Chapter 3

Managing perishable and imperfect quality products in supply chains

Product deterioration have a negative impact on successful and smooth supply chain operation and so managers have difficulties to make right decisions. Deterioration is the result of various effects on stock, some of which are damage, spoilage, obsoles, decay, decreasing usefulness and many more. While kept in store fruits, vegetables, etc suffer from depletion by decent spoilage. Through a gradual loss of potential or utility with the passage of time, electronic goods, grain, radioactive substances deteriorate. Gasoline, alcohol etc undergo physical depletion over time through the process of evaporation. Liu and Shi [95] have classified perishability and deteriorating inventory models into two major categories, namely decay models and finite lifetime models. Finite lifetime models assume a limited lifetime for each item. Blood cells, cans of fruit, foodstuffs, cosmetics, drugs, etc are examples of items having fixed lifetimes. Decaying products are of two types. Products which deteriorate from beginning and products which start to worsen after a certain time. For perishable products, disposal cost and salvage value have significant impact on order quantity. Some products like fruits, vegetables have low disposal cost. On contrary electronic goods, radioactive substances have high disposal cost. For some products (eg. backed goods, books, etc.) there are salvage values because products can be used alternatively such as sale of the product for scrap, recycling, donation of the product for charity, etc. Thus, perishability of a product has an impact on overall profit because it often impels the managers to adjust order quantity and/or price of the product suitably. Noting these under estimated difficulties for supply chains this chapter developed two different supply chain models to manage perishable product.

On the other hand, one of the major issues in supply chain is to maintain goodwill with the customers by delivering good quality products. Imperfect quality product found at the delivery time to the customers causes bad impression/impact of the seller that loses goodwill with customers to some extent. Advances in science and technology, improved mechanization
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and automation for large-scale production have made outstanding contributions to improve the quality of product. But, the percentage of good quality products in any manufacturing system could not reach at 100%. Consequently, it is necessary to separate imperfect quality products from the whole lot by screening process to sustain the existence of a business sector in a competitive market. Pricing is also an important decision factor which affects on the profit of the enterprise and it plays a significant role in demand [77]. Lee and Rosenblatt [91] investigated the effect of an imperfect production process on the optimal production cycle time. They assumed that defective items could be reworked instantly at a cost and they found that the presence of defective products motivated smaller lot sizes. Ben-Daya et al. [14] developed integrated inventory inspection models with and without replacement of nonconforming items found during inspection. Salameh and Jaber [122] developed an EOQ (Economic Order Quantity) model for items received with imperfect quality under the assumption that defective items could be sold as a single batch at the end of 100% screening process. They found that the economic lot size increases with increasing value of percentage of defective products. Goyal et al. [59] suggested that the defective items are produced throughout a production process and poor quality products detected in the screening process of a lot are sold at a discounted price. Wee et al. [145] developed an optimal inventory model for items with imperfect quality and shortage backordered, where poor-quality items exist during production. Poor quality items are picked up during the screening process and are withdrawn from stock instantaneously.

This chapter depicts the problems of perishable and imperfect quality products in supply chain in three different sections. First two sections consider the issue of product deterioration on supply chain modelling while the last one demonstrates the operation of a supply chain that deals with a percentage of imperfect quality product.

Section 3.1 considers a manufacturer-distributor-retailer chain that operates for a perishable product. It is assumed the retailer disposes the perished product. To compensate a percentage of the retailer’s disposal cost, the manufacturer and the distributor form a coalition that deals with the retailer. The purpose is to examine the ability of a specific side payment contract - compensation on disposal cost of deteriorated products, for cutting out channel conflict. Further, the model also discusses how the surplus profit can be divided among the channel members through a nested bargaining. First, the retailer and the coalition bargain and then the manufacturer and the distributor bargain within the coalition for surplus division. Since, the surplus profit in the channel is generated through disposal cost sharing, it is the key decision variable in the bargaining process. In particular the section answers of the following questions. First, does this side-payment contract expel channel conflict? Second, if it does then in which proportion does the coalition share the compensation? In which proportion do the manufacturer and the distributor compensate the retailer? How does it affect the profits of the channel members? Third, is the compensation, provided by the coalition to the retailer, higher than the disposal cost? Fourth, if the manufacturer or the distributor solely provides the compensation then is it possible to coordinate the channel? As such how the surplus profit will be divided among the channel members.

Section 3.2 extends the channel structure of section 3.1 to a conventional distribution channel, that consists of a manufacturer, multiple distributors and multiple retailers corresponding to each distributor. The manufacturer produces a perishable product and sells it to the cus-
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омers through the distribution channel. The research proposes a hybrid, quantity discount-compensation on deterioration cost, contract and examine its efficiency in resolving both horizontal and vertical conflict. The manufacturer provides quantity discount to each distributor independently. Each distributor compensates a percentage of deterioration cost of each of its retailer independently. Finally, Nash bargaining product is used to divide the benefit, which is obtained for synchronized decision of the channel members. Since, the distributors’ costs and benefit shares are dependent on the manufacturer’s and the retailers’ decisions simultaneously, the section proposes two approaches, namely backward contract-bargaining process and forward contract-bargaining process. In the former first the retailers bargain with their distributors independently in a pre-defined sequence. Based on the outcome, each distributor bargains with the manufacturer independently in a pre-specified sequence. In forward contract-bargaining, the channel members bargain in reverse order.

