
operates an online channel. The manufacturer, as the
leader of the channel, considers stakeholders’ welfare
through CSR and influences the downstream channel
members to behave socially. Instead of the manufac-
turer’s CSR activity, we also incorporate the effect of
CSR in the form of consumer surplus in its profit
function. In a manufacturer-Stackelberg game, setting
apart the discussion of the effects of CSR in decen-
tralized and centralized decision-making, we apply all
unit quantity discounts with the agreement of a fran-
chise fee to resolve channel conflict and to find out
win–win profits of the channel members.

Literature review

In the recent trends of global business scenarios, the
dual-channel supply chain has a significant importance
in supply chain management. In addition to a retail
channel, an Internet channel of the manufacturer has
the potential to reduce retailers’ dominance, addressing
different customer segments in order to gain a higher
profit margin. For instance, with the popularity of the
Internet, many top manufacturers, such as IBM, Cisco,
Nike and Estee Lauder, have started selling online
directly. Several electronics manufacturers, including
Sony, PalmOne and Samsung, have setup boutique-
style outlets in upscale locations. The largest
English-language publisher, Random House, has pub-
licly declared that it may sell books directly to the
readers, putting them in direct competition with Barnes
and Noble and Amazon.com [49].

Meanwhile, traditional companies are expanding
their business through online access at retail stores. Dell
has installed kiosks in shopping malls and now sells its
computers through Costco [30]. These studies suggest
that more consumers are embracing multiple channels
to satisfy their shopping needs [47]. Some customers
prefer purchasing online, whereas others prefer shop-
ing in retail stores. As a result, manufacturers redesign
their traditional channel structures by engaging in direct
sales to reach different customer segments that cannot
be reached by the traditional retail channel, giving birth
to dual channels. Several issues in dual-channel supply
chains have been addressed by researchers. Hua et al.
[21] analysed the effect of delivery lead-time on the
pricing decisions in a dual-channel supply chain. Dan
et al. [14] determined the optimal retail service and
prices in a dual-channel supply chain. Chen et al. [8]
developed a dual-channel supply chain and proposed
pricing strategies that maximize decentralized dual-
channel performance. Sharma and Mehrotra [46]
claimed that the dual-channel setup increases channel
conflict, though it has the potential to increase cus-
tomers’ demand. Yan [50] developed a dual-channel
supply chain and analysed the effect of differentiated
branding. Chiang et al. [9] proposed a model that
demonstrates that a dual-channel supply chain can be
used to control retailers’ prices. Panda et al. [39,42]
analysed pricing and replenishment decisions in a
dual-channel supply chain considering continuous unit
cost decrease. All of the proposed models mentioned
above have addressed pricing and replenishment pol-
cies, channel conflict and channel competition, mainly
between brick-and-mortar and Internet channels, but
they do not focus on corporate social responsibility.
Corporate social responsibility is corporate self-regulation, which currently does not have a unique definition. Dahlsrud [15] analysed 37 definitions of CSR and developed five dimensions of CSR: environmental, social, economic, stakeholder, and voluntariness. Dylick and Hockert [17] defined CSR as “meeting the needs of a firm’s direct and indirect stakeholders (e.g., shareholders, employees, clients, pressure groups, communities etc.), without compromising its ability to meet the needs of future stakeholders as well.” Applications of CSR in supply chains have emerged in the last two decades. Considering a socially responsible supply chain, Murphy and Poist [32] suggested a total responsibility approach by adding social issues to the traditional economy. Carter and Jennings [7] explained the necessity of CSR consideration in supply chain decision-making through a case study and survey research. Analysing a French sample data set, Ageron et al. [1] derived several conditions for successful sustainable supply chain management. Cruz [11] traced the equilibrium condition for an environmentally responsible supply chain network by using a multi-criteria decision-making approach. Cruz and Wakolbinger [13] extended the model to a multi-period setting for measuring long-term effects of CSR. Considering a socially responsible supply chain network, Hsueh and Chang [25] showed that social responsibility sharing through monetary transfer leads to channel optimization. Cruz [12] developed a decision support system framework for modelling and analysis of a CSR supply chain network. Ni et al. [34] investigated a two-echelon supply chain by assuming the dominant upstream channel member’s CSR cost, which is shared by the downstream channel member through a wholesale price contract. Ni and Kevin [34] investigated a two-echelon supply chain by assuming individual CSR costs for each channel member. They examined the effects of strategic interactions between the channel members via a game theoretical approach. Panda [36] and Panda et al. [38] considered CSR supply chains and used different contracts to resolve channel conflict. They used a Nash bargaining product to divide surplus profit between the channel members. Coordination among channel members is imperative for improving channel-wide performance because it neutralizes the difference between the decentralized and centralized outcomes. The main objective of a coordination mechanism is the transfer of money from one channel member to another while they act coherently. Existing literature has rich content in this regard for two-echelon supply chains. Quantity discounts [38], two-part tariffs [26], revenue sharing [36], mail-in rebates [45], buyback [16], disposal cost sharing [37,43], profit sharing [31], etc. are used to resolve double marginalization in a two-tier supply chain. However, there are a few papers that have focused on cutting out channel conflict in dual-channel supply chains. Chen et al. [8] used wholesale prices and manufacturers’ direct channel price contracts for channel coordination. They also suggested that two-part tariffs and profit sharing in a range coordinates the channel and that the channel members’ profits follow a win–win strategy. Agrawal et al. [2] showed that sales effort resolves channel conflict when the channels do compete with each other. Cai [6] proposed a hybrid revenue sharing and linear online retail prices relationship to cut out channel conflict and examined the influence of channel coordination on the supplier. Panda [40,41] used revenue sharing contracts and Nash bargaining for channel coordination and under quantity-price and time-price dependent demand. Boyacci [5] showed that revenue sharing, buyback and wholesale price contracts are unable to resolve double marginalization. He mentioned that a penalty contract can coordinate a dual-channel supply chain, though it is difficult to implement. However, CSR in the modelling of channel coordination has not been considered in the models reported above.

