Chapter 4

Managing Dual-channel supply chains for continuous unit cost decrease and corporate social responsibility

In the recent trends of global business scenario, dual channel supply chain receives significant importance and becomes a hot research and application topic because besides retail channel, an internet channel of the manufacturer has the potential to reduce retailer’s dominance, address different customer segments, gain higher profit margin etc. Some customers prefer purchasing online, while others prefer shopping in retail stores. As a result manufacturers redesign their traditional channel structures by engaging in direct sales to reach different customer segments that cannot be reached by the traditional retail channel, giving birth to the dual channel.

The dazzling growth of e-commerce impels the manufacturers to introduce direct online channels because of sustainability in highly competitive global market. Increasing reach to the customers of geographically diverse locations, reduced cost of searching, reduced time consumption for purchasing through e-channel propel a manufacturer to augment its market coverage and hence market share through internet sale. As a result in Czech Republic 24% of the country’s total turnover generated via online channel in 2010. In 2012, global e-commerce sales topped $1 trillion for the first time in the history [65]. In Western Europe e-commerce spending hits 128 billion euros in 2013, up 14.3% from that in 2012. It is forecasted that e-commerce spending in 2017 will reach 191 billion euros in Western Union a compound annual growth rate about 11% [66]. E-commerce sales in US is increased by 15.8% in 2013 in comparison to that in 2011\(^1\). Thus, it is important for a manufacturer to restructure its traditional brick-and-mortar channel by engaging in direct sales through internet channel because the customers always prefer alternatives to choose the one that is better suited for their needs.

\(^1\)For detail discussion on the growth of e-commerce and its implication in supply chain see the review paper [135]
This chapter analyzes two dual-channel supply chain models in two different sections. First one of them considers unit cost decreasing nature of hi-tech product and the other addresses corporate social responsibility. Main objectives of the chapter is to analyzes how online channel can be managed along with retail channel, that is, how to determine prices, replenishment policies for profit maximization in dual channel supply chain and how product compatibility effects on dual-channel supply chain.

Section 4.1 considers a retail-e-tail channel supply chain, where the retail channel has two echelons. The section discusses the pricing and replenishment issues of a product that experiences continuous unit cost decrease in its short life span. As the product has short life time, the section considers the number of replenishment over the planning horizon is a decision variable along with selling prices. Assuming the manufacturer as the Stackelberg leader, the optimal pricing and replenishment policy is identified. It is found that there is a severe price competition between the retail and online channel and product compatibility has a significant impact on the pricing policy. In particular (i) customers’ higher retail channel preference above a threshold leads to non-coexistence of dual-channel, (ii) the dual-channel is non-profitable for product compatibility outside an interval and (iii) higher or lower retail price in comparison to online price is dependent on product compatibility. Also, the retailer’s higher setup cost may lead to non-existence of online channel. Finally, a profit sharing mechanism through wholesale price adjustment resolves channel conflict.

Section 4.2 introduces a corporate social responsible two-echelon dual-channel supply chain. Besides operating online channel the manufacturer intends to swell stakeholders welfare by exhibiting CSR. The section develops the pricing decisions of decentralized and centralized policy. It also examines the effect of degree of concern of the manufacturer about CSR on product compatibility and discusses feasibility for successful operation dual-channel supply chain. Finally, it demonstrates the issues like channel coordination (through all unit quantity discount with the agreement of franchise fee) and surplus profit division.

### 4.1 Managing dual-channel supply chain for continuous unit cost decreasing product

This section\(^2\) demonstrates a dual-channel supply chain for continuous unit cost decreasing product. Decreasing component price and diminishing demand over time due to introduction of upgraded versions of components are now important characteristics of high-tech industrial market. In high-tech industries such as communication and computers, some component cost is declining at about one percent per week \([19]\). Thus, production and sale one week earlier or later leads to about 1% loss or gain. As a matter of fact, in decreasing unit cost environment the decision maker always remains in searching the appropriate selling price because it has a considerable impact on demand and hence on optimal ordering policy. Various aspects of

\(^2\)This section is based on the paper “Pricing and replenishment policies in dual-channel supply chain under continuous unit cost decrease”, published in *Applied Mathematics Computation*, 256 (2015), 913-929.”
dual-channel supply chain, such as advantages and disadvantages of online channel in addition to brick-and-mortar channel, when to open an online channel, pricing policies, replenishment policies, price competition, retail services, sales effort in retail channel, return policies, etc have been explored extensively in supply chain literature. Interestingly, there is no research till date that has discussed pricing and replenishment policies for the hi-tech products, whose unit costs decrease continuously in their short life span. Hi-tech products have high online compatibility and tech savvy customers that generally considers the specifications of the products through online channels and compare the retail prices with the products in online manufacturers’ suggested retail prices. In such situation there is a need for the manufacturer to identify online price and replenishment/production policy of a product that effectively reduces total channel cost and increases channel profit.

In inventory literature, there are a few models concerning continuous cost changing. Both Buzacott [20] and Erel [45] have proposed two inventory models where the unit cost of the product increases under inflationary situation. Erel [45] has developed the model under the assumption that the unit cost of the product increases in compound nature. Whereas, Buzacott [20] has assumed compound increments of both unit cost and setup cost. Goyal et al. [60] have developed inventory models under decreasing feature of unit cost. Khouja and Goyal [81] have suggested that the model of Buzacott [20] can be used for continuous unit cost change, if the rate of change of unit cost is same as the ordering cost. Erel’s [45] model is also applicable in this purpose. But, if the rate of inflation is less than 10%, then the models provide wrong approximation. Teng and Yang [139] and Teng et al.[138] have developed inventory models under partial backlogging where demand and cost fluctuate over time. In both the models, optimal replenishment policy and optimal purchasing policy have been determined to minimise system running cost. They have claimed that this policy fits for todays high-tech market. Khouja and Goyal’s [81] model may be considered as a special case of Teng and Yang [139] and Teng et al. [138] with constant demand and unit cost dependent holding cost. Concerning unit cost decrement, interested readers may consult the paper of Khouja and Park [83], which provides interesting review of the literature associating it with the existing industrial scenario. Khouja and Park [83] have developed an inventory model to determine optimal operating policy in which the unit cost of the product decreases continuously by a constant percentage. Under the restriction of equal cycle lengths for finite time horizon, they have derived an approximate close form value for the optimal cycle length to minimize system operating cost. Panda [114] has determined the optimal pricing and replenishment policy in a declining time and price sensitive market where the unit cost of the product decreases linearly with time.

The purpose of this section is to determine optimal pricing and replenishment policy in a supply chain while the manufacturer operates an online channel besides retail channel. The manufacturer sales the product through retail and online channels simultaneously. As the product has a short life time and it’s cost decreases continuously with respect to time, demand of the products in both the channels is sensitive with price that results in different selling prices per unit. In conventional inventory models with pricing strategy, the number of price changes is pre-determined, i.e., the times of price changes are known earlier over a finite time horizon. Relaxing this assumption, this section assumes that several replenishments of equal cycle lengths may be done over a finite time horizon and the decision maker has the opportunity to adjust the unit selling prices in each of the replenishment cycle to maximize the profit. Generally, in
decentralized decision making, the retailer decides the number of replenishment cycle that maximizes its profit, whereas, in centralized channel, the manufacturer proposes the replenishment number. Moreover, the section applies negotiation of profit sharing mechanism to determine the optimal wholesale price of the manufacturer that resolves channel conflict. The research reported in this model differs from the prior works is the following aspects. First, A product is considered, whose unit cost decreases continuously with time and becomes obsolete after its finite life time, which the earlier studies have not considered. Second, instead of considering a single replenishment/production cycle, the section considers the system over the finite lifetime of the product. As a result, in addition to selling prices in the channels, the optimal number of replenishment cycles in decentralized and centralized channel are different. The section uses profit sharing mechanism to align the replenishment cycles of both the channel structure. Third, it addresses the effect of product compatibility on the optimal selling prices and optimal order quantity of the channel.

4.1.1 Model formulation and basic analysis

Assume that besides the retail channel, a manufacturer sells a hi-tech product through an online channel. The channel operates over the finite time horizon $L$ in which the retailer replenishes $n$ times after every time interval $T$ such that $nT = L$. As in Khouja and Park [83], Khouja and Goyal [81], Panda [114], the unit cost of the product decreases continuously with respect to time at a rate $c(t) = u_1 - u_2 t$, $t \in (0, u_1/u_2)$. $u_1$ is the introductory unit cost of the product and $u_2$ is the unit cost’s time sensitive parameter. It is quite reasonable to assume that $u_1/u_2 > L$, i.e., the unit cost of the product is positive over the planning horizon $L$. In the $i$th ($i=1,2,\ldots,n)$ replenishment cycle the demands in the retail and direct channels are linear in unit selling prices ([149], [153]) and are of the forms

$$D^r = k_1 a - b_1 p_r^i + r_1 (p_r^i - p_d^i), \quad i = 2, \ldots, n$$

and

$$D^d = (1 - k_1) a - b_2 p_d^i + r_2 (p_r^i - p_d^i), \quad i = 1, 2, \ldots, n.$$ 

Where, $a > 0$ is the market potential. $k_1$, $(0 \leq k_1 \leq 1)$ is the compatibility of the product with the retail channel. Product compatibility is how the consumer perceives the product into the customers lifestyle choices. When the product closely matches the individuals needs, wants, beliefs, values, and consumptions patterns of the customers, it can be considered highly compatible with the consumers’ choice. The percentage of the primary demand $a$ that goes to the retail channel is $k_1$ and when the value of $k_1$ is greater, the product’s compatibility with the retail channel is larger and more consumers would purchase the product from the retail channel. Computer-related products, books, information, magazines, and digital products have more compatibility with the direct channel than the products like water, rice, gasoline, and milk. Here, $b_1 > 0$ and $b_2 > 0$ are the price sensitivity factors in retail channel and online channel respectively. $r_1 > 0$ and $r_2 > 0$ are the cross-price effects, which reflect the degree of price competition between the channels. According to Yan [149], it is assumed that the price sensitivity factors and cross-price effect in the retail channel and in the online channel are equal, i.e., $b_1 = b_2 = b$ and $r_1 = r_2 = r$. It is quite reasonable that $b > r$, i.e., the effect of price sensitivity of a channel is greater than cross-price effect. Thus, in the $i$th ($i=1,2,\ldots,n)$
replenishment cycle, the demands of the product in the retail channel and in the online channel are respectively as follows.

\[ D^r = k_1a - (b + r)p^r_i + rp^d_i, \quad i = 1, 2, \ldots, n \]  

(4.1)

and

\[ D^d = (1 - k_1)a - (b + r)p^d_i + rp^r_i, \quad i = 1, 2, \ldots, n. \]  

(4.2)

Quite often in the ith (i=1,2,......, n) replenishment cycle, the selling price in the online channel is higher than the manufacturer's wholesale price, i.e., \( p^d_i > w_i, \quad i = 1, 2, \ldots, n \). Otherwise, the retailer purchases the product through the online channel rather than from the manufacturer. Also, for profitability of the retailer, in the ith (i=1,2, ....., n) replenishment cycle, the selling price of the retailer is higher than the manufacturer’s wholesale price, i.e., \( p^r_i > w_i, \quad i = 1, 2, \ldots, n \). Under this model setting, our objective is to find out optimal decisions in decentralized and decentralized systems.