Section 3.3 considers a three-echelon supply chain with duopolistic retailers in the downstream and the retailers may play Cournot, Collusion and Stackelberg games. The manufacturer supplies a product in a single lot to the distributor that contains a random proportion of defective items. After receiving the lot, distributor separates the imperfect items by screening process and sells in a secondary market with a discounted price. At the end of the screening process, the distributor satisfies the demand of two retailers with perfect quality products only. The purpose of the section is to (i) examine the effect of imperfect quality product on the optimal decisions, (ii) verify whether the hybrid contract mechanism resolves the channel conflict or not and (iii) demonstrates how a nested bargaining process depicts win-win profits for the channel members after channel coordination.

3.1 Coordinating a three-echelon supply chain through disposal cost sharing and bargaining for surplus profit division

This section\(^1\) considers a three-echelon supply chain consisting of a manufacturer, a distributor and a retailer. All the channel members are risk neutral and seek to maximize profits. The manufacturer produces and supplies the product to the distributor in a single lot. The distributor supplies it to the retailer. The product deteriorates at a constant rate \(\theta\) and shortages are not allowed. Deteriorated product cannot be reworked and disposed by the retailer without any salvage value. Under this model setting the objective of each channel member is to maximize own profit margin though the channel profit is suboptimal. In such case, the channel profit and the profits of the channel members will depend on the retailers own profit maximizing order quantity, \(Q_{ds}\). To overcome this problem the model assumes the channel members are willing to accept a common order quantity \(Q\) that is larger than \(Q_{ds}\) provided all the channel members receive at least their decentralized profits. These restrictions are ensured by the commitment of the manufacturer-distributor coalition to share either minimum or maximum fraction of the retailer’s disposal cost jointly. As such, some surplus profit will be generated at either the

\(^1\)This section is based on the paper “Disposal cost sharing and bargaining for coordination and profit division in a three-echelon supply chain.” published in International Journal of Management Science & Engineering Management, 9(4), 276-285.
retailer’s or the coalition’s end. This surplus will be shared by the channel members through bargaining. Since the surplus profit in the channel is generated through disposal cost sharing, the disposal cost sharing fraction is the key decision variable in the bargaining process.

The present study differs from previous work in the following ways. First, unlike Pasternack [115], here it is assumed that, as an inducement tool, compensation for the disposal costs of deteriorated products is used. The compensation is provided by the manufacturer-distributor coalition. Second, instead of considering sub-supply chain coordination in a three-echelon supply chain as in Seifert et al. [127], the study discusses how channel conflict can be resolved by a side-payment contract. Third, the present study explains a procedure to divide the surplus that is created through coordination by applying Nash bargaining to the products. The disposal cost compensation contract needs the upstream channel members to offer a higher compensation package for the disposal cost of deteriorated inventories. So, the retailer would have enough incentive to choose a higher target quantity, by its possibly benefiting all parties by increased profits compared with those in a decentralized setting. Since inventory decisions depend on sharing the disposal cost, it becomes a key contract parameter involved in such a supply chain coordination contract.

### 3.1.1 Model formulation and basic analysis

In decentralized decision making at the beginning of the replenishment cycle the retailer has $Q_{ds}$ units of inventory. As time progresses, the inventory level decreases due to uniform demand $D$ and deterioration. The inventory level reaches to zero level after time $T_{ds}$. Then, the next replenishment cycle starts. If $q(t)$ is the instantaneous inventory level of the retailer at time $t$ then the governing differential equation is

$$\frac{dq(t)}{dt} + \theta q(t) = -D$$

with the initial and terminal conditions $q(0) = Q_{ds}$ and $q(T_{ds}) = 0$ respectively. Solution of the differential equation is

$$q(t) = Q_{ds}e^{-\theta t} + \frac{D}{\theta}(e^{-\theta T} - 1)$$

Using the terminal condition the order quantity of the retailer is found as

$$Q_{ds} = \frac{D}{\theta}(e^{\theta T_{ds}} - 1)$$

Total average profit of the retailer in the replenishment cycle is

$$\pi_{ds}^r = \frac{1}{T_{ds}} \left[ p_r D T_{ds} - s^r - p_d Q_{ds} - h \int_0^{T_{ds}} q(t) dt - c_d \left( Q_{ds} - \int_0^{T_{ds}} D dt \right) \right]$$

The first, second, third, fourth and fifth terms in the above expression represent sales revenue, ordering cost, purchase cost, holding cost and disposal cost of the retailer respectively. On
simplification it yields

\[ \pi_{ds} = D \left( p + \frac{h}{\theta} + c_d \right) - \frac{s^r}{T_{ds}} - D \left( w^d + \frac{h}{\theta} + c_d \right) \frac{e^{\theta T_{ds}} - 1}{\theta T_{ds}} \]  

(3.4)

From the necessary condition \( d\pi_{ds}/dT_{ds} = 0 \) for the existence of optimal solution yields^2

\[ T_{ds}^* = \frac{2\sqrt{s^r}}{\sqrt{2D(\theta c_d + \theta w^d) + \theta \sqrt{s^r}}} \]  

(3.5)

Substituting \( T_{ds}^* \) of (3.5) in (3.3) one can find the retailer’s optimal EOQ in decentralized decision making.