The research reported in this paper differs from the prior works in many aspects, as follows. First, unlike the natural intention of maximizing the channel members’ profits, the objectives of the channel members are to engage in CSR and to find the effects of CSR on the dual-channel supply chain. The outcomes of the paper indicate that the profits of the members of the chain are always higher than their individual pure profits when the channel members concentrate more on CSR. Second, the paper discusses the effect of CSR on the pricing issues of a product. Third, the paper analyses the effect of customers’ channel preference on the channel competition for optimal prices. Fourth, it examines the effect of CSR on product compatibility and discusses feasibility for successful operation of a dual-channel supply chain. Fifthly, the paper uses all unit quantity discounts with agreement of franchise fees as the contract mechanism to resolve channel conflict. It cuts out channel conflict but is unable to depict win–win profits for the channel members. Sixthly, the paper uses a Nash bargaining product to divide the surplus profit between the channel members, where integrated profits of all of the members of the chain are win–win.
Model

We consider a dual-channel supply chain comprising a manufacturer and a retailer. The manufacturer produces and sells products through retail as well as e-tail/online channels. Here, the price-demand relationship is deterministic and known. Following Yue and Liu [49], Kurata et al. [28] and Huang and Swaminathan [22], we assume that, for model simplicity, the channel demand functions in the two channels are linear in self-price and cross-price effects. The forms of the demand functions in retail and e-tail (online) channels are

\[
D_R = \alpha_R \cdot p_R + \alpha(p_T - p_R) \quad (1)
\]

and

\[
D_O = (1 - \theta) \cdot \alpha_R \cdot p_R + \alpha(p_T - p_O) \quad (2)
\]

where \(\alpha > 0\) is the market potential. The parameter \(\theta\) \((0 \leq \theta \leq 1)\) is the compatibility of the product with the retail channel. How the product is perceived within the context of the customers' lifestyles choices is dependent on the compatibility of the product. When the product closely matches the needs, wants, beliefs, values and consumption patterns of the customers, it can be considered highly compatible with the consumers' choices. The percentage of the primary demand that goes to the retail channel is \(\theta\), and, when the value of \(\theta\) is greater, the product's compatibility with the retail channel is larger and more consumers purchase the product from the retail channel.

Retail products, books, information, magazines and digital products have more compatibility with the direct channel than products such as water, rice, gasoline and milk. Here, \(b_1(>0)\) and \(b_2(>0)\) are the price sensitivity factors in the retail channel and online channel, respectively. The parameter \(\alpha(>0)\) is the cross-price effect, which reflects the degree of price competition between the channels. Quite often, the selling price in the online channel is higher than the manufacturer's wholesale price, i.e., \(p_O > w_m\). Otherwise, the retailer would purchase the product through the online channel rather than from the manufacturer. Additionally, for the profitability of the retailer, the selling price of the retailer is higher than the manufacturer's wholesale price, i.e., \(p_R > w_m\).

Many leading brands face intense pressure for socially responsible supply chain management [3]. A commonly noted response to this pressure is to maintain the code of conduct given to the business partners who are socially responsible [44]. For quantitative analysis, the paper considers only the effect of CSR in the form of consumer surplus rather than the CSR activities, which are performed by the socially responsible channel members. It is well established that a firm's social responsibility is accounted for through the consumer surplus that is accrued from its stakeholders [18, 19, 27, 29, 34, 35]. The consumer surplus is the difference between the maximum price that the consumers are willing to pay for a product and the market price that they actually pay for the product.

Thus, in the present model, the consumer surplus can be found as

\[
CS = \int_{P_{\text{res}}}^{P_{\text{max}}} D_R dp_R + \int_{P_{\text{res}}}^{P_{\text{max}}} D_O dp_O = \frac{D_R^2}{2(b_1 + \alpha)} + \frac{D_O^2}{2(b_2 + \alpha)} \quad (3)
\]

where \(P_{\text{res}} = (b_2 + \alpha + \delta) / (b_1 + \alpha)\) and \(P_{\text{max}} = (b_2 + \alpha + \delta) / (b_1 + \alpha)\) denote the market prices of the product in retail and online channels, respectively, and \(P_{\text{inc}} = (b_2 + \alpha + \delta) / (b_1 + \alpha)\) denote the maximum prices that consumers are willing to pay for a product in retail and online channels, respectively.

If \(0 \leq \tau \leq 1\) is the degree of CSR that is the socially responsible manufacturer's concern, then it incorporates \(\tau CS\) as the consumer surplus in its profit. The value of \(\tau = 0\) implies that the manufacturer is the pure profit maximizer, and \(\tau = 1\) represents that the manufacturer is the perfect welfare maximizer. Because the manufacturer is socially responsible, its profit function consists of pure profit that is received by selling the product and consumer surplus through CSR practices. Under this setting, we first derive the centralized and decentralized decisions of the channel members.

Decentralized decisions

In decentralized decision-making, the channel members operate independently and optimize their individual goals. Interactions between the manufacturer and the retailer are considered as a Stackelberg game. The manufacturer acts as the Stackelberg leader of the channel, and the retailer is its follower. In a Stackelberg game, the leader makes the first move, and the follower then reacts by playing the best move consistent with the available information. In this way, the manufacturer first announces the wholesale price.
and selling the product in the online channel.