4.1.1.1 Decentralized policy

Interactions between the manufacturer and the retailer are considered as a Stackelberg game. The manufacturer acts as the stackelberg leader of the channel and the retailer is it’s follower. In this way, the manufacturer first announces the wholesale price and selling price of the product in the online channel. Based on the manufacturer’s decision, the retailer determines the retail price and orders \( n^r \) replenishments to the manufacturer each of length \( T^r \) over \( L \) such that \( n^rT^r = L \). In the ith (i=1,2,......, n) replenishment cycle, profit function of the retailer is

\[ \pi^{r/ds}(p^r_i) = D^r_iT^rw_i - D^r_iT^r(s^r - hp_i^r T^r - T^r(1 - s^r)), \quad i = 1, 2, \ldots, n^r \]  

(4.3)

In the decentralized decision making, the profit function of the manufacturer is the sum of two profit functions. One is for the quantities that it sold through the retail channel and the other is for the online channel. The profit function of the manufacturer, in the ith replenishment cycle, is

\[ \pi^{m/ds}(w_i, p^d_i) = D^r_iT^rw_i - D^r_iT^r c(i - 1)T^r + D^d_iT^r p^d_i - D^d_iT^r \frac{h^m T^r}{2} - D^d_iT^r c(i - 1)T^r - s^m, \quad i = 1, 2, \ldots, n^r \]  

(4.5)

In equation (4.5) first and second terms represent the manufacturer’s net profit from the retail channel and the remaining terms represent the manufacturer’s profit from the online channel. Total profit of the manufacturer over the planning horizon is
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\[ \pi^{m/ds}(w_i, p_i^{d/ds}) = \sum_{i=1}^{n_r} \pi_i^{m/ds}(w_i, p_i^{d/ds}) = \frac{L}{n_r} \sum_{i=1}^{n_r} \left[ D_i^r(w_i - c[i(1 - L)]^n) \right. \\
\left. + D_i^d(p_i^{d/ds} - c[i(1 - L)]^n) - n^r s^n \right] \] \tag{4.6}

Total profit of the retailer in the planning horizon is a function of \( p_{i}^{r/ds}, i = 1, 2, \ldots, n^r \) and \( n^r \), where \( n^r \) is a discrete variable. On the other hand, the profit function of the manufacturer is a function of \( w_i \) and \( p_i^{d/ds}, i = 1, 2, \ldots, n^r \). Since the manufacturer is the Stackelberg leader of the channel, it first determines the wholesale price and selling price of the product through the online channel, where it assumes that the retailer will consider multiple replenishment cycles. As a follower, the retailer then sets the retail price and number of replenishment cycles in the time horizon. Now, for the concavities of the profit functions the proposition follows.

**Proposition 4.1**

(i) For given \( n^r \), the manufacturer’s profit function over \( L \) is a concave function of \((w_i, p_i^{d/ds}), i = 1, 2, \ldots, n^r \) and optimal wholesale price and retail price in the online channel are

\[ w_i^r = \frac{a[i + k_1 b]}{2((b + r)^2 - r^2)} - \frac{Lh^r}{4n^r} + \frac{1}{2} \left[ (i - 1) \frac{L}{n^r} \right], \quad i = 1, 2, \ldots, n^r \] \tag{4.7}

\[ p_i^{d/ds} = \frac{a[i + (1 - k_1) b]}{2((b + r)^2 - r^2)} + \frac{Lh_i^m}{4n^r} + \frac{1}{2} \left[ (i - 1) \frac{L}{n^r} \right], \quad i = 1, 2, \ldots, n^r \] \tag{4.8}

(ii) For given \( n^r \) and the manufacturer’s optimal \((w_i, p_i^{d/ds}), i = 1, 2, \ldots, n^r \) pair, the retailer’s profit function is concave over \( L \) and it’s optimal retail price is

\[ p_{i}^{r/ds}(w_i, p_i^{d/ds}) = \frac{w_i}{2} + \frac{r}{2(b + r)} \frac{p_i^{d/ds}}{2} + \frac{a k_1}{2(b + r)} + \frac{L h_i^r}{4n^r}, \quad i = 1, 2, \ldots, n^r \] \tag{4.9}

\[ p_i^{r/ds} = \frac{a}{2(b + 2r)} \left[ \frac{r}{b} + \frac{(3b + 4r) k_1}{2(b + r)} \right] + \frac{L}{8n^r} \left[ h_i^r + \frac{r}{b + r} \right] \\
+ \left[ \frac{b + 2r}{4(b + r)} \right] c \left[ (i - 1) \frac{L}{n^r} \right], \quad i = 1, 2, \ldots, n^r \] \tag{4.10}

**Proof:** See appendix.

Using the optimal selling prices of the retail channel and online channel, optimal order quantities, optimal profits in the ith replenishment cycle and over the entire time horizon can be found and are presented in table-4.1. Note from (4.9) that \( \partial p_{i}^{r/ds}(w_i, p_i^{d/ds})/\partial p_i^{d/ds} = r/[2(b + r)] > 0 \), i.e., in the ith replenishment cycle, the optimal selling price of the retailer decreases when the selling price of the online channel decreases. Further, \( \partial p_{i}^{r/ds}(w_i, p_i^{d/ds})/\partial w_i = 1/2 > 0 \), i.e.,
the optimal retail price is also decreasing with decreasing optimal wholesale optimal of the manufacturer. As the selling price of the retailer decreases with decreasing \( p^{d/ds}_i \) and \( w_i \) in the \( i \)th replenishment cycle, the manufacturer has the option to control the retailer’s selling price by introducing an online channel, where it sets the wholesale price \( w_i \) and unit selling price \( p^{d/ds}_i \). This result is quite similar to Chiang et al.\[25\]. Also, observe that

\[
\frac{\partial \pi^{r/ds}(p^{r/ds}_i)}{\partial p^{d/ds}_i} = \frac{L}{n'} \sum_{i=1}^{n'} \left[ D^{r/ds}_i \frac{r}{2(b+r)} + \frac{r}{2} \left( p^{r/ds}_i - w_i - \frac{Lh^r}{2n'^{r}} \right) \right] > 0.
\]

That is, the optimal total profit of the retailer over \( L \) increases with increasing optimal online selling price. The intuitive reason is straightforward. When the online selling price increases, it forces some demands of the online channel to switch to the traditional retail channel. As a result the retailer’s profit increases. On the other hand,

\[
\frac{\partial \pi^{r/ds}(p^{r/ds}_i)}{\partial w_i} = -\frac{L}{n'} \sum_{i=1}^{n'} \left[ D^{r/ds}_i \frac{r}{2(b+r)} + \left( \pi^{r/ds} - w_i - \frac{Lh^r}{2n'^{r}} \right) \right] \frac{(b+r)}{2} < 0.
\]

That is, the optimal total profit of the retailer decreases with increasing wholesale price of the manufacturer and it is quite obvious. However, the optimal selling prices and wholesale prices satisfy the following proposition.

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Over the sales season $L$

<table>
<thead>
<tr>
<th>Order quantity</th>
<th>Manufacturer</th>
<th>Retailer</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{a L}{2} \left[ 1 - \frac{bk_1}{2(b+r)} \right] - \frac{L b}{8n^2} \left[ \frac{(2b+3r)m}{16n^2} + h^r \right] - \frac{b(b+4r)L}{32n^2} \left( 1 - \frac{1}{n^r} \right) \left( u_1 - u_2 L \right)$</td>
<td>$\frac{a L}{2} \left[ 1 - \frac{bk_1}{2(b+r)} \right] - \frac{L b}{8n^2} \left[ \frac{(2b+3r)m}{16n^2} + h^r \right] - \frac{b(b+4r)L}{32n^2} \left( 1 - \frac{1}{n^r} \right) \left( u_1 - u_2 L \right)$</td>
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</tr>
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</table>

<table>
<thead>
<tr>
<th>Profit</th>
<th>Manufacturer</th>
<th>Retailer</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-\frac{a L}{2} \left( 2(b+r) - 2(2 - k_1)bk_1 + \frac{b^2k_1^2}{2} \right) - \frac{L^2 (Ah^r - Bh^m)}{32n^2}$</td>
<td>$-\frac{a L}{2} \left( 2(b+r) - 2(2 - k_1)bk_1 + \frac{b^2k_1^2}{2} \right) - \frac{L^2 (Ah^r - Bh^m)}{32n^2}$</td>
<td>$-\frac{a L}{2} \left( 2(b+r) - 2(2 - k_1)bk_1 + \frac{b^2k_1^2}{2} \right) - \frac{L^2 (Ah^r - Bh^m)}{32n^2}$</td>
</tr>
</tbody>
</table>

$\textbf{Table 4.1:}$ Optimal solutions in decentralized and centralized decision making

(Where, $c \left[ (i - 1) \frac{L}{n^r} \right] = C_i$, $A = \left[ rh^m - (b+r)h^r \right]$ and $B = \left[ \frac{(2b+3r)^2 - r^2}{b+r} (h^m - (b+r)h^r) \right]$)
Proposition 4.2  In the planning horizon \( L \), for given \( n^r \), (i) \( p_1^{r/ds} > p_2^{r/ds} > \ldots > p_{n^r}^{r/ds} \), (ii) \( p_1^{d/ds} > p_2^{d/ds} > \ldots > p_{n^r}^{d/ds} \), and (iii) \( w_1 > w_2 > \ldots > w_{n^r} \).

Proof: See appendix.

Proposition 4.2 indicates that the optimal selling prices in the channels and optimal wholesale price of the manufacturer decrease with the increasing replenishment number. That is, the optimal prices in the \( i \)th cycle are higher than those of \((i+1)\)th, \((i=1, 2, ..., n^r - 1)\) cycle. The reasonable explanation is as follow. As the product has limited lifetime, its unit cost decreases continuously with time. So, the wholesale price of the product decreases because it depends on the unit cost of the product. As a result the selling prices in the channels also decrease because the channel members are willing to sell more product by lowering the selling prices before the products obsolesce.

As assumed, the optimal pricing strategy of table-4.1 is acceptable to the channel members only when \( p_i^{r/ds} > w_i \) and \( p_i^{d/ds} > w_i \) for \( i = 1, 2, ..., n^r \). Now, \( p_i^{r/ds} > w_i \) if

\[
 k_1 > \frac{b}{a} \left[ (i-1) \frac{L}{n^r} \right] - \frac{L(rh^m + 3(b + r)h^r)}{2an^r} \tag{4.11}
\]

Also, \( p_i^{d/ds} > w_i \) if

\[
 k_1 < 0.5 + \frac{L(b + 2r)(h^r + h^m)}{4an^r} = k_1^{max}, \quad (say) \tag{4.12}
\]

Right hand side of (4.11) depends on \( n^r \) and \( i \), whereas right hand side of (4.12) is dependent on \( n^r \) but independent of \( i \). Since, the system operates over the planning horizon \( L \), where \( n^r \) replenishment cycles are made, the maximum value of the right hand side of (4.11) is at \( i = 1 \), i.e.,

\[
 \frac{bh_1}{a} - \frac{L(rh^m + 3(b + r)h^r)}{2an^r} = k_1^{min}, \quad (say)
\]

and the Lemma follows.

Lemma 4.1  For given \( n^r \), the manufacturer will participate in the dual-channel for a product that’s unit cost decreases continuously over \( L \) if \( k_1 \in (k_1^{min}, k_1^{max}) \).