Furthermore,

\[ \frac{d^2 \pi_{ds}}{dT_{ds}^2} = -\left( \frac{s^r}{T_{ds}^*} + \frac{4\theta D(h + \theta c_d + \theta w^d)}{(2 - \theta T_{ds})^3} \right) \]  

(3.6)

The right hand side of the above expression is negative if \( 2 - \theta T_{ds} > 0 \). Now using \( T_{ds}^* \) of (3.5) and simplifying it is found that \( 2 - \theta T_{ds} = \frac{\sqrt{2D(h + \theta c_d + \theta w^d) + \theta \sqrt{s^r}}}{(2 - \theta T_{ds})^3} > 0 \). Thus, \( d^2 \pi_{ds}/dT_{ds}^2 < 0 \) and hence \( \pi_{ds} \) is concave.

The distributor’s and the manufacturer’s profit functions in decentralized decision making for the retailer’s EOQ are given by

\[ \pi^d_{ds} = \frac{1}{T_{ds}^*} \left[ (w^d - w^m) Q_{ds} - s^d \right] \]  

(3.7)

\[ \pi^m_{ds} = \frac{1}{T_{ds}^*} \left[ (w^m - c) Q_{ds} - s^m \right] \]  

(3.8)

As pointed out, in a multi-echelon vertical manufacturing system, where the product is transferred to the customers downward through different echelon, the set up cost of the manufacturer is substantially higher than the ordering costs of the distributor and the retailer. Thus, when the manufacturer opts for lot-for-lot production policy, its objective is to produce as much units as possible. By doing so the manufacturer reduces its set up cost for per unit product and makes more profit in a single production run. So, the manufacturer will always try to receive order of larger lot than \( Q_{ds} \) from the distributor. Under the channel structure assumed in this study, the distributor simply acts as an intermediator, because it receives the lot from the manufacturer and transfer it to the retailer, and makes profit. Thus, when the manufacturer produces larger lot the distributor receives more profit. But the retailer has no reason to accept an order quantity \( Q \) larger than its decentralized EOQ because its system running cost in such case will increase. As such, to impel the retailer to order larger quantity \( Q \) than \( Q_{ds} \), the manufacturer and the distributor form a coalition that provides a compensation on disposal cost of the retailer. If all the channel members agree for a common order quantity \( Q \) then the retailer’s new average profit in a production cycle is found as

\[ \pi^r = D \left( p + \frac{h}{\theta} + c_d \right) - \frac{s^r}{T} - D \left( w^d + \frac{h}{\theta} + c_d \right) \frac{e^{\theta T} - 1}{\theta T} \]  

(3.9)

^2To derive \( T_{ds}^* \), the approximation \( e^{\theta T_{ds}} = (2 + \theta T_{ds})/(2 - \theta T_{ds}) \) is used.
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Loss of the retailer’s profit for accepting $Q$ instead of $Q_{ds}$ is

$$X = \pi_{ds}^{r} - \pi^{r} \quad (3.10)$$

The average profits of the distributor and the manufacturer for common order quantity $Q$ are respectively as

$$\pi^{d} = \frac{1}{T}[(w^{d} - w^{m})Q - s^{d}] \quad (3.11)$$

$$\pi^{m} = \frac{1}{T}[(w^{m} - c)Q - s^{m}] \quad (3.12)$$

The manufacturer-distributor coalition compensates the loss of the retailer’s profit for common order quantity $Q$. Thus, after providing the compensation to the retailer the profit function of the coalition for order quantity $Q$ is

$$Y = \pi^{m} + \pi^{d} - X \quad (3.13)$$

The first order condition $dY/dT = 0$ for the existence of optimal solution yields (using the approximation $e^{\theta T} = (2 + \theta T)/(2 - \theta T)$ in (3.13) )

$$T^{\ast} = \frac{2\sqrt{s^{r} + s^{d} + s^{m}}}{\sqrt{2D(h + \theta c_{d} + \theta c) + \theta \sqrt{s^{r} + s^{d} + s^{m}}} \quad (3.14)}$$

Moreover,

$$\frac{dY}{dT} = \left[\frac{s^{r} + s^{d} + s^{m}}{T^{3}} + \frac{4\theta D(h + \theta c_{d} + \theta c)}{(2 - \theta T)^{3}}\right]$$

The expression on the right hand side is negative if $2 - \theta T > 0$. But

$$2 - \theta T = 2\sqrt{2(s^{r} + s^{d} + s^{m})D(h + \theta c_{d} + \theta c)/[2D(h + \theta c_{d} + \theta c) - \theta^{2}(s^{r} + s^{d} + s^{m})]} > 0$$

Hence, the joint profit function of the coalition after providing the retailer’s loss to it is concave. Proceeding in the same way as in the decentralized decision making the optimal common order quantity of the channel is found as

$$Q^{\ast} = \frac{D}{\theta} (e^{\theta T^{\ast}} - 1) \quad (3.15)$$

The retailer will show interest to order $Q$ rather than $Q_{ds}$ if it receives at least its decentralized profit. The manufacturer-distributor coalition compensates any loss of the retailer’s profit due to changed ordering policy through compensation on disposal cost. If $\lambda_{\min}$ is the minimum percentage of disposal cost that compensates the retailer’s profit due to coordinated order quantity $Q$ then using (3.10) the following expression can be found.