Based on the manufacturer's decision, the retailer determines the retail price. The profit function of the retailer is

\[ \pi_r = (p_r - w_m)(0 - b_1 p_r + z(p_d - p_r)) \]  

(4)

Note that \( \frac{\partial^2 \pi_r}{\partial p_r^2} = -2(b_1 + \alpha) < 0 \). That is, \( \pi_r \) is a concave function of \( p_r \). Substituting the value of \( p_r \) into the total profit function of the manufacturer and the necessary conditions \( \frac{\partial \pi_m}{\partial w_m} = 0 \) and \( \frac{\partial \pi_m}{\partial p_d} = 0 \) for optimization of \( w_m \) yields

\[ \pi_m = (w_m - c)(0 - b_1 p_d + z(p_d - p_m)) \]  

(5)

The total profit function of the manufacturer is

\[ \pi_m = \pi_a + \tau \pi_S \]  

(6)

where \( 0 \leq \tau \leq 1 \).

The second-order partial derivatives of the profit function, which are as follows

\[ \frac{\partial^2 \pi_m}{\partial w_m^2} = \frac{-4b_2 a^2 + b_1 \beta_1 (4 + \tau) + a(b_2 + 2z)\tau}{4b_2^2} \]  

(10)

Differentiating \( \pi_r \) with respect to \( p_r \) and equating it to zero, we have

\[ p_r = \frac{0 - a + b_1(a + \alpha)w_m + 2\alpha p_d}{2(b_1 + \alpha)} \]  

(7)

\[ \frac{\partial^3 \pi_m}{\partial w_m^2 \partial p_d} = \frac{a - (3b_3 \beta_3 + b_1 \beta_3)\tau}{4b_2^2 \beta_2} \]  

(12)
Clearly, \( \frac{\partial^2 v_a}{\partial w_a^2} < 0 \) and \( \frac{\partial^2 v_a}{\partial p_a^2} < 0 \) for any \( r \in [0, 1] \). Now, the necessary condition for concavity of \( v_a \) is 

\[
\frac{\partial^2 v_a}{\partial w_a^2} \frac{\partial^2 v_a}{\partial p_a^2} - \left( \frac{\partial^2 v_a}{\partial w_a \partial p_a} \right)^2 > 0
\]

\[
(b_2 z + b_1 \beta_z) (2 - r) (4 z \beta_z + b_1 \beta_z (4 - r) - b_2 x r)
\]

For any \( r \in [0, 1] \), \( \frac{\partial^2 v_a}{\partial w_a^2} \frac{\partial^2 v_a}{\partial p_a^2} - \left( \frac{\partial^2 v_a}{\partial w_a \partial p_a} \right)^2 > 0 \), and thus the profit function of the manufacturer is a concave function of \( p_a \) and \( w_a \). Using (8) and (9) in (7) and simplifying, we obtain the optimal retail price of the product with decentralized decisions as follows

\[
p_a^* = \frac{\beta_z (b_1 c \beta_z + b_1 c z (3 b_2 + 2 x) + a (2 (a + b_2) x + 3 a b_2) + a b (3 b_2 + a (2 + x)))}{(b_2 z + b_1 \beta_z) (4 z \beta_z + b_1 \beta_z (4 - r) - b_2 x r)}
\]

From (8), (9) and (13), we have the optimal demands of the product in retail and online channels, the profit of the retailer, and the pure and total profit of the manufacturer with decentralized decisions as follows

\[
D^*_a = \frac{\beta_z (2 a b_2 - b_1 c \beta_z (2 - r) + b_1 c z (4 x + b_1 (4 - r) - b_2 x r))}{(2 - r) (4 x + b_1 (4 - r) - b_2 x r)}
\]

\[
D^*_r = \frac{\beta_z (2 a b_2 - b_1 c \beta_z (2 - r) + b_1 c z (4 x + b_1 (4 - r) - b_2 x r))}{(2 - r) (4 x + b_1 (4 - r) - b_2 x r)}
\]

\[
\pi^*_a = \frac{\beta_z (2 a b_2 - b_1 c \beta_z (2 - r) + b_1 c z (4 x + b_1 (4 - r) - b_2 x r))}{(2 - r) (4 x + b_1 (4 - r) - b_2 x r)}
\]

\[
\pi^*_r = \frac{\beta_z (2 a b_2 - b_1 c \beta_z (2 - r) + b_1 c z (4 x + b_1 (4 - r) - b_2 x r))}{(2 - r) (4 x + b_1 (4 - r) - b_2 x r)}
\]

Please cite this article in press as: N.M. Modak et al., Corporate social responsibility, coordination and profit distribution in a dual-channel supply chain, Pacific Science Review (2015), http://dx.doi.org/10.1016/j.pscr.2015.05.001
where
\[ F_1 = 4b_1 \beta_1 (b_1 c x + b_1 c \beta_2 - a(x + b_2 \theta)) + (b_1 x + b_1 \beta_2)(b_1 c x + b_1 c \beta_2 - a(x + b_2 \theta)) \tau^2 + \left( -4b_1^2 \beta_2^2 + a (a - b_1 c x) (3b_2 + 2a) \right. \\
+ \left. a(b_1(4b_2 + 2a) \theta) + b_1 \beta_2 (-c x (7b_2 + 2a)) + a (4b_2 \theta + a (3 + 0))) \right) \tau \]
and
\[ F_2 = c(b_2 x + b_1 \beta_2)(2x (2a - 2t) + b_2 (4 - 4t) (1 - t) - t) + b_2 (2a (2 - 4t) + b_2 (4 - 4t) (1 - t)) \]
\[ + a (b_2 \beta_1 (-1 + 1) (4 - 4t) (1 - t)) + a^2 (2 \theta (2 - 4t) - b_2 (4 + 4t) (5 + 0 + 2r)) \]
\[ + a(b_2 (4 \beta_1 (-2 + 0) + (10b_2 + 7a) - 3 (2b_2 + a) \theta) + (-a + b_2 (-2 + 0) \theta) \tau) \]

Here, the consumer can buy the product from both retail and online channels. The retailer operates its channel in competition with the online channel. The retailer has to buy the product from the manufacturer and then sell it to the consumers, and, in doing so, it makes a profit. Thus, the concept of dual channels will be feasible only when the selling prices in both retail and online channels are higher than the wholesale price of the manufacturer. That is, the optimal pricing strategy of the decentralized system is acceptable to the channel members only when \( p_r^* > w_r^* \) and \( p_o^* > w_o^* \).