Lemma 4.1 indicates that the customer’s channel preference is one of the determining factors for operating an online channel besides the traditional retail channel. When \( k_1 < k_1^{min} \), the retailer can’t do business because it’s selling price is less than the manufacturer’s wholesale price. On the other hand, for \( k_1 > k_1^{max} \), the manufacturer can’t set the optimal selling price as the online price. Note that, the model considers the maximum of lower threshold of product’s compatibility with the retail channel. As there are multiple replenishment cycles over \( L \), the retailer’s optimal selling prices are profitable in these cycles only when the lower limit of \( k_1 \) is maximum among all of these values. Online selling price is higher than the wholesale price. It does not ensure that the retailer will participate in the profit making retail-e-tail channel for a product that experiences continuous unit cost decrease over \( L \) because of its setup cost. In the
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ith (i=1, 2, ..., \(n^r\)) replenishment cycle, the retailer will participate in the dual-channel only when its profit is positive, i.e., \(\pi_i^{r/ds} > 0\), i.e., if

\[
k_1 > \frac{1}{a} \left[ 4 \sqrt{\frac{n^r(b + r)s^r}{L}} + \frac{L[(b + r)h^r - rh^m]}{2n^r} + bc[(i - 1) \frac{L}{n^r}] \right] = k_{i1}^r, \quad i = 1, 2, \ldots, n^r
\]

But, the value of \(k_{i1}^r\) depends on \(n^r\) and \(i\) and attains its maximum value when \(i=1\). That is

\[
k_1 = \frac{1}{a} \left[ 4 \sqrt{\frac{n^r(b + r)s^r}{L}} + \frac{L[(b + r)h^r - rh^m]}{2n^r} + bu_1 \right] \quad (4.13)
\]

On the other hand, the manufacturer will operate the online channel until the demand in the online channel is positive\(^3\) for \(i = 1, 2, \ldots, n^r\) i.e., \(D_i^{r/ds} > 0\), if,

\[
k_1 < \frac{2(b + r)}{2b + r} - \frac{[(2(b + r)^2 - r^2)h^m - r(b + r)h^r]L}{2an^r(2b + r)} - \frac{b(2b + 3r)}{(2b + r)a} \left( i - 1 \right) \frac{L}{n^r} = k_{i1}^m
\]

\(k_{i1}^m\) also depends on \(n^r\) and \(i\) and attains its minimum value when \(i=1\), i.e.,

\[
k_1 = \frac{2(b + r)}{2b + r} - \frac{[(2(b + r)^2 - r^2)h^m - r(b + r)h^r]L}{2an^r(2b + r)} - \frac{b(2b + 3r)u_1}{(2b + r)a} \quad (4.14)
\]

Eq.(4.13) and (4.14) suggest when the customers' retail channel preference lies in between \(k_1\) and \(\overline{k}_1\), the manufacturer can successfully operate a profitable dual-channel. Notice that

\[
k_1 - k_1^{min} = \frac{1}{a} \left[ 4 \sqrt{\frac{n^r(b + r)s^r}{L}} + \frac{2L(b + r)h^r}{n^r} \right] > 0
\]

and

\[
k_1 - k_1^{max} = \frac{(2b + 3r)(a - 2bu_1)}{2a(2b + r)} + \frac{(2b^2 + 7br + 4r^2)h^r - (2b^2 + 3br + r^2)h^m}{4an^r(2b + r)} > 0.
\]

That is, \(\max\{k_1, k_1^{min}\} = k_1\) and \(\min\{k_1, k_1^{max}\} = k_1^{max}\), i.e., \((k_1, k_1^{max})\) are nested in \((k_1^{min}, \overline{k}_1)\). This result is quite obvious because the manufacturer must operate the online channel only when both the channels are profitable. The retail channel of the manufacturer will be profitable not only for the retail price is equitable to the wholesale price but also reasonably higher because it must over compensate the costs related entire retail channel running cost. It is possible only when the customers’ retail channel preference is higher than \(k_1\) but lower than \(k_1^{max}\). Similarly, the retailer will participate in the dual-channel and will make profit if \(k_1 > k_1\). Thus, the proposition follows.

\(^3\)Unlike the retailer’s case, for positive profit of the manufacturer the model considers the positive online demand because the setup cost for operating the online channel is included in the system from where the manufacturer supplies the product. It is assumed that the setup cost of the manufacturer consists of setup cost for manufacturing the product and setup cost for operating the online channel because of the concentration only on overall profitability (profit from the retail channel and profit from the online channel) of the manufacturer.
Proposition 4.3  Over the planning horizon \( L \), for given \( n^r \), the manufacturer can operate a profitable retail-online channel when the customers’ retail channel preference \( k_1 \in (k_{11}, k_1^{\text{max}}) \). However, as far as the competition between the retail channel and online channel is concerned, the proposition follows.

Proposition 4.4  For given \( n^r \), in the \( i \)th \((i = 1, 2, ..., n^r)\) replenishment cycle the optimal retail price is higher than online selling price if \( k_1 \in (k_{11}, k_1^{\text{dc}}) \), while reverse may be noted for \( k_1 \in (k_1^{\text{dc}}, k_1^{\text{max}}) \).

Proof: See appendix.

Proposition 4.4 suggests the customer’s channel preference intensifies the channel competition. To attract more customers, the retailer sets the retail price less than online selling price when the customers’ preference for the retail channel is below a threshold of the products channel compatibility. On the other hand, the retailer sets the retail price greater than the online selling price for gaining higher profit margin when \( k_1 \) is above the threshold \( k_1^{\text{dc}} \). Thus, before making any pricing decision the channel members must consider the customers’ channel preference as a deciding factor. This result can be further justified as follows.

\[
\frac{dp_i^d}{dk_1} = -\frac{ab}{2[(b + r)^2 - r^2]} < 0
\]

\[
\frac{dp_i^r}{dk_1} = \frac{a(3b + 4r)}{4(b + r)(b + 2r)} > 0
\]
i.e., selling price in the retail channel increases and online selling price decreases when the customers’ preference for the retail channel increases. As more profit gains are the objectives of the channel members, the channel members decide about the selling prices based on the product compatibility. Also note that

\[
\frac{dw_i}{dk_1} = \frac{ab}{2[(b + r)^2 - r^2]} > 0
\]
i.e., the wholesale price of the manufacturer increases with the increment of the customers’ retail channel preference. This is quite reasonable. Since, the retailer sets higher selling price because of customers’ higher preference for the retail channel, the manufacturer also increases its wholesale price in order to acquire some margins from the retailer’s gained profit. However, the channel members can apply such strategy and counter strategy until the product compatibility with the retail channel lies in \((k_{11}, k_1^{\text{max}})\). It is very interesting to note that \( \frac{dk_1}{ds^r} = (2/a)\sqrt{(b + r)n^r/Ls^r} > 0 \), i.e., the lower threshold of profitable product compatibility with the retail channel is directly proportional to the retailer’s set up cost.
As such in the $i$th replenishment cycle $k_{dc}^{i} < k_1$ if

\[ s^r > \frac{L}{n^r(b+r)} \left[ \frac{ak_{dc}^{i}}{4} - \frac{L[(b+r)h^r - rh^m]}{8n^r} - \frac{bu_1}{4} \right]^2 \]  

(4.15)

In such case, $k_{dc}^{i} < k_1 < k_1^{\text{max}}$, i.e., the lower limit of customers retail channel preference is higher than the upper limit of product compatibility in between which the manufacturer can set higher online price in comparison to the retail price. Obviously in this case the manufacturer can not run the online channel because it cannot set the profitable online selling price. Thus the manufacturer can operate the profitable online only when the retailer’s set up cost is reasonably low and the proposition follows.

**Proposition 4.5** The manufacturer cannot operate the online channel if

\[ s^r > \frac{L}{n^r(b+r)} \left[ \frac{ak_{dc}^{i}}{4} - \frac{L[(b+r)h^r - rh^m]}{8n^r} - \frac{bu_1}{4} \right]^2 \]

Notice that $d(\sum LD_i^{d/ds}/n^r)/dk_1 = L\alpha/4 > 0$ and $d(\sum LD_i^{d/ds}/n^r)/dk_1 = -L\alpha(2b + r)/[4(b + r)] < 0$, i.e., over the planning horizon $L$, the optimal order quantity of the retail channel increases, whereas the optimal order quantity of the online channel decreases with the increment of the customer’s retail channel preference. This result is quite obvious because more customers will buy the product from the retail channel. As a result, the retailer’s order quantity will increase. Interestingly, $dQ^{ds}/dk_1 = -Lab/[4(b + r)] < 0$, i.e., the total order quantity over $L$ decreases with increasing product compatibility with the retail channel. Total order quantity is the sum of product sold through the retail channel and online channel. The decrement of sold product through the online channel with increment of the customer’s retail channel preference is higher than the product selling through the retail channel. As a result, the volume of total order quantity decreases with the increment of the customers retail channel preference.

Observe that

\[ \frac{d}{dn^r} \left( \frac{1}{n^r} \sum_{i=1}^{n^r} w_i \right) = \frac{L}{4n^r^2} (h^r - u_2) < 0 \quad \text{if} \quad h^r < u_2 \]

That is, average wholesale price of the manufacturer over the planning horizon $L$ decreases as the retailer’s number of replenishments increases. This is quite reasonable because the unit cost of the product decreases continuously over $L$. As $n^r$ increases, the manufacturer supplies the product to the retailer for shorter replenishment cycles. Since, the wholesale of the manufacturer is dependent on the product’s unit cost, multiple replenishments lead to decrement of average wholesale price. Now

\[ \frac{d}{dn^r} \left( \frac{1}{n^r} \sum_{i=1}^{n^r} (b_i^{d/ds}) \right) = -\frac{L}{4n^r^2} (h^m + u_2) < 0 \]
\[
\frac{d}{dn^r} \left( \frac{1}{n^r} \sum_{i=1}^{n^r} p_i r/ds \right) = -\frac{L}{8n^r} \left[ h^r + \frac{r}{b + r} h^m + u_2 \frac{(b + 2r)}{b + r} \right] < 0
\]

That is, average retail price and online price over \( L \) decrease as the number of replenishment increases. The selling prices in the channels are dependent on the wholesale price and unit cost of the product. Unit cost of the product decreases with increasing \( n^r \). As a result, the selling prices also decrease. On the other hand,

\[
\frac{dQ^{ds}}{dn^r} = \frac{L^2b}{8n^r} \left[ h^r + \left( \frac{2b + 3r}{b + r} \right) h^m + u_2 \frac{(3b + 4r)}{b + r} \right] > 0
\]

That is, optimal order quantity of the channel increases as the number of replenishment increases. As the selling prices in the channels decrease for increasing \( n^r \), the customers buy more and hence \( Q^{ds} \) increases over \( L \). As far as the optimal number of replenishment over the time horizon \( L \) is concerned, the proposition follows.

Proposition 4.6  
Over the selling season \( L \) the retailer’s profit is maximum for \( n^r_0 \) number of replenishments, where \( n^r_0 \) is given by

\[
n^r_0 = \begin{cases} \lfloor n_0 \rfloor & \text{if } \pi^{r/\text{ds}}([n_0]) > \pi^{r/\text{ds}}([n_0] + 1) \\ \lfloor n_0 \rfloor + 1 & \text{otherwise} \end{cases}
\]

and

\[
n_0 = (-d + \sqrt{d^2 + b^3})^{\frac{1}{3}} + (-d - \sqrt{d^2 + b^3})^{\frac{1}{3}}
\]

where \( b = L^2(A-bu_2)2a_k - b(2u_1 - u_2 L)/96(b+r)s^r \) and \( d = L^3[3A(A - 2bu_2) + 2b^2u_2^2]/192(b+r)s^r \); \([n_0]\) denotes largest integer not greater than \( n_0 \).

**Proof:** See appendix.

Proposition-4.6 suggests that, based on the manufacturer’s wholesale price and online selling price, the retailer chooses \( n^r_0 \) number of replenishment over \( L \) that maximizes its profit. As the manufacturer is dependent on the retail channel, it will follow the retailer’s replenishment policy and based on it the manufacturer will apply the online pricing schedule.

### 4.1.1.2 Centralized policy

The traditional centralized policy views the system as single entity where there is one central planner who makes all decisions so as to maximize the profit of the whole system. The centralized policy determines suitable selling prices of the product for both retail and direct channel as well as production cycle so as to maximize the total system profit. The relevant costs considered for the retailer and the manufacturer in this policy are similar to those in the decentralized replenishment policy. Profit function of the integrated channel in the ith replenishment cycle is

\[
\pi^c_i = D^c_i T^c(p^c_i - \frac{h^c T^c}{2} - c[(i-1)T]) + D^d T^c(p^d_i - \frac{h^m T^c}{2} - c[(i-1)T]) - s^r - s^m \quad (4.17)
\]
The optimal solution of (4.17) is presented in the following proposition.