$$X = \lambda_{\min} \left[c_{d}D \left(\frac{e^{\theta T} - 1}{\theta T^{3}} - 1\right)\right]$$
On simplification the above expression yields

\[ \lambda_{\text{min}} = \frac{\pi_{ds}^r - \pi^r}{c_dD \left( \frac{e^{\theta T} - 1}{\theta T} - 1 \right)} \]  

(3.16)

Therefore, the retailer’s minimum profit for coordinated order quantity is

\[ \pi_{\text{min}} = D \left( p + \frac{h}{\theta} + c_d \right) - s^r \frac{T}{D} - D \left( w^d + \frac{h}{\theta} + c_d \right) \left( \frac{e^{\theta T} - 1}{\theta T} \right) + \lambda_{\text{min}} \left[ c_dD \left( \frac{e^{\theta T} - 1}{\theta T} - 1 \right) \right] \]  

(3.17)

When the manufacturer-distributor coalition provides minimum compensation on the disposal cost it’s profit for coordinated order quantity will be maximum. As such, the basic question is, in which proportion the manufacturer and the distributor within the coalition will share the compensation on disposal cost. Without loss of any generality assume the simplest case that the distributor provides \( k_{\text{min}} \lambda_{\text{min}} \) and the manufacturer provides \((1 - k_{\text{min}}) \lambda_{\text{min}}\). Then maximum profit of the distributor and maximum profit of the manufacturer for coordinated order quantity are respectively as

\[ \pi_{\text{max}}^d = \frac{1}{D} \left( (w^d - w^m)Q - s^d \right) - k_{\text{min}} \lambda_{\text{min}} \left[ c_dD \left( \frac{e^{\theta T} - 1}{\theta T} - 1 \right) \right] \]  

(3.18)

\[ \pi_{\text{max}}^m = \frac{1}{D} \left( (w^m - c)Q - s^m \right) - (1 - k_{\text{min}}) \lambda_{\text{min}} \left[ c_dD \left( \frac{e^{\theta T} - 1}{\theta T} - 1 \right) \right] \]  

(3.19)

The value of the parameter \( k_{\text{min}} \) determines the maximum profits that the distributor and the manufacturer can gain. Thus, the parameter \( k_{\text{min}} \) is crucial to both the parties and it depends on their negotiation powers. The member having more negotiation power than the other will gain more profit for the coordinated order quantity.

The manufacturer-distributor coalition can provide compensation on the retailer’s disposal cost for coordinated order quantity as long as it’s profit is not less than the decentralized profit. If \( \lambda_{\text{max}} \) is the maximum percentage of compensation on disposal cost that the manufacturer-distributor coalition provides then it follows that

\[ \pi^m + \pi^d - (\pi_{ds}^m + \pi_{ds}^d) = \lambda_{\text{max}} \left[ c_dD \left( \frac{e^{\theta T} - 1}{\theta T} - 1 \right) \right] \]

On simplification, the maximum percentage of compensation on disposal cost that the manufacturer-distributor coalition can provide to the retailer, is found as

\[ \lambda_{\text{max}} = \frac{\pi^m + \pi^d - (\pi_{ds}^m + \pi_{ds}^d)}{c_dD \left( \frac{e^{\theta T} - 1}{\theta T} - 1 \right)} \]  

(3.20)

Similar to the previous case, if the distributor and the manufacturer share \( k_{\text{max}} \lambda_{\text{max}} \) and \((1 - k_{\text{max}}) \lambda_{\text{max}}\) then their minimum profits are found as

\[ \pi_{\text{min}}^d = \frac{1}{D} \left( (w^d - w^m)Q - s^d \right) - k_{\text{max}} \lambda_{\text{max}} \left[ c_dD \left( \frac{e^{\theta T} - 1}{\theta T} - 1 \right) \right] \]  

(3.21)
\[ \pi_{\text{min}} = \frac{1}{T} [(w^m - c)Q - s^m] - (1 - k_{\text{max}})\lambda_{\text{max}} \left[ c_d D \left( \frac{e^{\theta T} - 1}{\theta T} - 1 \right) \right] \] (3.22)

Consequently, the retailer’s maximum profit is
\[ \pi_{\text{max}} = D \left( p + \frac{h}{\theta} + c_d \right) - s^r - D \left( w^d + \frac{h}{\theta} + c_d \right) \left( \frac{e^{\theta T} - 1}{\theta T} \right) + \lambda_{\text{max}} \left[ c_d D \left( \frac{e^{\theta T} - 1}{\theta T} - 1 \right) \right] \] (3.23)

Note from equations (3.18) and (3.21) that the value of \( k, i = \{\text{min}, \text{max}\} \) is determined through the negotiation between the manufacturer and the distributor. But (3.19) and (3.21) do not ensure that the distributor’s profit in both the cases are greater than its decentralized profit, i.e., \( \pi_i^d \geq \pi_{ds}^d, i = \{\text{min}, \text{max}\} \). To ensure in both the cases the distributor receives at least its decentralized profit, simplifying the inequality the upper limit of \( k_i \) can be found as
\[ k_i \leq \frac{\pi_i^d - \pi_{ds}^d}{\lambda_i \left[ c_d D \left( \frac{e^{\theta T} - 1}{\theta T} \right) \right]}, i = \{\text{min}, \text{max}\} \]