Now, \( p_r^* > w_r^* \) if \( \theta < \theta_{max} \), where
\[
\theta_{max} = \frac{b_1 c (b_1 + 2a) (b_2 x + b_1 \beta_2) \tau + a (b_1^2 \beta_2^2 (4 - 4t) (1 - t) - 2b_2 a \tau + b_2 (4a (1 - t) + b_2 (4 - (7 - 7t) \tau)))}{a (4(b_1 + b_2) \beta_2 \beta_1 - (5b_1 \beta_2 + 4b_2 a \beta_2 + b_1 (4b_2 + 11b_2 a + 4a \tau)) + (b_1 + b_2) (b_1 x + b_1 \beta_2) \tau^2)} \tag{19}
\]

and \( p_o^* > w_o^* \) if
\[
\theta > \frac{b_1 c (2 - \tau) + (a - b_1 c) \alpha \tau}{2a b_1 \beta_2 - a b_2 \tau} = \theta_{min} \tag{20}
\]

Note that, both \( \theta_{min} \) and \( \theta_{max} \) depend on \( \tau \), i.e., on the degree of social concern of the manufacturer. It is observed that this range will be valid until \( \theta_{max} > \theta_{min} \) and, solving \( \theta_{max} = \theta_{min} = 0 \), we have three values of \( \tau \), \( \frac{2a b_1 \beta_2 - a b_2 \tau}{b_1 c (2 - \tau) + (a - b_1 c) \alpha \tau} = \theta_{min} \). Among these three values of \( \tau \), only \( \frac{b_1 c (2 - \tau) + (a - b_1 c) \alpha \tau}{2a b_1 \beta_2 - a b_2 \tau} \) is acceptable because \( \tau \in (0, 1) \). That is, \( \theta_{max} > \theta_{min} \).

If \( \tau \in \left( \frac{b_1 c (2 - \tau) + (a - b_1 c) \alpha \tau}{2a b_1 \beta_2 - a b_2 \tau}, \frac{b_1 c (2 - \tau) + (a - b_1 c) \alpha \tau}{2a b_1 \beta_2 - a b_2 \tau} \right] \), \( \theta_{max} < \theta_{min} \). From the above discussion, we have the following proposition.

**Proposition 1.** The pricing policy of the decentralized socially responsible dual-channel supply chain can be operated successfully if the product compatibility parameter \( \theta \in (\theta_{min}, \theta_{max}) \) and the degree of social concern of the manufacturer \( \tau \in (0, \frac{b_1 c (2 - \tau) + (a - b_1 c) \alpha \tau}{2a b_1 \beta_2 - a b_2 \tau}) \). From Fig. 1, one can observe that, to operate the dual-channel successfully, the manufacturer cannot set its degree of social concern, i.e., the value of \( \tau \), above the threshold \( \frac{b_1 c (2 - \tau) + (a - b_1 c) \alpha \tau}{2a b_1 \beta_2 - a b_2 \tau} \) and because, when \( \tau \in \left( \frac{b_1 c (2 - \tau) + (a - b_1 c) \alpha \tau}{2a b_1 \beta_2 - a b_2 \tau}, \frac{b_1 c (2 - \tau) + (a - b_1 c) \alpha \tau}{2a b_1 \beta_2 - a b_2 \tau} \right) \), the pricing policy of the dual channels becomes infeasible. Thus, from **Proposition 1**, we can conclude that the manufacturer cannot show CSR concern over a threshold to operate a decentralized dual-channel supply chain. It also indicates that the customer's channel preference is one of the determining factors for operating an online channel in addition to the traditional retail channel. When \( \theta < \theta_{max} \), the retailer cannot do business because its selling price is less than the manufacturer's wholesale price. Alternately, for \( \theta > \theta_{max} \), the manufacturer cannot set the optimal selling price as the online price exists. The online selling price is higher than the wholesale price. This does not ensure that the retailer will participate in the profit-making retail/e-tail channel; the retailer will participate in the dual channels only when its demand in the retail channel is positive, i.e., \( D_r > 0 \), i.e., if
\[
\theta > \frac{b_1 c \beta_1 (2 - \tau) + (a - b_1 c) \alpha \tau}{2a b_1 \beta_2 - a b_2 \tau} = \theta_{min} \tag{21}
\]
Note that \( \theta_r = \theta_{min} \). Alternately, the manufacturer will operate the online channel until the demand in the online channel is positive, i.e., \( D_o > 0 \), i.e., if
Eqs. (21) and (22) suggest that the manufacturer can successfully operate a profitable dual-channel supply chain when the customer’s retail channel preference lies between $\theta_{\min}$ and $\min(\theta_{\max}, \theta_{\max})$. This result is quite obvious because the manufacturer will operate the online channel only when both the channels are profitable. Thus, another proposition is as follows.