**Proposition 4.7** (i) For given $n$, the profit function of the integrated channel over $L$ is a concave function of $(p_{rc}^i, p_{dc}^i), i = 1, 2, \ldots, n$ and the optimal selling price of the product for retail and direct channel are respectively

$$p_{rc}^i = \frac{a(r + bk_1)}{2b(b + 2r)} + \frac{b^r L}{4n^c} + \frac{1}{2}c \left( (i - 1) \frac{L}{n^c} \right), i = 1, 2, \ldots, n$$  \hspace{1cm} (4.18)

and

$$p_{dc}^i = \frac{a[r + b(1 - k_1)]}{2b(b + 2r)} + \frac{h^m L}{4n^c} + \frac{1}{2}c \left( (i - 1) \frac{L}{n^c} \right), i = 1, 2, \ldots, n$$  \hspace{1cm} (4.19)

**Proof:** See appendix.

Using the optimal values of $p_{rc}^i$ and $p_{dc}^i$, the demand of the product in retail and direct channel in the $i$th, $i = 1, 2, \ldots, n$, replenishment cycle and multiplying these with cycle length one can get the amount of quantity sold through retail and online channel respectively which are displayed in table-4.2. Also, the order quantity that the manufacturer produces, is equal to the product of total demand and cycle length, the profit of the integrated channel in $i$th, $i = 1, 2, \ldots, n$, replenishment cycle as well as over the planning horizon are displayed in table-4.2. Now the optimal values of the centralized decision satisfy the following properties.

**Proposition 4.8** In the centralized channel over the planning horizon $L$, for given $n$, (i) $p_{1c}^i > p_{2c}^i > \ldots > p_{nc}^i$, (ii) $p_{1c}^{dc} > p_{2c}^{dc} > \ldots > p_{nc}^{dc}$

**Proof:** See appendix.

This result is quite similar to the decentralized decision making process. As the unit cost of the product decreases continuously, higher number of replenishment always leads to decrement of selling prices in both the channels. Now, the manufacturer would be interested on opening the online channel only when online channel demand in the $i$th, $i = 1, 2, \ldots, n$, replenishment cycle is positive, i.e., $D_{dc}^i > 0$, which, after simplification, yields

$$k_1 < 1 - \left[ \frac{b + r h^m - rh^r L}{2a n^c} \right] - \frac{b}{a} c \left( (i - 1) \frac{L}{n^c} \right) = k_{i, max}^c$$

where $k_{i, max}^c$ depends on $n$ and $i$ and it attends minimum value for $i = 1$ i.e.,

$$k_1 < 1 - \left[ \frac{(b + r h^m - rh^r L)}{2a n^c} \right] - \frac{b u_1}{a} = k_i^c$$.  \hspace{1cm} (4.20)

Although in centralized channel the manufacturer and the retailer co-operate and take decision jointly, the product compatibility has an impact on the manufacturer’s decision for opening the online channel. In the $i$th replenishment cycle, if the customers’ retail channel preference is higher than threshold $k_i^c$, then the manufacturer’s decision for opening an online channel is not profitable because its online demand is negative in such case. On the other hand, there must be a competition between the retail channel and the online channel in the centralized process though the channel members co-operate. The
### Centralized scenario

<table>
<thead>
<tr>
<th></th>
<th>Direct channel</th>
<th>Retail channel</th>
</tr>
</thead>
<tbody>
<tr>
<td>Selling price</td>
<td>( \frac{a[r + b(1-k_1)]}{2(b+2r)} + \frac{b^mL}{4n^c} + \frac{1}{2} C_{ci} )</td>
<td>( \frac{a(r + bk_1)}{2(b+2r)} + \frac{b^rL}{4n^c} + \frac{1}{2} C_{ci} )</td>
</tr>
<tr>
<td>Demand</td>
<td>( \frac{1}{2} \left[ a(1-k_1) - \frac{(b+r)h^mL}{2n^c} + \frac{rh^rL - bC_{ci}}{2n^c} \right] )</td>
<td>( \frac{1}{2} \left[ ak_1 - \frac{(b+r)h^rL}{2n^c} + \frac{rh^mL - bC_{ci}}{2n^c} \right] )</td>
</tr>
<tr>
<td>Channel profit</td>
<td>( \frac{L}{2n^c} \left[ \frac{r+b-2bk_1(1-k_1)a^m}{2(b+2r)} - \frac{ak_1h^rL}{2n^c} - \frac{a(1-k_1)h^mL}{2n^c} + \frac{(b+r)(h^r + h^m) - 2rh^r h^m}{8n^c} \right] )</td>
<td>( \frac{L}{4n^c} \left[ \left( a - \frac{b(h^r + h^m)L}{2n^c} \right) C_{ci} - bC_{ci} \right]^2 - s^m - s^r )</td>
</tr>
</tbody>
</table>

Over the sales season \( L \)

| Order quantity          | \( \frac{L}{2} \left[ a - \frac{bkL(h^m + h^r)}{2n^c} - 2b \left( u_1 - \frac{u_2}{2} \left( 1 - \frac{1}{n^c} \right) \right) \right] \) | \( \frac{L}{2} \left[ \frac{r+b-2bk_1(1-k_1)a^m}{2(b+2r)} - \frac{ak_1h^rL}{2n^c} - \frac{a(1-k_1)h^mL}{2n^c} + \frac{(b+r)(h^r + h^m) - 2rh^r h^m}{8n^c} \right] \) |
| Channel profit          | \( \frac{L}{2} \left[ \frac{r+b-2bk_1(1-k_1)a^m}{2(b+2r)} - \frac{ak_1h^rL}{2n^c} - \frac{a(1-k_1)h^mL}{2n^c} + \frac{(b+r)(h^r + h^m) - 2rh^r h^m}{8n^c} \right] \) | \( \frac{L}{2} \left[ \frac{r+b-2bk_1(1-k_1)a^m}{2(b+2r)} - \frac{ak_1h^rL}{2n^c} - \frac{a(1-k_1)h^mL}{2n^c} + \frac{(b+r)(h^r + h^m) - 2rh^r h^m}{8n^c} \right] \) |
|                         | \( + \frac{L}{2} \left[ u_1^2 - u_1 u_2 L(1 - \frac{1}{n^c}) + \frac{u_2^2 L^2}{6} (1 - \frac{1}{n^c})(2 - \frac{1}{n^c}) - n^c s^m - n^c s^r \right] \) | \( + \frac{L}{2} \left[ u_1^2 - u_1 u_2 L(1 - \frac{1}{n^c}) + \frac{u_2^2 L^2}{6} (1 - \frac{1}{n^c})(2 - \frac{1}{n^c}) - n^c s^m - n^c s^r \right] \) |

**Table 4.2:** Optimal solutions in centralized decision making

(Where, \( c \left[ (i-1) \frac{L}{n^c} \right] = C_{ci} \), \( A = [rh^m - (b+r)h^r] \) and \( B = \frac{2(b+r)^2 - r^2 h^m - r(b+r)h^r}{b+r} \))
channel members take decision jointly but the market potential remains same. When the manufacturer operates an online channel, some customers switches to the online channel. As a result the retailer’s demand decreases and it earns less profit. Besides the selling prices of the retail and online channel, the customers’ channel preference determines the divisions of potential market demand. So, like the decentralized decision making process, here also the selling prices of the retail channel may be higher than the online channel, i.e., \( p_i^c > p_i^d \), \( i = 1, 2, \ldots, n^c \), i.e., if

\[
k_1 > 0.5 - \frac{(b + 2r)(h^r - h^m) L}{4an^c}
\]

(4.21)

Also, note that for any \( i = 1, 2, \ldots, n^c \)

\[
\frac{c_i}{k_1} - k_1 = \frac{1}{2} - \frac{bu_1}{a} + \frac{(b + 4r)h^r - (3b + 4r)h^m}{4an^c} > 0
\]

(4.22)

Thus from (4.25), (4.26) and (4.27), the proposition follows.

**Proposition 4.9** For given \( n^c \), the manufacturer will operate an online channel if \( k_1 \in (0, \overline{k_1}) \). The online selling price is higher than retail price for any \( k_1 \in (0, k_1^1) \) and the retail price is higher than the online price for any \( k_1 \in (k_1^0, \overline{k_1}) \).

Proposition 4.9 demonstrates that, when the channel members cooperate and take decision jointly, the manufacturer’s decision to open an online channel is profitable only when the customers’ retail channel preference is below the threshold \( \overline{k_1} \). Interestingly below this threshold of the product compatibility there exists a price competition between the retail channel and the online channel. If the customers’ retail channel preference is in \( (0, k_1^1) \), then the online price will be higher than the retail price and the reverse is set for \( k_1 \in (k_1^0, \overline{k_1}) \). Thus, for a profitable centralized retail-online channel, the channel will set the selling prices according to the customers’ channel preference.

Observe that \( dQ_{dc}/dk_1 = La'/2 > 0 \) and \( dQ_{dc}/dk_1 = -La'/2 < 0 \), i.e., in centralized channel over the time horizon \( L \), optimal order quantity in the retail channel increases, whereas it decreases in the online channel when the customers’ retail channel preference increases. This is quite obvious. But \( dQ_{dc}/dk_1 = dQ_{rc}/dk_1 + dQ_{dc}/dk_1 = 0 \), i.e., customers’ channel preference has no overall impact on the order quantity when the channel members cooperatively operate a profitable retail-online channel. The other characteristics of the selling prices and order quantity with respect to the number of replenishment in the centralized channel are same as the decentralized decision making process. As far as the optimal number of replenishment over the planning horizon \( L \) is concerned, the proposition follows.

**Proposition 4.10** Over the selling season \( L \), the number of replenishments for which system’s profit is maximum is given by

\[
n_{0^c}^e = \begin{cases} 
[n_0^c] & \text{if } \pi^{r/c}([n_0^c]) > \pi^{r/c}([n_0^c] + 1) \\
[n_0^c] + 1 & \text{otherwise}
\end{cases}
\]

and

\[
n_0^c = (-d_c + \sqrt{d_c^2 + b_c^1})^{\frac{1}{2}} + (-d_c - \sqrt{d_c^2 + b_c^1})^{\frac{1}{2}}
\]

(2.23)

where \( b_c = -[2aL^2(u_2 + k_1h^r + (1 - k_3)h^m) - hL^2(2u_2 - u_2 L)(2u_2 + h^r + h^m)]/24(s^r + s^m) \),

\( d_c = L^3[3((b + r)(h^r + h^m) - 2rh^r + h^m) + 4bu_2]/24(s^r + s^m) \) and \( [n_0^c] \) denotes the largest integer not greater than \( n_0^c \).
CHAPTER 4. MANAGING DUAL-CHANNEL SUPPLY CHAINS

Proof Same as proposition-4.6.

Note that \( \pi^c > \pi^r/ds + \pi^m/ds \), i.e., the channel is not coordinated. This is quite obvious as indicated in supply chain literature that cooperative integrated decision is always more profitable than decentralized self centered decision. The next section demonstrates a profit sharing mechanism assuming that the manufacturer and the retailer jointly take the centralized decision, which is the channel best decision and share the total channel profit in a portion that ensures win-win profit.

4.1.2 Channel coordination using Profit sharing

The difficulties for coordination in this model are due to different optimal cycle length and hence different pricing policy for centralized and decentralized scenarios. For accepting the centralized cycle length, which is less than the decentralized cycle length, the retailer’s cost will increase and there is no reason that the retailer will adopt centralized policy unless proper incentive. As an incentive manufacturer can offer the retailer to share the surplus profit if the retailer adopt centralized decisions. Under profit sharing mechanisms, the system performance is first optimized and the resultant benefit is then shared between the manufacturer and the retailer. This solution can be considered as a cooperative solution. Its implementation, however, depends on the development of a profit sharing scheme that is acceptable to both parties.

The manufacturer provides incentive to the retailer for accepting centralized selling price, \( p^c_r \) and cycle length, \( L/n_0 \) by offering him to the surplus profit proportionally according to their decentralized profit. To obtain centralized channel profit manufacturer has also sell the product at a price \( p^c_d \) through the online channel. Surplus profit for accepting centralized policy over the planning horizon \( L \) is \( \pi_{sp} = \pi^c - (\pi^r/ds + \pi^m/ds) \). The manufacturer and the retailer will get additional profits \( [\pi^m/ds/(\pi^r/ds + \pi^m/ds)]\pi_{sp} \) and \( [\pi^r/ds/(\pi^r/ds + \pi^m/ds)]\pi_{sp} \) respectively over the planning horizon \( L \). Now the question is how they implement the profit sharing policy in different cycle. For this the model proposes that the surplus profit can be shared between them by just adjusting wholesale price properly. Thus, for a particular cycle the manufacturer and the retailer get additional profit \( [\pi^m/ds/n_0(\pi^r/ds + \pi^m/ds)]\pi_{sp} \) and \( [\pi^r/ds/n_0(\pi^r/ds + \pi^m/ds)]\pi_{sp} \) respectively.