Similarly, the range of \( k_i, i = \{\text{min}, \text{max}\} \), which ensures that the manufacturer’s profit for coordinated order quantity after sharing compensation on disposal cost with the distributor is at least equal to its decentralized order quantity can be determined from the inequality \( \pi_i^m \geq \pi_{ds}^m, i = \{\text{min}, \text{max}\} \). Simplifying the inequality the lower limit of \( k_i \) can be found as
\[ k_i \geq 1 - \frac{\pi_i^m - \pi_{ds}^m}{\lambda_i \left[ c_d D \left( \frac{e^{\theta T} - 1}{\theta T} \right) \right]}, i = \{\text{min}, \text{max}\} \]

Thus, after providing compensation on disposal cost to the retailer, the distributor and the manufacturer share compensation in the proportion \( k_i, i = \{\text{min}, \text{max}\} \) between them such that it always ensures that both parties will receive at least their decentralized profits. The range of the compensation cost sharing fraction is given by
\[ 1 - \frac{\pi_i^m - \pi_{ds}^m}{\lambda_i \left[ c_d D \left( \frac{e^{\theta T} - 1}{\theta T} \right) \right]} \leq k_i \leq \frac{\pi_i^d - \pi_{ds}^d}{\lambda_i \left[ c_d D \left( \frac{e^{\theta T} - 1}{\theta T} \right) \right]}, i = \{\text{min}, \text{max}\} \] (3.24)

\( \lambda_{\text{min}} \) and \( \lambda_{\text{max}} \) are two bounds of the compensation on disposal cost of deteriorated product. For any \( \lambda \in (\lambda_{\text{min}}, \lambda_{\text{max}}) \) profit of the manufacturer-distributor coalition and the profit of the retailer are win-win. Since \( k \) depends on any \( \lambda \in (\lambda_{\text{min}}, \lambda_{\text{max}}) \), the range of \( k \) for which the profit of the manufacturer and the manufacturer’s profit are win-win is found as
\[ 1 - \frac{\pi_i^m - \pi_{ds}^m}{\lambda \left[ c_d D \left( \frac{e^{\theta T} - 1}{\theta T} \right) \right]} \leq k \leq \frac{\pi_i^d - \pi_{ds}^d}{\lambda \left[ c_d D \left( \frac{e^{\theta T} - 1}{\theta T} \right) \right]}, \lambda \in (\lambda_{\text{min}}, \lambda_{\text{max}}) \] (3.25)

From (3.25) following results can be realized. First, for \( \lambda = \lambda_{\text{min}} \), the range of \( k \), using (3.16) can be found as
\[ 1 - \frac{\pi_i^m - \pi_{ds}^m}{\pi_{ds}^r - \pi^r} \leq k_{\text{min}} \leq \frac{\pi_i^d - \pi_{ds}^d}{\pi_{ds}^r - \pi^r} \] (3.26)

Note that, the lower bound of \( k_{\text{min}} \) may be negative when \( \pi_i^m - \pi_{ds}^m > \pi_{ds}^r - \pi^r \). The intuitive reason is as follows. When the manufacturer-distributor coalition provides minimum compen-

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sation on disposal cost, the channel is coordinated and it receives entire surplus profit of the channel. If the manufacturer and the distributor share the compensation within the coalition in such a way that the manufacturer’s share of surplus profit is greater than the retailer's loss of profit due to coordinated order quantity then the lower bound of $k_{min}$ is negative. Similarly, the upper bound of $k_{min}$ may be greater than one when the portion of surplus profit received by the distributor is higher than the loss of profit of the retailer due to coordinated order quantity. However, whether $k_{min}$ is negative or greater than one that depends on two factors, viz, the retailers profit loss for coordinated order quantity and surplus profit generated in the channel for coordination. Second for $\lambda = \lambda_{max}$, using (3.20), instead of a range $k_{max}$ is found as

$$k_{max} = \frac{\pi^d - \pi^d_{ds}}{(\pi^m - \pi^m_{ds}) + (\pi^d - \pi^d_{ds})}$$

(3.27)

The intuition is straightforward. When the manufacturer-distributor coalition provides maximum compensation on disposal cost it's profit is the sum of the decentralized profits. Obviously the retailer will take away entire surplus profit. So, for a particular disposal cost compensation split the coalition’s decentralized profit will be ensured. Fig-3.1 represents the range of $k$ for $\lambda \in (\lambda_{min}, \lambda_{max})$. It indicates the same characteristics those are mentioned above.

If all channel members operate jointly then under centralized decision making the channel profit is

$$\pi = \frac{1}{T} \left[ (p - c)D - D \left( \frac{h}{\theta} + c_d \right) \left( e^{\theta T} - 1 \right) - \left( s^r + s^d + s^m \right) \right]$$

(3.28)

Optimal cycle length and optimal order quantity can be found as (3.14) and (3.15) respectively. That is, when the manufacturer-distributor coalition compensates the retailer's loss of profit through sharing disposal cost for changed order quantity, the system will be coordinated. Now, in which ratio the surplus profit will be shared by the channel members that depends on their bargaining powers. Several bargaining models are available in the literature to divide the surplus among the channel members. Here the study will consider the Nash bargaining product to discuss the issue.