**Proposition 2.** The manufacturer can operate a profitable retail-online channel when the customers’ retail channel preference $\theta \in (\theta_{\min}, \min(\theta_{\max}, \theta_{\max}))$. Further, comparing selling prices of retail and online channels, we have $p^*_r > p^*_o$ if $\theta > \theta_1$, where

$$\theta_1 = \frac{c(b_1 \alpha + b_3 \beta_1 - (b_1 - b_2)\alpha \tau + a(\Delta^2 + (b_2 + 2\alpha)\tau + b_2 + 2\alpha)\tau)}{a(\Delta^2 + (b_2 + 2\alpha)\tau + b_2 + 2\alpha)\tau} \left[ \frac{1}{\theta_{\min}} \right]$$

(23)

Now, combining the above results with Proposition 1, we obtain the following proposition.

**Proposition 3.** In a socially responsible decentralized dual-channel supply chain, the optimal retail price is higher than the online selling price if $\theta \in (\theta_1, \min(\theta_{\min}, \theta_{\max}))$, and the reverse may be noted for $\theta \in (\theta_{\min}, \theta_1)$. From Fig. 2, one may note that $p^*_r \geq w^*_r$ if $\theta > \theta_{\min}$ and $p^*_o \geq w^*_o$ if $\theta \leq \theta_{\max}$. The optimal online selling price ($p^*_o$) decreases with increasing product compatibility, whereas the optimal retail price ($p^*_r$) increases with increasing product compatibility. The optimal online selling price coincides with the retail price when $\theta = \theta_1$. In the next subsection, we shall study the centralized decisions of the socially responsible dual-channel supply chain when the channel members act as a single entity.

**Centralized decisions**

When the channel members cooperate and find the decision that maximizes the supply chain performance, it requires a centralized decision-making process. It may be assumed that there is a single decision-maker who produces and sells the product to the customers. The total profit of the channel is the sum of the pure profit ($\pi_c$) and the consumer surplus ($CS_c$) that the channel accrues from the stakeholders. The profit function of the channel is

$$\nu_c = \pi_c + CS_c$$

(24)

where $\pi_c = (p_r - c)D_r + (p_o - c)D_o$ and $CS_c = \frac{\partial p_r}{\partial p_o}$. The necessary conditions $\frac{\partial \pi_c}{\partial p_r} = 0$ and $\frac{\partial \nu_c}{\partial p_r} = 0$, for the existence of the optimal solution, yield the optimal values of the selling prices as

$$p^*_r = p^*_o$$

and $w^*_r = w^*_o$. This result is quite obvious because the manufacturer will operate the online channel only when both the channels are profitable.

Fig. 1. Graphical representation of $\theta_{\min}$ and $\theta_{\max}$ with respect to $\theta$. 

$$\theta < \frac{a_2 \beta_1 (4 - \tau) - b_1 \beta_2 (4 - \tau) - b_1 (2a \alpha - b_2 \alpha \tau)}{a(2a + b_2 (4 - \tau))} = \theta_1$$

(22)
follows

\[ p_{w} = \left( \beta_{1}(b_{1}c_{1} + b_{1}c_{2} + a(b_{1} + b_{1}c_{1})) + a(b_{1} + b_{1}c_{1})(b_{1} + a_{1}) \right) \right) \]

and

\[ p_{w} = \left( \frac{2b_{1}c_{1}b_{1}(b_{1}c_{1} + b_{1}c_{2} + a(b_{1} + b_{1}c_{1})) + (b_{1} + b_{1}c_{1})(b_{1} + a_{1})}{(b_{1}c_{1} + b_{1}c_{2} + a(b_{1} + b_{1}c_{1})) + a(b_{1} + b_{1}c_{1})(b_{1} + a_{1})} \right) \]

To check the concavity of the total centralized channel profit function, we take the second-order partial derivatives, which are as follows

\[ \frac{\partial^{2}V_{c}}{\partial p^{2}} = -b_{1}(2 - \tau) + a \left( \frac{a_{1}}{b_{1} + a_{1}} \right) \]

\[ \frac{\partial^{2}V_{c}}{\partial p_{1}^{2}} = -b_{1}(2 - \tau) + a \left( \frac{a_{1}}{b_{1} + a_{1}} \right) \]

\[ \frac{\partial^{2}V_{c}}{\partial p_{3}^{2}} = 2a(1 - \tau) \]

Hence, we have

\[ \frac{\partial^{2}V_{c}}{\partial p_{1}^{2}} - \left( \frac{\partial^{2}V_{c}}{\partial p_{3}^{2}} \right) \]

\[ \left( \frac{\partial^{2}V_{c}}{\partial p_{1}^{2}} - \left( \frac{\partial^{2}V_{c}}{\partial p_{3}^{2}} \right) \right) \]

Clearly, \( \frac{\partial^{2}V_{c}}{\partial p^{2}} < 0 \) and \( \frac{\partial^{2}V_{c}}{\partial p_{1}^{2}} - \left( \frac{\partial^{2}V_{c}}{\partial p_{3}^{2}} \right) > 0 \) for all \( \tau \in [0, 1] \). Thus, the total profit function of the centralized channel \( (V_{c}) \) is a concave function of \( p_{w} \) and \( p_{r} \). Hence, the optimal selling prices \( (p_{w} \text{ and } p_{r}) \) provide the global maximum to (24). From (25) and (26), we obtain the optimal values of the demand of the product in the retail and online channels for the centralized decision, which are as follows

\[ V_{c}(p_{w}, p_{r}) = \left( \frac{\partial^{2}V_{c}}{\partial p_{w}^{2}} - \left( \frac{\partial^{2}V_{c}}{\partial p_{3}^{2}} \right) \right) > 0 \]