In the \( i \)th \( i = 1, 2, \ldots, n_0 \) replenishment cycle, profit of the retailer is given by

\[
\frac{LD^c_i}{n_0} \left( p^c_i - w^p_i \right) - \frac{Lh^r}{2n_0} - s^r = \pi^r/ds + \left( \frac{\pi^r/ds}{n_0(\pi^r/ds + \pi^m/ds)} \right) \pi_{sp} \tag{4.24}
\]

On simplification, the wholesale price of the product in the \( i \)th \( i = 1, 2, \ldots, n_0 \) replenishment cycle can be found as

\[
w^p_i = p^c_i - \frac{Lh^r}{2n_0} - \frac{n_0 \pi^r/ds}{LD^c_i} \left( \pi^r/ds + \left( \frac{\pi^r/ds}{n_0(\pi^r/ds + \pi^m/ds)} \right) \pi_{sp} + s^r \right) \tag{4.25}
\]

Thus, through proper choice of wholesale price, profit sharing mechanism can be implemented and the decentralized channel can achieve profit equal to centralized profit also assure win-win outcomes for all the channel members.
4.1.3 Numerical illustration

Assume that a manufacturer sales a hi-tech product and decides the sales season L as 120 days. At the beginning of the sales season the unit cost of the product is \( u_1 = $50 \) and the cost decreases at a rate \( u_2 = $0.25 \) per day. Other parameter values are \( a = 100, b = 0.4, r = 0.1, h^r = $0.15, h^m = $0.12 \) per unit per day, \( s^m = $5000, s^r = $1000 \) per replenishment cycle. Also assume that the customer’s retail channel preference is \( k_1 = 0.5 \). The optimal values are presented in table-4.3.

In decentralized setting the retailer replenishes thrice over the planning horizon whereas in centralized channel total number of replenishment is 4. As indicated, the wholesale price and the selling prices in retail and online channel decrease in every next replenishment cycle. Also from table-4.3, one can easily observe that the manufacturer’s and the retailer’s profit increase 12.8% and 15.4% respectively from the decentralized channel when they adopt centralized policy through profit sharing mechanism. Both in centralized and decentralized channel optimal order quantities over the planning horizon increase in each and every next replenishment. Optimal online channel price and retail price are higher than the manufacturer’s wholesale price for \( k_1 \in (0.1526, 0.5162) \). Also, \( k_1 = 0.8564 \) and \( k_1 = 0.354 \).

Thus for profitable retail-online channel customers’ retail channel preference must be \((0.354, 0.5126)\). Interestingly as indicated, customers’ retail channel preference intensifies price competition between the channel. In the present model selling in decentralized setting for \( k_1 > 0.4295, 0.4203, 0.4111 \) in the 1st, 2nd and 3rd replenishment cycles, the retail prices are higher than the online selling prices (see fig-4.1(a)). In centralized channel there is also price competition that is presented in fig-4.1(b). However, as in the decentralized case, the manufacturer can operate a profitable integrated channel when the customers’ retail channel preference \( k_1^C \in (0, 0.791) \). Also, the retail price is higher than the online price for customers’ retail channel preference in \((0.4982, 0.791)\). Notice that the product compatibility has lesser impact on centralized channel when compared with decentralized channel.

The setup cost of the retailer has an impact on the manufacturer’s decision for operating the retail-online channel. As the lower threshold of product compatibility is directly is proportional to the retailer’s setup cost, \( k_1 \) may be higher than \( k_1^{DC} \). In such case the manufacturer can not set the online selling price and hence can not operate the retail-online channel. In the present model setting if \( s^r > 2361 \) then the manufacturer can not operate the online channel (see fig-4.2). Thus, for a profitable retail-online channel, apart from the customers’ retail channel preference the setup cost of the retailer should be reasonably low. From table-4.3 notice that profit sharing mechanism coordinate the channel
<table>
<thead>
<tr>
<th>System</th>
<th>n</th>
<th>$w_i$</th>
<th>retail price</th>
<th>online price</th>
<th>order quantity</th>
<th>$\pi^{r/ds}$</th>
<th>$\pi^{m/ds}$</th>
<th>Total Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Decentralized</td>
<td>1</td>
<td>83</td>
<td>105.1</td>
<td>91.1</td>
<td>2582</td>
<td>9312</td>
<td>81819</td>
<td>91131</td>
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<tr>
<td></td>
<td>2</td>
<td>85.2, 77.7</td>
<td>103.8, 99.3</td>
<td>89.3, 81.8</td>
<td>3019</td>
<td>12652</td>
<td>109758</td>
<td>122410</td>
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<tr>
<td></td>
<td>3</td>
<td>86, 81, 76</td>
<td>103.4, 100.4, 97.4</td>
<td>88.7, 83.7, 78.7</td>
<td>3165</td>
<td>13239</td>
<td>116685</td>
<td>129924</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>86.4, 82.6, 78.9, 75.1</td>
<td>103.2, 100.9, 98.6, 96.4</td>
<td>88.4, 84.6, 80.9, 77.2</td>
<td>3237</td>
<td>13058</td>
<td>117826</td>
<td>130884</td>
</tr>
<tr>
<td>Centralized</td>
<td>1</td>
<td>-</td>
<td>92</td>
<td>91.1</td>
<td>3211</td>
<td>-</td>
<td>-</td>
<td>101444</td>
</tr>
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<td></td>
<td>2</td>
<td>-</td>
<td>89.8, 82.2</td>
<td>89.3, 81.8</td>
<td>3765</td>
<td>-</td>
<td>-</td>
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<td>89, 84, 79</td>
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<td>3950</td>
<td>-</td>
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<td>-</td>
<td>88.6, 84.9, 81.1, 77.4</td>
<td>88.4, 84.6, 80.9, 77.2</td>
<td>4043</td>
<td>-</td>
<td>-</td>
<td>147942</td>
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<tr>
<td>Profit sharing</td>
<td>4</td>
<td>75.8, 71.2, 66.5, 61.8</td>
<td>88.6, 84.9, 81.1, 77.4</td>
<td>88.4, 84.6, 80.9, 77.2</td>
<td>4043</td>
<td>15075</td>
<td>132867</td>
<td>147942</td>
</tr>
</tbody>
</table>

Table 4.3: Optimal values in decentralized channel, centralized channel and for profit sharing mechanism.
and divides the surplus profit between the channel members. Also the average wholesale price of the manufacturer in profit sharing mechanism is 68.8 and it is lower than the average wholesale price, 83 of the decentralized decision making. This is quite obvious.

This section considers a retail-etail channel supply chain for a product that experiences continues unit cost decrease over the planning horizon. Existing literature in this direction discusses the replenishment and pricing policy over the planning horizon for a single business entity like retailer or manufacturer. But as indicated, in the current business scenario, the overall channel performance optimization is prevalingly in practice and coexistence of retail and online channels are quite common. In such scenario the present model proposes pricing and replenishment policy and analyzes how the customers’ channel preference affects the individual and integrated decisions of the channel members. It also discusses, in decentralized decision making how the channel performance can be maximized through a transfer pricing policy.

### 4.2 Managing a socially responsible dual-channel supply chain

This section\(^4\) introduces a corporate social responsible two-echelon dual-channel supply chain. Besides operating online channel, the manufacturer intends to swell stakeholders welfare by exhibiting CSR. The pricing decisions for both the cases of the decentralized and centralized scenarios are studied analytically as well as numerically. It also examines the effect of degree of concern of the manufacturer about CSR on product compatibility and discusses feasibility of the successful operations on dual-channel supply chain. It also discusses the issues like channel coordination\(^5\), surplus profit division (using Nash bargaining) etc.

\(^4\)This section is based on the paper “Corporate social responsibility, coordination and profit distribution in a dual-channel supply chain.” published in Pacific Science Review, Article in press (2015)

\(^5\)Using all unit quantity discount with the agreement of franchise fee
in the literature earlier. Second, it examines the effect of CSR on product compatibility and discusses feasibility for successful operation dual-channel supply chain. Third, it addresses the effect of product compatibility and degree of the manufacturer’s social concern on the optimal selling prices and optimal order quantity of the channel.

### 4.2.1 Model formulation and basic analysis

Consider a dual-channel supply chain comprise of a socially responsible manufacturer and a retailer similar as section 4.1. Instead of multiple replenishment cycle of previous model, this model only considers a single cycle and so dropping ‘i’ from demand function of the previous model one can obtain the demand functions in retail and e-tail (online) channel as

\[ D_r = k_1 a - b_1 p^r + r(p^d - p^r) \]  
\[ D_d = (1 - k_1) a - b_2 p^d + r(p^r - p^d) \]

Differently from the previous one, this model considers that consumers may have different price sensitivity i.e., \( b_1 \neq b_2 \). As indicated, CSR is accounted through consumer surplus that a channel member accrues from the stakeholders. In the present model setting the consumer surplus can be found as

\[
CS = \int_{p^r/\text{mkt}}^{p^r/\text{max}} D_r dp^r + \int_{p^d/\text{mkt}}^{p^d/\text{max}} D_d dp^d \\
= \int \frac{(k_1 a + r p^d)}{(b_1 + r)} D_r dp^r + \int \frac{((1-k_1) a + r p^r)}{(b_2 + r)} D_d dp^d \\
= \frac{D_r^2}{2(b_1 + r)} + \frac{D_d^2}{2(b_2 + r)}
\]

Where \( p^r/\text{mkt} \) and \( p^d/\text{mkt} \) respectively denotes market price of the product in retail and online channel, while \( p^r/\text{max} \) and \( p^d/\text{max} \) respectively denotes maximum price that the consumers are willing to pay for a product in retail and online channel. If \( \alpha \in [0, 1] \) is the degree of CSR that is socially responsible manufacturers concerned then it incorporates \( \alpha((D_r^2/(2(b_1 + r)) + (D_d^2/(2(b_2 + r)))) \) as consumer surplus in its profit. \( \alpha = 0 \) implies that the manufacturer is the pure profit maximizer and \( \alpha = 1 \) represents that the manufacturer is the perfect welfare maximizer. Since the manufacturer is socially responsible, its profit function consists of pure profit that is receives by selling the product and consumer surplus through CSR practice. Under this setting the model first derives the centralized and decentralized decisions of the channel members.