### 3.1.2 Surplus sharing

The Nash bargaining model that has been used in various contexts, is an axiomatic derivation of bargaining solution. The axiomatic derivation leaves out the actual process of negotiations while focusing on the expected outcome based on pre-specified solution procedures. Also the axioms does not reflect the rationale of the agents or the process in which the agreement is reached. One of the important characteristics of the Nash solution concept is that the outcome is random because it depends on the participating players negotiation powers. In Nash bargaining model the objective function is the product of the players benefit from cooperation and it must be maximized. Each players benefit is the difference between the negotiated profit and profit under decentralized decision making.

As assumed throughout the model, the manufacturer and the distributor form a coalition and the coalition as a single entity provides the compensation on disposal cost to the retailer.
The bargaining process that is used in this study to divide the channel surplus is a nested bargaining process, where a bargaining takes place on the basis of the previous bargaining outcome. It consists of two steps. First, the manufacturer-distributor coalition bargains with the retailer. Second, once the coalition and the retailer finalize their share, the manufacturer and the distributor bargain within the coalition to divide the surplus that is received by their coalition from the first step, between them. In the first step, since any value of $\lambda \in (\lambda_{\text{min}}, \lambda_{\text{max}})$ eliminates channel conflict, the participating parties will bargain to determine an acceptable $\lambda \in (\lambda_{\text{min}}, \lambda_{\text{max}})$ that will maximize the Nash bargaining product. The Nash bargaining product can be found as

$$\max_{\lambda_{\text{min}} \leq \lambda \leq \lambda_{\text{max}}} \left[ (\pi^m(\lambda) + \pi^d(\lambda)) - (\pi^m_{ds} - \pi^d_{ds}) \right]^{(1-\alpha_r)}$$

(3.29)

Where $\alpha_r$ and $(1 - \alpha_r)$, $0 \leq \alpha_r \leq 1$ are negotiation powers of the manufacturer-distributor coalition and the retailer respectively and

$$\pi^r(\lambda) = D \left( p + \frac{h}{\theta} + c_d \right) - \frac{s^r}{T} - D \left( w^d + \frac{h}{\theta} + c_d \right) \left( \frac{e^{\theta T} - 1}{\theta T} \right) + \lambda \left[ c_d D \left( \frac{e^{\theta T} - 1}{\theta T} - 1 \right) \right]$$

$$\pi^d(\lambda) = \frac{1}{T} \left[ (w^d - w^m)Q - s^d \right] - k\lambda \left[ c_d D \left( \frac{e^{\theta T} - 1}{\theta T} - 1 \right) \right]$$

$$\pi^m(\lambda) = \frac{1}{T} \left[ (w^m - c)Q - s^m \right] - (1 - k)\lambda \left[ c_d D \left( \frac{e^{\theta T} - 1}{\theta T} - 1 \right) \right]$$

The necessary optimality condition with respect to compensation on disposal cost yields

$$\lambda^* = \alpha_r \lambda_{\text{min}} + (1 - \alpha_r)\lambda_{\text{max}}$$

(3.30)

Thus, $\lambda^*$ determines division of channel surplus between the retailer and the manufacturer-distributor coalition.

As soon as, the manufacturer-distributor coalition receives its share of the channel surplus, the manufacturer and the distributor bargain between them to share it. Since, the manufacturer and the distributor compensate the retailer’s disposal cost in the proportions $k$ and $(1 - k)$ within the range (3.25) of $\lambda^*$, the Nash bargaining product will be maximized with respect to $k$. The Nash bargaining product is found as

$$\max_k [\pi^m(k) - \pi^m_{ds}]^{\beta_{md}} [\pi^d(k) - \pi^d_{ds}]^{(1-\beta_{md})}$$

(3.31)

Where $\beta_{md}$ and $(1 - \beta_{md})$, $0 \leq \beta_{md} \leq 1$, are negotiation powers of the manufacturer and the distributor respectively and

$$\pi^d(k) = \frac{1}{T} \left[ (w^d - w^m)Q - s^d \right] - k\lambda^* \left[ c_d D \left( \frac{e^{\theta T} - 1}{\theta T} - 1 \right) \right]$$

$$\pi^m(k) = \frac{1}{T} \left[ (w^m - c)Q - s^m \right] - (1 - k)\lambda^* \left[ c_d D \left( \frac{e^{\theta T} - 1}{\theta T} - 1 \right) \right]$$
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The necessary condition for the existence of optimal solution yields,

\[ k = (1 - \beta_{md}) + \frac{\beta_{md}(\pi^d - \pi^d_{ds}) - (1 - \beta_{md})(\pi^m - \pi^m_{ds})}{(1 - \alpha_r)(\pi^m + \pi^d - \pi^d_{ds} - \pi^m_{ds}) - \alpha_r(\pi^r - \pi^r_{ds})} \]  (3.32)