Fig. 2. Graphical representation of optimal decentralized prices with respect to 0.
\[ D_{c} = \frac{\beta_{1}(2a\beta_{1}\theta + b_{c}\beta_{2}(2 - \tau) + b_{c}c\tau - a(a + b\theta)\tau)}{a(b_{1}(2 - \tau) + 4\delta(1 - \tau)) + b_{1}\beta_{2}(2 - \tau)^{2}} \]  

and

\[ D_{a} = \frac{\beta_{1}(-2\delta_{1}(b_{c}\alpha + a(-1 + 0)) + (b_{c}\alpha - a\delta_{1} + b_{c}\beta_{1} + ab\theta)\tau)}{a(b_{1}(2 - \tau)^{2} + 4\delta(1 - \tau)) + b_{1}\beta_{2}(2 - \tau)^{2}} \]  

The optimal pure and total profits of the centralized channel are as follows

\[ \pi_{c}^{*} = (p_{c} - \epsilon)D_{c} + (p_{a} - \epsilon)D_{a} \]  

and

\[ v_{c}^{*} = (p_{c} - \epsilon)D_{c} + (p_{a} - \epsilon)D_{a} + \frac{DC_{1} + DC_{2}}{2\beta_{1}} \]  

In the context of centralized decision-making, the manufacturer would be interested in opening the online channel only when the online channel demand is positive, i.e., \( D_{c} > 0 \), which, after simplification, yields

\[ \theta < \frac{\beta_{1}(2 - \tau) - 2b_{c}\alpha\delta_{1} + c(b_{c}\alpha + b\beta_{1})\tau}{2a\delta_{1} - ab\tau} = \theta_{\text{max}} \]  

Alternately, the minimum level of product compatibility for operating a centralized retail channel can be found from \( D_{c} > 0 \). This, after simplification, yields

\[ \theta > \frac{b_{c}\beta_{2}(2 - \tau) + (a + b\epsilon)\alpha\tau}{2a\delta_{1} - ab\tau} = \theta_{\text{min}} \]  

Although the manufacturer and the retailer co-operate and make decisions jointly in the centralized channel, the product compatibility has an impact on the manufacturer's decision to open the online channel. In the \( i \)-th replenishment cycle, if the customers' retail channel preference is higher than the threshold \( \theta_{\text{max}} \), then the manufacturer's decision to open an online channel is not profitable because its online demand is negative in this case. However, there must be competition between the retail channel and the online channel in the centralized process even though the channel members co-operate. The channel members make decisions jointly, but the market potential remains the same. When the manufacturer operates an online channel, some customers switch to the online channel; as a result, the retailer's demand decreases, and it earns less profit. In addition to the selling prices of the retail and online channels, the customers' channel preference determines the divisions of the potential market demand. Thus, like in the decentralized decision-making process, here also the selling prices of the retail channel may be higher than the online channel, i.e., \( p_{c} > p_{a} \). Hence, if \( \theta > \theta_{2} \), where

From the above discussion, we have the following proposition.

**Proposition 4.** The manufacturer can operate a centralized dual-channel supply chain if \( \theta \in (\theta_{\text{max}}, \theta_{\text{max}}) \). The online selling price is higher than the retail price for any \( \theta \in \theta_{\text{max}} \), and the retail price is higher than the online price for any \( \theta \in (\theta_{2}, \theta_{\text{max}}) \). Proposition 4 demonstrates that, when the channel members cooperate and make decisions jointly, the manufacturer's decision to open an online channel is profitable only when the customers' retail channel preference lies between \( \theta_{\text{max}} \) and \( \theta_{\text{max}} \). Interestingly, between these thresholds of the product compatibility, there exists a price competition between the retail channel and the online channel. If the customers' retail channel preference is within \( (\theta_{\text{max}}, \theta_{2}) \), then the online price will be higher than the retail price, and the reverse is true for \( \theta \in (\theta_{2}, \theta_{\text{max}}) \). Fig. 3 also
justifies our analytical findings. Thus, for a profitable centralized retail/online channel, the channel will set the selling prices according to the customers’ channel preference. Observe that both \( \theta_{\max} \) and \( \theta_{\max}^* \) depend on the degree of the manufacturer’s social concern, i.e., on \( \tau \). A centralized dual-channel supply chain will be feasible only when \( \theta_{\max} \leq \theta_{\max}^* \). Solving \( \sqrt{2b_1b_2}/(\sqrt{b_1} + \sqrt{b_2}) = \alpha \) and \( \sqrt{2b_1b_2}/(\sqrt{b_1} + \sqrt{b_2} + \alpha) \). Clearly, both values are greater than 1 because \( \sqrt{b_1} \sqrt{b_2} > \alpha \). However, \( \tau \in (0, 1) \), hence we have the following proposition.

**Proposition 5.** A centralized dual-channel supply chain will be feasible for any degree of the manufacturer’s social concern. Proposition 5 shows the feasibility of a centralized dual-channel supply chain, i.e., that positive demands in both channels exist in the centralized scenario for any degree of the manufacturer’s social concern. This is in contrast to the decentralized scenario, for which the manufacturer cannot exhibit CSR above a certain threshold. Hence, channel coordination is essential not only for profit enhancement but also for the performance of a high level of social responsibility.

In the next section, we shall analyse the issue of channel coordination.