### 4.2.2 Decentralized policy

In decentralized decision making, the channel members operate independently and optimize their individual goals. Interactions between the manufacturer and the retailer are considered as a Stackelberg game. The manufacturer acts as the Stackelberg leader of the channel and the retailer is it’s follower. In Stackelberg game, leader makes first move and follower then reacts by playing the best move consistent with available information. In this way, the manufacturer first announces the wholesale price and selling price of the product in the online channel. Based on the manufacturer’s decision, the retailer
determines the retail price. Profit function of the retailer is
\[ \pi_r = (p^r - w)(k_1a - b_1p^r + r(p^d - p^r)) \]  

Pure profit function of the manufacturer is
\[ \pi_m = (w - c)(k_1a - b_1p^r + r(p^d - p^r)) + (p_d - c)((1 - k_1)a - b_2p_d + r(p^r - p^d)) \]  

Total profit function of the manufacturer is
\[ v_m = \pi_m + \alpha CS \]
\[ = (w - c)D_r + (p^d - c)D_d + \alpha \left[ \frac{D^2_r}{2(b_1 + r)} + \frac{D^2_d}{2(b_2 + r)} \right] \]

Where \( \alpha \in (0, 1) \)
Differentiating \( \pi_r \) with respect to \( p^r \) and equating to zero gives
\[ p^* = \frac{k_1a + (b_1 + r)w^m + rp^d}{2(b_1 + r)} \]  

Note that, \( d^2 \pi_r / dp^r = -2(b_1 + r) < 0 \). That is, \( \pi_r \) is a concave function of \( p^r \). Substituting the value of \( p^* \) into the total profit function of the manufacturer and the necessary conditions \( \partial v_m / \partial w^m = 0 \) & \( \partial v_m / \partial p^d = 0 \) for optimization of \( v_m \) yields
\[ w^{m*} = \frac{4\beta_1\beta_2(b_2cr + b_1c/\beta_2 + a(r + b_2k_1) + a(b_2r + b_1\beta_2)(r + b_2k_1)\alpha^2}{((b_2r + b_1\beta_2)(2 - \alpha)2\beta_2(4r + b_1(4 - \alpha)) - b_2r\alpha)} \]
\[ - \left( \frac{2b_1^2\beta_2^2 + r((a + b_2c)r(3b_2 + 2r)+ ab_2(4b_2 + 3r)b_1k_1)}{(b_2r + b_1\beta_2)(2 - \alpha)(\beta_2(4r + b_1(4 - \alpha)) - b_2r\alpha)} \right) \]
\[ = \left( \frac{(b_1\beta_2(5b_2 + 2r) + a(4b_2k_1 + r(3 + k_1))\alpha}{(b_2r + b_1\beta_2)(2 - \alpha)(\beta_2(4r + b_1(4 - \alpha)) - b_2r\alpha)} \right) \]

\[ p^{d*} = \frac{4\beta_1\beta_2(b_2cr + b_1c/\beta_2 + a(\beta_1 - b_1k_1) + a(b_2r + b_1\beta_2)(\beta_1 - b_1k_1)\alpha^2}{(b_2r + b_1\beta_2)(2 - \alpha)(\beta_2(4r + b_1(4 - \alpha)) - b_2r\alpha)} \]
\[ - \left( \frac{\beta_1(c(2b_2 + 2r)(b_2r + b_1\beta_2) + a(5b_1\beta_1 + r(5b_2 + 2r)))\alpha}{(b_2r + b_1\beta_2)(2 - \alpha)(\beta_2(4r + b_1(4 - \alpha)) - b_2r\alpha)} \right) \]
\[ + \left( \frac{a(5b_1\beta_2 + 3b_1(2b_2 + r))b_1k_1\alpha}{(b_2r + b_1\beta_2)(2 - \alpha)(\beta_2(4r + b_1(4 - \alpha)) - b_2r\alpha)} \right) \]

Where, \( \beta_1 = b_1 + r \) and \( \beta_2 = b_2 + r \).

The optimal solutions will maximize the manufacturer’s total profit function if the profit function is concave with respect to its decision variables. Checking of the concavity of total profit function of the manufacturer are as follows
\[ \frac{\partial^2 v_m}{\partial w^m \partial w^m} = \frac{-4\beta_2 + b_1\beta_2(-4 + \alpha) + r(b_2 + 2r)\alpha}{4\beta_2} \]
\[ \frac{\partial^2 v_m}{\partial p^d \partial p^d} = \frac{-4\beta_1\beta_2(2b_1\beta_2 + r(2b_2 + r)) + (4b_1^2\beta_2^2 + b_1r\beta_2(8b_2 + 5r) + r^2(4b_2^2 + 5b_2r + 2r^2))\alpha}{4\beta_1^2\beta_2} \]
Clearly, \( \frac{\partial^2 v_m}{\partial p^2 \partial w_m} < 0 \) and \( \frac{\partial^2 v_m}{\partial p^2 w_m} < 0 \) for any \( \alpha \in [0, 1] \). Now, the necessary condition for concavity of \( v_m \) is

\[
\left( \frac{\partial^2 v_m}{\partial w_m^2} \right) \times \left( \frac{\partial^2 v_m}{\partial p^2 w_m} \right) - \left( \frac{\partial^2 v_m}{\partial p^2 \partial w_m} \right)^2 > 0
\]

Clearly, for any \( \alpha \in [0, 1] \), \( \frac{\partial^2 v_m}{\partial w_m^2} \times \frac{\partial^2 v_m}{\partial p^2 w_m} - \left( \frac{\partial^2 v_m}{\partial p^2 \partial w_m} \right)^2 > 0 \) and thus the profit function of the manufacturer is a concave function of \( p^d \) and \( w_m \). Using (43.33) and (43.34) in (43.35) and simplification yields the optimal retail price of the product in decentralized decision as follows

\[
p^*_r = \frac{\beta_2 \left( b_1^2 \beta_2 + b_1 c \epsilon r(3b_2 + 2r) + r(2 \alpha + b_2 c) + (r + b_2 k_1) + a b_1 (3b_2 k_1 + r(2 + k_1)) \right)}{(b_2 r + b_1 \beta_2)(b_2(4r + b_1(4 - \alpha)) - b_2 r \alpha)} - \frac{a(b_2 r + b_1 \beta_2)(r + b_2 k_1) \alpha}{(b_2 r + b_1 \beta_2)(b_2(4r + b_1(4 - \alpha)) - b_2 r \alpha)}
\]

Now using (43.33), (43.34) and (43.35) one can get optimal demand of the product in retail and online channel, profit of the retailer, pure and total profit of the manufacturer in decentralized decision as follows

\[
D^*_r = \frac{\beta_1 (2a \beta_2 k_1 - b_1 c \beta_2 (2 - \alpha) + b_2 \epsilon r \alpha - a(r + b_2 k_1) \alpha)}{(2 - \alpha)(b_2(4r + b_1(4 - \alpha)) - b_2 r \alpha)}
\]

\[
D^*_d = \frac{\beta_2 \left( c(2b_1(2b_2 + r) - b_2 r(4 - \alpha) + b_1 \beta_2 \alpha) + a(b_1(1 - k_1)(4 - \alpha) + r(4 - 2k_1 - \alpha)) \right)}{(2 - \alpha)(b_2(4r + b_1(4 - \alpha)) - b_2 r \alpha)}
\]

\[
\pi^*_r = \frac{1}{\beta_1} \left[ \frac{\beta_1 (2a \beta_2 k_1 - b_1 c \beta_2 (2 - \alpha) + b_2 \epsilon r \alpha - a(r + b_2 k_1) \alpha)}{(2 - \alpha)(b_2(4r + b_1(4 - \alpha)) - b_2 r \alpha)} \right]^2
\]

\[
v^*_m = \frac{2a^2 \beta_2 [ \beta_2^3 - 4b_1 \beta_1 k_1 + (b_1(2b_1 + b_2) + (b_1 + b_2)r) k_1^2]}{(2b_2 r + b_1 \beta_2)(2 - \alpha)(b_2(4r + b_1(4 - \alpha)) - b_2 r \alpha)} - \frac{a^2 (\beta_1(b_1 \beta_2 + r(b_2 + 2r)) + 2b_1(b_2 r - b_1 \beta_2) k_1) \alpha}{(2b_2 r + b_1 \beta_2)(2 - \alpha)(b_2(4r + b_1(4 - \alpha)) - b_2 r \alpha)}
\]

\[
+ \frac{a^2 \left( \beta_2 \left( b_1^2 \beta_2 + b_1 c \epsilon r \alpha \right) \right) + 2a b_2 (b_1 \beta_2)(2b_2 - b_1 \beta_2) k_1 + (\beta_1(b_2 + 2r) - (b_1 - b_2) r k_1) \alpha}{(2b_2 r + b_1 \beta_2)(2 - \alpha)(b_2(4r + b_1(4 - \alpha)) - b_2 r \alpha)}
\]

Where, \( F_1 = 4 \beta_1 \beta_2 (b_2 c r + b_1 c \beta_2 - a(r + b_2 k_1)) + (b_2 r + b_1 \beta_2)(b_2 c r + b_1 c \beta_2 - a(r + b_2 k_1)) \alpha^2 + (-4b_1 c \epsilon r + (a - b_2 c r)(3b_2 + 2r) + a b_2(4b_2 + 3r) k_1) + b_1 \beta_2(-c r(7b_2 + 2r) + a(4b_2 k_1 + r(3 + k_1))) \alpha \) and \( F_2 = c(b_2 r + b_1 \beta_2) \left( r(2r(2 - \alpha) + b_2(4 - \alpha)(1 - \alpha)) + b_1 \left( r(2 - \alpha)^2 + b_2(4 - \alpha)(1 - \alpha) \right) + a(b_2^2 \beta_2(-1 + k_1)(4 - \alpha)(1 - \alpha) + r^2(2r(-2 + \alpha) - b_2(4 + \alpha(-5 + k_1 + \alpha))) + a b_1 r(4b_2^2(-2 + k_1) + (10b_2 + 7r - 3(2b_2 +
Consumer can buy the product from both retail and online channel. The retailer operates its channel competing with the online channel. The retailer has to buy the product from the manufacturer and then sell it to the consumers and doing so it makes profit. Thus, concept of dual channel will be feasible only when selling price in both retail and online channel are higher than the wholesale price of the manufacturer. That is, the optimal pricing strategy of decentralized system is acceptable to the channel members only when \( p^* > w^m \) and \( p^{d*} > w^m \). Now, \( p^{d*} > w^m \) if \( k_1 < k_{1\text{max}} \). Where

\[
k_{1\text{max}} = \frac{b_2c(b_1+2r)(b_2r+b_1\beta_2)\alpha + a (b_1^2\beta_2(4-\alpha)(1-\alpha) - 2b_2r^2\alpha + b_1r(4r(1-\alpha) + b_2(4 - (7-\alpha)\alpha)))}{a(4(b_1+b_2)\beta_1\beta_2 - (5b_1^2\beta_2 + 4b_2r\beta_2 + b_1(4b_2^2 + 11b_2r + 4r^2))\alpha + (b_1 + b_2)(b_2r + b_1\beta_2)\alpha^2)}
\]

And \( p^* > w^m \) if

\[
k_1 > \frac{b_1c\beta_2(2-\alpha) + (a-b_2c)r\alpha}{2a\beta_2 - ab_2\alpha} = k_{1\text{min}}
\]

Note that, both \( k_{1\text{min}} \) and \( k_{1\text{max}} \) depends on \( \alpha \), i.e., on the degree of social concern of the manufacturer. Clearly, This range will be valid until \( k_{1\text{max}} > k_{1\text{min}} \) and solving \( k_{1\text{max}} - k_{1\text{min}} = 0 \) provides three values of \( \alpha \) as \( \{ 2, (b_1b_2 + b_1r)/(b_1b_2 + b_1r + b_2r), (4(b_1b_2 + b_1r + b_2r + r^2))/(b_1b_2 + b_1r + b_2r) \} \). Among three values of \( \alpha \) only \((b_1b_2 + b_1r)/(b_1b_2 + b_1r + b_2r) \in (0, 1)\), and hence acceptable because \( \alpha \in [0, 1] \). That is, \( k_{1\text{max}} > k_{1\text{min}} \) if \( \alpha \in [0, (b_1b_2 + b_1r)/(b_1b_2 + b_1r + b_2r)] \). From the above discussion the proposition follows.

**Proposition 4.11** Pricing policy of the decentralized socially responsible dual-channel supply chain can be operate successfully only if the product compatibility, \( k_1 \in (k_{1\text{min}}, k_{1\text{max}}) \) and the degree of social concern of the manufacturer, \( \alpha \in [0, (b_1b_2 + b_1r)/(b_1b_2 + b_1r + b_2r)] \).

![Figure 4.3: Graphical representation of \( k_{1\text{min}} \) and \( k_{1\text{max}} \) with respect to \( \alpha \)](image-url)
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CSR concern over a threshold to operate a decentralized dual-channel supply chain. It also indicates that the customer’s channel preference is one of the determining factors for operating an online channel besides the traditional retail channel. When \(k_1 < k_{1\text{min}}\), the retailer can’t do business because it’s selling price is less than the manufacturer’s wholesale price. On the other hand, for \(k_1 > k_{1\text{max}}\), the manufacturer can’t set the optimal selling price as the online price.