Therefore, optimal profits of the retailer, the distributor and the manufacturer are respectively as

\[ \pi^*_2 = \pi^*_ds + (1 - \alpha_r)[(\pi^m + \pi^d + \pi^r) - (\pi^m_{ds} + \pi^d_{ds} + \pi^r_{ds})] \]  (3.33)

\[ \pi^d_2 = \pi^d_{ds} + \alpha_r(1 - \beta_{md})[(\pi^m + \pi^d + \pi^r) - (\pi^m_{ds} + \pi^d_{ds} + \pi^r_{ds})] \]  (3.34)

\[ \pi^m_2 = \pi^m_{ds} + \alpha_r\beta_{md}[(\pi^m + \pi^d + \pi^r) - (\pi^m_{ds} + \pi^d_{ds} + \pi^r_{ds})] \]  (3.35)

From the above bargaining process three possible results are realized. First, the optimality criteria (3.30) interprets that the manufacturer-distributor coalition provides the weighted average of the minimum and maximum possible compensation on disposal cost. The weighted average is based on the negotiation powers. So, the player having more negotiation power than the other will acquire larger surplus share than the other. In particular, if the negotiation power of the coalition is significantly higher than the retailer, i.e., \( \alpha_r \to 1 \) then the coalition will realize entire channel surplus by providing \( \lambda^* \to \lambda_{min}^* \) to the retailer and vice versa. Second, when the manufacturer and the distributor bargain within the coalition to share the channel surplus, which is received by their coalition, the negotiated outcome is the weighted average of their bargaining powers. Also it depends heavily on the previous bargaining outcome and same conclusion can be drawn as in the previous case for the relation between the negotiation power and channel surplus. Third, the coalition and the retailer have same negotiation power, i.e., \( \alpha_r = 1/2 \) then \( \lambda^* = (\lambda_{min} + \lambda_{max})/2 \). As such, the coalition and the retailer will divide the channel surplus equitably. Moreover, if the manufacturer and the distributor have same negotiation power then they share the surplus equitably between them. As a result, in addition to the decentralized profit, the retailer receives half of the channel surplus and each of the manufacturer and the distributor realize quarter of the channel surplus in addition to their decentralized profits.

3.1.3 Numerical illustration

Assume that annual demand is \( D = 10000 \) units. The product deteriorates at a rate \( \theta = 0.4 \) per year. Set up cost of the manufacturer is \( S_m = \$10000 \) per production cycle. Ordering cost of the retailer and the distributor are respectively as \( S_r = \$3000 \) and \( S_d = \$4000 \). Unit production cost of the manufacturer is \( c = \$110 \). Selling prices of the manufacturer, the distributor and the retailer are \( w^m = \$130 \), \( w^d = \$140 \) and \( p = \$160 \) respectively. Holding cost and disposal cost of the retailer are \( h = \$2 \) per unit per year and \( c_d = 0.8c \) per unit respectively.

Under decentralized decision making the replenishment cycle length and order quantity are \( T^*_ds = 0.08451 \) year and \( Q^*_ds = 859.52 \) units. Optimal profit of the manufacturer, the distributor and the retailer are \( \pi^*_m = \$85085.9 \), \( \pi^*_d = \$54376.2 \) and \( \pi^*_r = \$124672 \) respectively. Total channel profit is \$264134.1

When the channel members operate jointly through the compensation on disposal cost, the
optimal cycle length is $T^* = 0.19658$ year and optimal order quantity is $Q^* = 2045.17$ units. Minimum and maximum compensations on the disposal cost offered by the manufacturer-distributer coalition are $\lambda^*_{\min} = 95.68\%$ and $\lambda^*_{\max} = 285.53\%$. For $\lambda^*_{\min}$ the optimal channel profit is $\pi^*_{\min} + \pi^d_{\min} + \pi^m_{\max} = \pi^*_{\min} + \pi^d_{\max} + \pi^m_{\min} = $331575.9. For $\lambda^*_{\max}$ the optimal channel profit is $\pi^*_{\max} + \pi^d_{\min} + \pi^m_{\max} = $331575.9. Optimal profits for both the cases are aligned with the optimal profit of the centralized decision making. For minimum compensation on disposal cost the manufacturer-distributer coalition receives entire channel surplus $67441.8$ and for $\lambda^*_{\max}$ the retailer receives the same.

![Figure 3.1: Range of k with respect to $\lambda$](image)

If the manufacturer-distributer coalition provides $\lambda^*_{\min}$ fraction of compensation on disposal cost then for any $k \in (0.86242, 2.12181)$ the manufacturer’s profit and the distributer’s profit are greater than their decentralized profits. The manufacturer and the distributer will receive profits within the ranges $\pi^*_{\min} \in ($85085.9, $152528)$ and $\pi^d_{\min} \in ($54376.2, $121818)$ and the retailer will get it’s decentralized profit. However, if the manufacturer-distributer coalition provides $\lambda^*_{\max}$ compensation on disposal cost then $k^* = 0.289$ and the manufacturer and the distributer will receive their decentralized profits.

When the retailer and the manufacturer-distributer coalition coalition bargain with equal negotiation powers, i.e., $\alpha_r = 0.5$ and then the manufacturer and the distributer bargain within the coalition with equal negotiation powers, i.e., $\beta_{md} = 0.5$ then $\lambda^* = 190.61\%$ and $k^* = 149.21\%$. Profits of the manufacturer, the distributer and the retailer are $101946.35$, $71236.65$ and $158392.6$ respectively. The retailer receives half of the channel surplus. The manufacturer and the distributer receive quarters of it. Profits of the manufacturer, the distributer and the retailer increase 19.82%, 31% and 27.04% respectively from decentralized decision making. From the numerical example following managerial implications are realized.