### Coordination through all unit quantity discount contracts

Coordination among the channel members is essential to optimize system performance. Thus, a key issue in supply chain management is to develop mechanisms that can align channel members and objectives and coordinate their activities to achieve centralized channel profit. To implement a centralized pricing policy, the manufacturer offers to the retailer that online selling prices will be adjusted according to a centralized policy, as well provides all unit quantity discounts as an incentive to the retailer. The online selling price of the manufacturer under the coordinated scenario is \( p_M - R \), and the discounted wholesale price of the manufacturer is \( w_M^* \). Under this mechanism, the profit function of the retailer and total profit function of the manufacturer are, respectively, as follows

\[
\begin{align*}
\pi_{\text{ret}} &= (p_F - \phi w_F^*)(\theta_1 - b_1p_F + \alpha(p_F - R - p_D)) \quad (34) \\
\pi_{\text{wco}} &= (\phi w_F^* - c)(\theta_1 - b_1p_F + \alpha(p_F - R - p_D)) \\
&+ (p_F - R - c)((1 - \theta)a - b_2(p_F - R)) \\
&+ \alpha(p_F - p_D + R) \\
&+ \tau \left[ \frac{(\theta_1 - b_1p_F + \alpha(p_F - R - p_D))^2}{2b_1} \\
&+ ((1 - \theta)a - b_2(p_F - R) + \alpha(p_F - p_D + R))^2}{2b_2} \right] \\
&+ (\phi w_F^* - c) \left[ \frac{\theta_1 - b_1p_F + \alpha(p_F - R - p_D)}{2b_1} \right] \left[ \frac{((1 - \theta)a - b_2(p_F - R) + \alpha(p_F - p_D + R))^2}{2b_2} \right] \quad (35)
\end{align*}
\]

In a decentralized dual-channel supply chain for a given wholesale price \( w_F^* \) of the manufacturer and price for the online channel \( p_F - R \), \( \pi_{\text{ret}} \) is concave in \( p_F \), hence solving \[ \frac{d\pi_{\text{ret}}}{dp_F} = 0 \], and we get

\[
p_F^* = p_M - R - \frac{\alpha b_1}{2b_1} + \frac{\phi w_F^*}{2b_1} \phi \\
\]

The channel will be coordinated only if the retailer’s self-optimized selling price \( p_F^* \) under the all unit
Thus, the retailer will agree to sell the product at the
centralized retail price if $\varphi = \varphi_{rc}$. However, one may
note that $\varphi_{rc}$ depends on the online selling price ($p_D$) of
the manufacturer. In the second stage, in response to
the retailer’s decision, the manufacturer will optimize
its total profit function. Hence, by solving $v_{nco} = 0$, the online selling price ($p_{nco}$) of the
manufacturer can be determined. In contrast, the
manufacturer has to adjust its online price for channel
coordination. Thus, the adjustment in the online selling
price that the manufacturer will consider, is given by
$R_{co} = (p_{nco} - p_D)$, which yields

$$V_{nco} = (p_{nco} - c)D_{nc} + (p_{nco} - R_{co})D_{nc}$$

$$+ \frac{D_{nc}^+}{2T_1} + \frac{D_{nc}^-}{2T_2}$$

(40)

Thus, profits of the retailer and the manufacturer are
as follows

$$\pi_{rc}^* = \frac{\beta_1(2a\beta_0 + b_1 c\beta_1 - 2 + \gamma) + b_1 c\sigma - a(\sigma + b_0)\gamma}{(b_1 \beta_z + \sigma(1 + 2\alpha))\{a(b_1(2 - \beta)^2 + 4\alpha(1 - \gamma)) + b_1 \beta_z(2 - \gamma)^2\}(2 - 2\beta)}$$

(39)

unit quantity discount can coordinate the supply chain
and allows the manufacturer to earn a positive profit.
Note that the difference in total profit of the manufac-
turer between coordinated and decentralized scenarios
is as follows

$$\pi_{rc}^* - \pi_{rc}^* = \frac{-2\beta_1(2a\beta_0 + b_1 c\beta_1 - 2 + \gamma) + b_1 c\sigma - a(\sigma + b_0)\gamma}{(a(b_1(2 - \beta)^2 + 4\alpha(1 - \gamma)) + b_1 \beta_z(2 - \gamma)^2)(2 - 2\beta)} < 0$$

(41)

Here, $\pi_{rc}^* + \pi_{nco}^* = \pi^*$, which means that the total
channel profit under the proposed contract mechanism
is exactly equal to the total centralized channel profit,
and we have the following proposition.

**Proposition 6.** The all unit quantity discount with the
set of contracts $(R_{co}, \pi_{rc})$ of the manufacturer can
coordinate a socially responsible dual-channel supply
chain. The above analysis shows that our proposed all

Please cite this article in press as: N.M. Modak et al., Corporate social responsibility, coordination and profit distribution in a dual-channel supply
chain, Pacific Science Review (2015), http://dx.doi.org/10.1016/j.pscr.2015.05.001
Thus, comparing the profits of the channel members with their respective decentralized profits, we find that, due to coordination, profits of the retailer are enhanced significantly, which improves the dual-channel supply chain efficiency but fails to provide benefit to the manufacturer. Next, we shall discuss the implementation of the contract with a complementary agreement between the manufacturer and the retailer that not only coordinates the dual-channel supply chain but also ensures a win-win strategy for both members of the chain.

All unit quantity discount with the agreement of franchise fees

In the previous section, we have already shown that an all unit quantity discount contract can coordinate the channel, but it cannot provide a win-win condition for the manufacturer. Thus, for successful implementation of the contract, suppose that the manufacturer charges a franchise fee \( F \) to the retailer. When a franchise fee \( F \) satisfies \( \pi_{\text{m}} - \pi_{\text{r}} \geq \pi_{\text{r}}^* \), the retailer will accept a \((R_{\text{m}}, q_{\text{r}}, F)\) contract, which yields

\[
F \leq \pi_{\text{m}}^* - \pi_{\text{r}}^* = \mathcal{F} \tag{42}
\]

Alternatively, the minimum franchise fee charged by the manufacturer to the retailer is given by

\[
F \geq \pi_{\text{m}}^* - \pi_{\text{r}}^* = \mathcal{F} \tag{43}
\]

Thus, we have the following proposition.