Online selling price is higher than the wholesale price. It does not ensure that the retailer will participate in the profit making retail-e-tail channel, the retailer will participate in the dual-channel only when it’s demand in retail channel is positive, i.e., \(D^*_r > 0\), i.e., if

\[
k_1 > \frac{b_1c\beta_2(2 - \alpha) + (a - b_2c)r\alpha}{2a\beta_2 - ab_2\alpha} = k_{1r}
\]

Note that, \(k_{1r} = k_{1\text{min}}\). On the other hand, the manufacturer will operate the online channel until the demand in the online channel is positive, i.e., \(D^*_o > 0\), i.e., if,

\[
k_1 < \frac{a\beta_1(4 - \alpha) - b_2cr(4 - \alpha) - b_1c(4b_2 + 2r - \beta_2\alpha)}{a(2r + b_1(4 - \alpha))} = k_{1m}
\]

Eq. (4.46) and (4.47) suggest when the customers’ retail channel preference lies in between \(k_{1\text{min}}\) and \(\min\{k_{1m}, k_{1\text{max}}\}\), the manufacturer can successfully operate a profitable dual-channel. This result is quite obvious because the manufacturer will operate the online channel only when both the channels are profitable. Thus, the proposition follows

**Proposition 4.12** The manufacturer can operate a profitable retail-online channel when the customers’ retail channel preference \(k_1 \in (k_{1\text{min}}, \min\{k_{1m}, k_{1\text{max}}\})\).

Further, comparison of selling prices of retail and online channel gives \(p^{\ast*} > p^{\ast\ast}\) if \(k_1 > k_{11}\) and \(p^{\ast*} < p^{\ast\ast}\) if \(k_1 < k_{11}\) where,

\[
k_{11} = \frac{c(b_2r + b_1\beta_2)(2b_1\beta_2 - (b_1 - b_2)r\alpha) + a(H_1 - b_2r^2\alpha + b_1r(r(4 - 3\alpha) + b_2(4 - (6 - \alpha)\alpha)))}{a(H_1 + b_2r(6r + b_2(3 - \alpha)(2 - \alpha) - 4r\alpha) + b_1(6\beta_2^2 - (b_2 + 2r)(5b_2 + 2r)\alpha + b_2(b_2 + 2r)\alpha^2))}
\]

Where, \(H_1 = b_1^2\beta_2(4 - \alpha)(1 - \alpha)\). Now combining the above results with the proposition-4.11, the proposition follows

**Proposition 4.13** In a socially responsible decentralized dual-channel supply chain, the optimal retail price is higher than online selling price if \(k_1 \in (k_{11}, \min\{k_{1m}, k_{1\text{max}}\})\), while reverse may be noted for \(k_1 \in (k_{1\text{min}}, k_{11})\).

From figure-4.4 one may note that, \(p^{\ast\ast} \geq w^{\ast\ast}\) if \(k_1 \geq k_{1\text{min}}\) and \(p^{\ast*} \geq w^{\ast*}\) if \(k_1 \leq k_{1\text{max}}\). The optimal online selling price decreases with increasing product compatibility, while the optimal retail price increases with increasing product compatibility. The optimal online selling price coincides with retail price when \(k_1 = k_{11}\). The next subsection explores centralized decisions of the socially responsible dual-channel supply chain when the channel members act as a single entity.
4.2.3 Centralized policy

When the channel members cooperate and find the decision that maximizes the supply chain performance, it is essentially the centralized decision making process. It may be assumed that there is a single decision maker, who produces and sales the product to the customers. The total profit of the channel is the sum of pure profit and the consumer surplus that the channel accrues from the stakeholders. The profit function of the channel is

\[ v_c = \pi_c + CS_c \]  \hspace{1cm} \text{(4.49)}

where

\[ \pi_c = (p^r - c)D_r + (p^d - c)D_d \]

and \[ CS_c = (D_r^2)/(2\beta_1) + (D_d^2)/(2\beta_2) \]. The necessary condition, \( dv_c/dp^r = 0 \) & \( dv_c/dp^d = 0 \), for the existence of the optimal solution yields the optimal value of selling prices as follows

\[ p^r_{ce} = \frac{2\beta_1\beta_2(b_2c_2 + a(r + b_2k_1)) + a(b_2r + b_1\beta_2)(r + b_2k_1)\alpha^2}{(b_2r + b_1\beta_2)(r(b_2(2-\alpha)^2 + 4r(1-\alpha)) + b_1\beta_2(2-\alpha)^2)} \] \hspace{1cm} \text{(4.50)}

\[ p^d_{ce} = \frac{2\beta_1\beta_2(b_2c_2 + a(\beta_1 - b_1k_1)) + a(b_2r + b_1\beta_2)(\beta_1 - b_1k_1)\alpha^2}{(b_2r + b_1\beta_2)(r(b_2(2-\alpha)^2 + 4r(1-\alpha)) + b_1\beta_2(2-\alpha)^2)} - \frac{\beta_1(c(b_2 + 2r)(b_2r + b_1\beta_2) + a(-3b_1\beta_2(-1 + k_1) + r(3b_2 + 2r - b_2k_1))\alpha}{(b_2r + b_1\beta_2)(r(b_2(2-\alpha)^2 + 4r(1-\alpha)) + b_1\beta_2(2-\alpha)^2)} \] \hspace{1cm} \text{(4.51)}

Second order partial derivatives of the total centralized channel profit function are taken to check the concavity of total centralized channel profit function as follows

\[ \frac{\partial^2 v_c}{\partial p^2} = b_1(-2 + \alpha) + r \left( -2 + \frac{(b_2 + 2r)\alpha}{b_2 + r} \right) \]
In the ith replenishment cycle, if the customers’ retail channel preference is higher than threshold $\alpha$, although in centralized channel the manufacturer and the retailer co-operate and take decision jointly, the optimal pure and total profit of the centralized channel are as follows. 

Clearly, $(\partial^2 v_c) / (\partial p^2) < 0$ and $(\partial^2 v_c) / (\partial p^2) < 0$ and $(\partial^2 v_c / \partial p^2) - (\partial^2 v_c / \partial p^2) > 0$ for all $\alpha \in [0, 1]$. Thus, the total profit function of the centralized channel $(v_c)$ is a concave function of $p^c$ and $p^i$. Hence the optimal selling prices $(p^c, p^i)$ provide global maximum to (4.44). Using (4.45) and (4.46), the optimal demand of the product in retail and online channel in centralized decision are found as follows.

\[
\begin{align*}
D_{rc} &= \frac{\beta_1(2\alpha \beta_2 k_1 + b_1 c \beta_2 (-2 + \alpha) + b_2 c r \alpha - a(r + b_2 k_1)\alpha)}{r(b_2(2 - \alpha)^2 + 4r(1 - \alpha)) + b_1 \beta_2 (2 - \alpha)^2} \\
D_{dc} &= \frac{\beta_2(-2\beta_1 (b_2 c + a(-1 + k_1)) + (b_2 c - a\beta_1 + b_1 c \beta_2 + ab k_1)\alpha)}{r(b_2(2 - \alpha)^2 + 4r(1 - \alpha)) + b_1 \beta_2 (2 - \alpha)^2}
\end{align*}
\]

Optimal pure and total profit of the centralized channel are as follows.

\[
\pi_c = (p^c - c)D_{rc} + (p^i - c)D_{dc}
\]

In centralized decision making context the manufacturer would be interested on opening the online channel only when online channel demand is positive, i.e., $D_{dc} > 0$, which, after simplification, yields

\[
k_1 < \frac{a\beta_1 (2 - \alpha) - 2b_2 c \beta_1 + c(b_2 r + b_1 \beta_2)\alpha}{2a_0 \beta_1 - ab_0 \alpha} = k_{1, max}^c
\]

On the other hand, minimum level of the product compatibility for operating centralized retail channel can be found from $D_{rc} > 0$, which, after simplification, yields

\[
k_1 > \frac{b_1 c \beta_2 (2 - \alpha) + (a - b_2 c) r \alpha}{2a_0 \beta_1 - ab_0 \alpha} = k_{1, min}^c
\]

Although in centralized channel the manufacturer and the retailer co-operate and take decision jointly, the product compatibility has an impact on the manufacturer’s decision for opening the online channel. In the ith replenishment cycle, if the customers’ retail channel preference is higher than threshold $k_{1, max}^c$, then the manufacturer’s decision for opening an online channel is not profitable because its online demand is negative in such case. On the other hand, there must be a competition between the retail channel and the online channel in the centralized process though the channel members co-operate. The channel members take decision jointly but the market potential remains same. When the manufacturer operates an online channel, some customers switch to the online channel. As a result the retailer’s demand decreases and it earns less profit. Besides the selling prices of the retail and online channel,
the customers’ channel preference determines the divisions of potential market demand. So, like the decentralized decision making process, here also the selling prices of the retail channel may be higher than the online channel, i.e., \( p_{rc} > p_{dc} \), i.e., if \( k_1 > k_{12} \) where

\[
k_{12} = \frac{2ab_1\beta_1\beta_2 - (b_1 - b_2)cr(b_2r + b_1\beta_2) + a\beta_1(b_2r + 3b_1\beta_2)\alpha + ab_1(b_2r + b_1\beta_2)\alpha^2}{a(2(b_1 + b_2)\beta_1\beta_2 - 3b_1^2\beta_2 + b_1(b_2 + 2r)(3b_2 + 2r) + b_2r(3b_2 + 4r))\alpha + (b_1 + b_2)(b_2r + b_1\beta_2)\alpha^2}
\]  

(4.58)

From the above discussion, the proposition follows.

**Proposition 4.14**  The manufacturer can operate a centralized dual-channel if \( k_1 \in (k_{1min}, k_{1max}) \). The online selling price is higher than retail price for any \( k \in (k_{1min}, k_{12}) \) and the retail price is higher than the online price for any \( k_1 \in (k_{12}, k_{1max}) \).

**Figure 4.5:** Nature of centralized prices with respect to product compatibility

Proposition 4.14 demonstrates that, when the channel members cooperate and take decision jointly, the manufacturer’s decision to open an online channel is profitable only when the customers’ retail channel preference lies in between \( k_{1min} \& k_{1max} \). Interestingly between this threshold of the product compatibility there exists a price competition between the retail channel and the online channel. If the customers’ retail channel preference is in \( (k_{1min}, k_{12}) \), then the online price will be higher than the retail price and the reverse is set for \( k_1 \in (k_{12}, k_{1max}) \). Figure-4.5 also justifies our analytical findings. Thus, for a profitable centralized retail-online channel, the channel will set the selling prices according to the customers’ channel preference.

Observe that, both \( k_{1min} \) and \( k_{1max} \) depend on the degree of the manufacturer’s social concern i.e., on \( \alpha \). Centralized dual-channel will be feasible only when \( k_{1min} \leq k_{1max} \). Solving \( k_{1max} - k_{1min} = 0 \) for \( \alpha \) one can obtain two values of \( \alpha \) as \( (\sqrt{2\beta_1\beta_2} - r) \) and \( (\sqrt{2\beta_1\beta_2} + r) \). Clearly, both values are greater than 1 because \( \sqrt{2\beta_1\beta_2} > r \). But \( \alpha \in (0, 1) \), hence the proposition follows.

**Proposition 4.15** Centralized dual-channel will be feasible for any degree of the manufacturer’s social concern.
CHAPTER 4. MANAGING DUAL-CHANNEL SUPPLY CHAINS

Proposition-4.15 shows that the feasibility of centralize dual-channel i.e., positive demand in both channel exist in centralized scenario for any degree of the manufacturer’s social concern. While in decentralized scenario the manufacturer can not exhibit CSR above a threshold. Hence, channel coordination is essential not only for profit enhancement but also for perform high level of social responsibility. The next section analyzes the issue of channel coordination.