Proposition 7. The all unit quantity discount with the agreement of franchise fees can coordinate a two-level socially responsible dual-channel supply chain and provide a win-win opportunity for the channel members for the franchise fee \( F \) if it satisfies the inequality \( \mathcal{F} \leq F \leq \mathcal{F} \). Proposition 7 suggests that a higher \( F \) benefits the manufacturer, whereas a lower \( F \) benefits the retailer. The value of \( F \) depends heavily on the bargaining power of the retailer in the supply chain. In the next subsection, we shall discuss the outcomes of bargaining.

Determination of franchise fees through bargaining

Bargaining refers to situations where two or more players who have the opportunity to collaborate form a mutual benefit in more than one way. To determine the exact value of franchise fees and profits of respective channel members, we use the generalized asymmetric Nash bargaining solution [33]. Nash proposed a basic framework to construct a negotiation model among players. Suppose the manufacturer and the retailer have bargaining powers of, respectively, \( \gamma \) and \( 1 - \gamma \) (\( \gamma \in (0, 1) \)). Let \( \Delta_m \) and \( \Delta_r \) denote the surplus profit share of the manufacturer and the retailer, respectively. The functional forms of \( \Delta_m \) and \( \Delta_r \) are as follows

\[
\Delta_m(F) = (\pi_{\text{r}}^* - \pi_{\text{m}}^*) = X_m + F \tag{44}
\]

\[
\Delta_r(F) = (\pi_{\text{m}}^* - \pi_{\text{r}}^*) = X_r - F \tag{45}
\]

where \( X_m = \pi_{\text{r}}^* - \pi_m^* \) and \( X_r = \pi_{\text{m}}^* - \pi_r^* \).

The total surplus profit generated through cooperation is equal to \( \Delta_m + \Delta_r = X_m + X_r \). According to generalized asymmetric Nash bargaining, we must maximize the following function

\[
\Delta(F) = \max_{F \geq 0} \gamma \Delta_m(F) + (1 - \gamma) \Delta_r(F) \tag{46}
\]

The equilibrium solution of the above Nash bargaining product can be obtained by solving \( \frac{\partial \Delta(F)}{\partial F} = 0 \) and is found as follows

\[
F^* = \gamma X_r - (1 - \gamma) X_m \tag{47}
\]

Using the bargaining solution of franchise fees, we secure the profit of the manufacturer and the retailer after bargaining as follows

\[
\pi_{\text{m}}^b = \pi_{\text{m}}^* + \gamma(X_m + X_r) \tag{48}
\]

\[
\pi_{\text{r}}^b = \pi_{\text{r}}^* + (1 - \gamma)(X_m + X_r) \tag{49}
\]

Note that, in particular, if all players involved in the bargaining procedure have equal bargaining power, i.e., \( \gamma = 1/2 \), then each and every one obtains an equal share \((X_m + X_r)/2\) of the total surplus. From the above results, we have the following proposition.

Proposition 8. (i) The bargaining outcome of franchise fees depends on channel members’ bargaining power. (ii) The surplus profit of the channel generated through coordination is distributed between the channel members according to the ratio of their bargaining powers.

Thus, the all unit quantity discount contract combined with the adjustment of the online selling price achieves channel coordination but fails to provide benefit to the manufacturer. However, with the agreement of the franchise fee, a win-win opportunity is provided to both members. Finally, through Nash asymmetric
bargaining, the members find their equilibrium profit within the win-win range depending on their bargaining power.

Summary and concluding remarks

In this paper, the analysis of a dual-channel socially responsible supply chain has been conducted. In a Stackelberg setting, the manufacturer, the leader of the channel, exhibits CSR. While formulating the model, we have incorporated only the effect of CSR in the form of consumer surplus in the socially responsible firm's profit function rather than the activities that it performs. The paper explores the pricing decision through both decentralized and centralized channels. The effect of CSR on the successful operation of dual channels is analysed. To coordinate the channel and obtain profit equal to the integrated system, a hybrid coordination mechanism is developed. The coordination mechanism not only coordinates the channel but also achieves a win-win outcome for the channel members. Finally, the manufacturer and the retailer share the surplus profit through a Nash bargaining solution.

This paper makes contributions to many aspects. First, the paper considers the effect of CSR on the dual-channel supply chain. Second, the paper develops decentralized and centralized models considering the effect of CSR on consumer surplus. Third, it examines the effect of CSR on product compatibility and discusses the feasibility of the successful operation of dual channels for both decentralized and centralized supply chains. It is analytically shown that the feasibility of a centralized dual-channel supply chain exists for any degree of the manufacturer's social concern. In the decentralized scenario, the manufacturer cannot exhibit CSR above a certain threshold and still operate the dual channels successfully. Fifth, an all unit quantity discount with the agreement of a franchise fee not only cuts out channel conflict but also provides a win-win opportunity for the channel members. Sixth, through Nash bargaining, the members find their equilibrium profit within the win-win range in the ratio of their bargaining powers. As far as the authors are aware, such a discussion within a single model has not yet been studied for supply chains.

Although the proposed model provides some ideas about a socially responsible supply chain that can be managed in the sense of pure profit maximization, it has some limitations. First, the demand is assumed to be deterministic and linear in price. This model can be extended immediately for uncertain price-dependent demand. Second, the CSR has a great impact on the channel members' pure profits. Under the present model settings, a threshold of CSR for the non-negative pure profit of the manufacturer is identified. However, detailed investigation is required to find a specific range of it. Third, the paper applies a transfer pricing policy for channel coordination and then uses a Nash bargaining product for profit division. Instead, strategic bargaining may be used for the same purpose, as indicated. These variations make the model robust and may discover a variety of characteristics of a socially responsible supply chain. Moreover, manufacturing disruptions due to reliability of the manufacturing system, asymmetric information, supply disruptions, cases of imperfect quality products, environmental issues for production, etc. are neglected here. These may be included in an extension of the proposed model in the future.

Uncited reference