4.2.4 Channel coordination using all unit quantity discount contract

To implement centralized pricing policy the manufacturer offers retailer that online selling prices will adjusted according to centralized policy as well as provides all unit quantity discount as incentive to the retailer. Online selling price of the manufacturer under coordinated scenario is \(p^d - R\) and discounted wholesale price of the manufacturer is \(\phi w^{m*} \ (\phi \in (0, 1))\). Under the mechanism, the profit function of the retailer and total profit function of the manufacturer are respectively as follows

\[ \pi_{rco} = (p' - \phi w^{m*})(k_1a - b_1p' + r(p^d - R - p')) \]  

(4.59)

\[ v_{nco} = (\phi w^{m*} - c)(k_1a - b_1p' + r(p^d - R - p')) \]

+ \((p_d - R - c)((1 - k_1)a - b_2(p_d - R) + r(p' - p_d + R))\)

+ \(\alpha((1 - k_1)a - b_2(p_d - R) + r(p' - p_d + R)^2)/2\beta_1\)

+ \(\alpha((1 - k_1)a - b_2(p_d - R) + r(p' - p_d + R)^2)/2\beta_2\)

(4.60)

In a decentralized dual-channel supply chain for a given wholesale price of the manufacturer and the price for the online channel \((p_d - R)\), \(\pi_{rco}\) is concave in \(p'_r\), hence solving \(d\pi_{rco}/dp'_r = 0\) gives

\[ p_{rco} = \frac{p'^r - Rr + ak_1 + w^{m*}\phi_1}{2\beta_1} \]  

(4.61)

The channel will be coordinated only if retailer’s self optimized selling price \(p_{rco}\) under the all unit quantity discount contract coincides with the centralized retail price i.e., \(p_{rco} = p'^c\). Solving \(p_{rco} - p'^c = 0\) for the fraction of wholesale price discount, \(\phi\)

\[ \phi_{co} = 2 \left( \frac{2\beta_1\beta_2(b_2c + b_1c\beta_2 + a(r + b_2k_1)) + a(b_2r + b_1\beta_2)(r + b_2k_1)a^2}{m^{*}(b_2r + b_1\beta_2)(r(b_2(2 - a)^2 + 4r(1 - a)) + b_1\beta_2(2 - a)^2)} \right) = \frac{2(p'^r - Rr + ak_1)}{w^{m*}\beta_1} \]

(4.62)

\[ = \frac{2\beta_2(b_2c + b_1c(3b_2 + 2r) + r(2(a + b_2c)r + 3ab_2k_1) + ab_1(3b_2k_1 + r(2 + k_1))}{w^{m*}(b_2r + b_1\beta_2 + (r(b_2(2 - a)^2 + 4r(1 - a)) + b_1\beta_2(2 - a)^2)} \]

Thus, The retailer will agree to sell the product at centralized retail price if \(\phi = \phi_{co}\). But one may note that, \(\phi_{co}\) is depends on the online selling price \(p'^d\) of the manufacturer. In the second stage, in response to the retailer’s decision, the manufacturer will optimize its total profit function. Hence by solving \(dv_{nco}/dp'^d = 0\), the online selling price \(p_{dco}\) of the manufacturer can be determined. But for channel coordination the manufacturer has to adjust its online price. Thus, the adjustment in online selling price that the manufacturer will consider, is given by \(R_{co} = p_{dco} - p'^d\) which on simplification yields

\[ R_{co} = \frac{2r\beta_1(2a\beta_2k_1 + b_1c\beta_2(-2 + \alpha) + b_2cra - a(r + b_2k_1)\alpha)}{(b_1\beta_2 + r(b_2 + 2r))(r(b_2(2 - a)^2 + 4r(1 - a)) + b_1\beta_2(2 - a)^2)(-2 + \alpha)} \]  

(4.63)
Thus, profits of the retailer and the manufacturer are as follows

\[
\pi^{*}_{rc} = \frac{\beta_{1}(2a\beta_{2}k_{1} + b_{1}c\beta_{2}(-2 + a) + b_{2}cr\alpha - a(r + b_{2}k_{1})\alpha^{2}}{(r(2b(2-\alpha)^{2} + 4r(1-\alpha)) + b_{1}\beta_{2}(2-\alpha)^{2})^{2}} \tag{4.64}
\]

\[
v^{*}_{mco} - v^{*}_{m} = \frac{-2\beta_{1}(2r\beta_{2} + b_{1}\beta_{2}(2-a) - b_{2}cr\alpha)(2a\beta_{2}k_{1} + b_{1}c\beta_{2}(-2 + a) + b_{2}cr\alpha - a(r + b_{2}k_{1})\alpha^{2}}{(r(2b(2-\alpha)^{2} + 4r(1-\alpha)) + b_{1}\beta_{2}(2-\alpha)^{2})^{2}} < 0 \tag{4.66}
\]

Thus, comparing the profits of the channel members with their respective decentralized profits, The section finds that, due to coordination, profits of the retailer enhance significantly which improves the dual-channel supply chain efficiency, but fails to provide benefit to the manufacturer.

Next, the section will discuss the implementation of the contract with a complementary agreement between the manufacturer and the retailer that not only coordinate the dual-channel supply chain but also ensure a win-win strategy for both the members of the channel.

### 4.2.5 All unit quantity discount with agreement of franchise fees

The previous section shows that all unit quantity discount contract can coordinate the channel, but it can not provide win - win profit for the manufacturer. Thus, for successful implementation of the contract suppose the manufacturer charges a franchise fee \( F \) to the retailer. When a franchise fee \( F \) satisfies \( \pi^{*}_{rc} - F \geq \pi^{*}_{r} \), the retailer will accept a \( (R_{co}, \phi_{co}, F) \) contract, which yields

\[
F \leq \pi^{*}_{rc} - \pi^{*}_{r} = F \tag{4.67}
\]

On the other hand, minimum amount of franchise fee charged by the manufacturer to the retailer is given by

\[
F \geq v^{*}_{m} - v^{*}_{mco} = F \tag{4.68}
\]

Thus, the proposition follows.

Proposition 4.17 The all unit quantity discount with agreement of franchise fees can coordinate a two-level socially responsible dual-channel supply chain and provide win-win opportunity for the channel members for the franchise fee \( F \) if it satisfy the inequality \( F \leq F \leq F \)
4.2.6 Determination of franchise fees through bargaining

To determine exact value of franchise fees and profits of respective channel members, the concept of the generalized asymmetric Nash bargaining solution \[104\] is used. Suppose the manufacturer and the retailer have bargaining power respectively \(\gamma\) and \(1 - \gamma\) (\(\gamma \in (0, 1)\)). Let \(\Delta_m\) and \(\Delta_r\), denote the surplus profit share of the manufacturer and the retailer respectively. The functional form of \(\Delta_m\) and \(\Delta_r\) are as follows

\[
\Delta_m(F) = v_{mc}^* + F - v_m^* = X_m + F
\]

\[
\Delta_r(F) = \pi_{rc}^* - F - \pi_r^* = X_r - F
\]

Where, \(X_m = v_{mc}^* - v_m^*\); and \(X_r = \pi_{rc}^* - \pi_r^*\).
The total surplus profit generated through cooperation is equal to \(\Delta_m + \Delta_r = X_m + X_r\).

The generalized asymmetric Nash bargaining product of the present model is given by

\[
\max_{F \leq F \leq F} \Delta(F) = \Delta_m \Delta_r^{(1-\gamma)}
\]

The equilibrium solution of the above Nash bargaining product can be obtained by solving \((\partial \log \Delta) / \partial F = 0\) as follows

\[
F^b = \gamma X_r - (1 - \gamma)X_m
\]

Using bargaining solution of franchise fees, the profit of the manufacturer and the retailer after bargaining found as follows

\[
v_{mc}^b = v_m^* + \gamma (X_m + X_r)
\]

\[
\pi_{rc}^b = \pi_r^* + (1 - \gamma)(X_m + X_r)
\]

Note that, in particular, if all players involved in bargaining procedure have equal bargaining power, i.e., \(\gamma = 1/2\), then each and everyone get equal share \((X_m + X_r)/2\) of total surplus. From the above results the proposition follows.

**Proposition 4.18:** (i) Bargaining outcome of franchise fees depends on channel members’ bargaining power. (ii) Surplus profit of the channel generated through coordination has been distributed between the channel members in the ratio of their bargaining power.

Thus, all unit quantity discount contract combined with adjustment of online selling price brings channel coordination but fails to provide benefit to the manufacturer. But with the agreement of franchise fee it provides win-win opportunity to both members. Finally, through Nash asymmetric bargaining the members find their equilibrium profit with in the win-win range depending on their bargaining power.

4.3 Managerial implications and concluding remarks

The chapter demonstrates how to manage dual-channel supply chain. It develops two dual-channel supply chain models, one of them considers unit cost decreasing nature of hi-tech product and the other addresses corporate social responsibility. The chapter demonstrates how online channel can be managed along with retail channel, that is, how to determine prices, replenishment policies for profit maximization in dual channel supply chain. The chapter also analyzes effect of product compatibility on dual-channel supply chain. The chapter highlighted following managerial insights.
CHAPTER 4. MANAGING DUAL-CHANNEL SUPPLY CHAINS

The model presented in section 4.1 considers that the channel is operating over a finite time horizon in which the retailer replenishes multiple times. Unit cost of the product decreases continuously with respect to time. In manufacturer-Stackelberg game setting the purpose of the model is to explore the effects of customers channel preference and number of replenishment on pricing and replenishment policies. It has following contributions. First, the retail price is directly proportional to online price and wholesale price. Thus, the manufacturer has the control, to some extent, on the retail price because it sets the wholesale price and online selling price. Second, the optimal selling prices and wholesale price of the product decreases continuously over the planning horizon because the products’ unit cost decreases continuously. As such, the manufacturer must determine a planning horizon shorter than the lifetime of the product. Third, product compatibility has significant impact on the successful operation of profitable retail-online channel. As indicated, if the customers retail channel preference lies in an interval then the manufacturer operate an online channel besides the traditional brick-and-mortar channel for a product that experiences continuous cost decrease. Interestingly, there is a price competition between the retail and online channel and customers’ channel preference intensifies the price competition. As such, when the customers’ retail channel preference is below a certain threshold, the manufacturer sets online price higher than the retail price. Otherwise the retail price is higher. In integrated channel also there is a price competition though the channel members cooperate. Thus, in centralized channel there is an interval of customers’ retail channel preference for profit making retail-online channel. Fourth, the setup cost of the retailer influences the manufacturer’s decision for opening the online channel. The manufacturer can’t operate the online channel when setup cost of the retailer is above a threshold because the manufacturer can’t set the online price in that case. Fifth, the profit sharing mechanism coordinates the channel and divides the surplus profit.

In the section 4.2, a analysis of a dual-channel socially responsible supply chain has been worked out. In stackelberg setting, the manufacturer leader of the channel exhibit CSR. While formulating the model only uses the effect of CSR in the form of consumer surplus in the socially responsible firm’s profit function rather than the activities, which it performs. It explores pricing decision of both decentralized and centralized channel. The effect of CSR on successful operation of dual channel is analyzed. To coordinate the channel and get profit equal to the integrated system, a hybrid coordination mechanism is developed. The coordination mechanism not only coordinates the channel but it brings win-win outcome for the channel members. Finally, the manufacturer and the retailer share the surplus profit through Nash bargaining solution. The model makes contributions to many aspects. First, it considers effect of CSR on channel supply chain, in authors best knowledge that has not addressed in the literature earlier. Second, it examines the effect of CSR on product compatibility and discusses feasibility for successful operation dual-channel on both decentralized and centralized supply chain. It is analytically showed that the feasibility of centralized dual-channel exist for any degree of the manufacturer’s social concern. While in decentralized scenario the manufacturer can not exhibit CSR above a threshold for operating dual channel successfully. Third, the all unit quantity discount with agreement of franchise fee not only cuted out channel conflict but also provided win-win opportunity for the channel members. Fourth, through Nash bargaining the members find their equilibrium profit within the win-win range in the ratio of their bargaining power.

The two dual-channel supply chain models presented in this chapter shows that product compatibility is a key factor of successful operations of retail-e-tail channel. The chapter explores that operational feasibility of dual-channel mainly depends on product compatibility whether it addresses decrement of unit cost of the product or socially responsibility. Although the chapter provides many important features of dual-channel supply chain but it has some limitations. First, the chapter considers linear price dependent demand function. Although price dependent demand functions are used extensively in economics and marketing, still it is necessary to examine whether the managerial implications found in this chapter can be generalized to other demand function or not. Second, it is assumed that the channel
members know all information though it is not common in practice. Thus, model may be developed by considering that the channel members have private information. Third, another interesting direction of future research is to consider multiple retailers where there exists retail price competition. Also, the manufacturer wants to sell the product through an online channel. Although members of replenishment cycle, price competition among the retailers make the model more complex in comparison to the present one, still it will be more dynamic and will be close to real business practice.