1. Range of the compensation fraction that the manufacturer-distributer coalition provides to the retailer indicates that the coalition may provide more than total disposal cost to the retailer as a compensation package still win-win outcome is achievable. This may happen if the surplus
created for channel coordination is substantially higher than total disposal cost of the retailer.

The manufacturer-distributor coalition compensates the loss of the retailer’s profit by providing \( \lambda^*_{min} \) percentage of the disposal cost. For channel coordination \((\lambda^*_{max} - \lambda^*_{min}) \times \text{disposal cost} \) amount of surplus is generated in the channel. Thus, to divide additional profit the coalition may provide \( \lambda^*_{min} \) plus a portion of \((\lambda^*_{max} - \lambda^*_{min}) \times \text{disposal cost}\), which may be greater than one.

\[
\lambda^*_{min} + \left(\lambda^*_{max} - \lambda^*_{min}\right) \times \text{disposal cost}
\]

\[\lambda^*_{max}\]

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**Figure 3.2:** Percentage change in the channel members profit with respect to the percentage change in: (a) demand; (b) holding cost; (c) deterioration; (d) retailers set-up cost; (e) distributors set-up cost; (f) manufacturers set-up cost.

2. The retailer’s profit for coordinated order quantity increases as the disposal cost compensation fraction \( \lambda \) increases in its range \((\lambda^*_{min}, \lambda^*_{max})\). Reverse trend may be noted for the manufacturer-distributor coalition. The result is obvious because channel profit remains same for any \( \lambda \in (\lambda^*_{min}, \lambda^*_{max}) \) and the surplus in the retailer’s profit beyond it’s decentralized profit is transferred from the manufacturer-distributor coalition as the compensation on disposal cost.
3. Profits of the manufacturer and the distributor are heavily influenced by the compensation fraction $\lambda$. As $\lambda \in (\lambda_{\text{min}}^*, \lambda_{\text{max}}^*)$ increases the surplus profit that is received by the coalition decreases. As a result the range of the cost sharing fraction $k$ with in the coalition shortened.

It ultimately provides a particular profit split when $\lambda = \lambda_{\text{max}}^*$, i.e., the manufacturer and the distributor receive their decentralized profits (see Fig-3.1). Further, the lower bound of $k$ may be negative and the upper bound of it may be greater than one. That is, instead of receiving a portion of channel surplus if the distributor provides a portion of its profit for coordinated order quantity then also it will receive at least its decentralized profit. The same conclusion can be drawn for the upper limit of $k$ for the manufacturer also.

4. The model is highly sensitive for the error in estimating $D$ and moderately sensitive for the change in $\theta$ (see also Fig-3.2). Low sensitivity is found for change in the parameter value $\mu$. When $D$ is increases with other parameter values fixed, it is found that the profits of the channel members increase. Reverse trends are found for the parameters $\theta$ and $\mu$.

5. If the ordering cost of the retailer increases with other parameter values fixed then (i) the retailer’s profit decreases, whereas the manufacturer’s profit and the distributor’s profit increases, (ii) $Q_{ds}^*$ and $Q^*$ increase and (iii) compensation fraction bounds increase. The intuition is straightforward. Since the ordering cost of the retailer is higher, it orders larger quantity to maximize profit and it has a negative impact on the retailer’s profit. To maximize the effect of ordering cost of the retailer, the channel EOQ will be high. Ratio of the channel EOQ and the retailer’s EOQ is low and it leads to low compensation fraction bounds. Reverse trend may be noted for the increment of either the manufacturer’s set up cost or the distributor’s ordering cost.

### 3.2 Coordinating a three-echelon distribution channel and benefit sharing considering perishable product

This section addresses the problems of coordination and benefit sharing in a conventional distribution channel, that consists of a manufacturer, multiple distributors and multiple retailers corresponding to each distributor. The manufacturer produces a perishable product and sells it to the customers through the distribution channel. The research proposes a hybrid, quantity discount-compensation on deterioration cost, contract and examine its efficiency in resolving both horizontal and vertical conflict. The manufacturer provides quantity discount to each distributor independently. Each distributor compensates a percentage of deterioration cost of each of its retailer independently. Finally, Nash bargaining product is used to divide the benefit, which is obtained for synchronized decision of the channel members. Since, the distributors’ costs and benefit shares are dependent on the manufacturer’s and the retailers’ decisions simultaneously, the study proposes two approaches, namely backward contract-bargaining process and forward contract-bargaining process. In the former first the retailers bargain with their  

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3This section is based on the paper entitled “Coordination and benefit sharing in a three-echelon distribution channel with perishable product.” communicated to *Applied Mathematical Modelling*. 
distributors independently in a pre-defined sequence. Based on the outcome, each distributor bargains with the manufacturer independently in a pre-specified sequence. In forward contract-bargaining, the channel members bargain in reverse order. Models dealt with more than one member in different stages are very few. Khouja [80] has developed a three-echelon supply chain, where at each stage there are multiple members and each member can supply to two or more buyers. Ben-Daya and Al-Nassar [13] have generalized Khouja [80] to the case, where shipment between stages can be made before a whole lot is completed. In another paper Khouja [82] has proposed synchronization of decisions that starts from supply of raw materials and ends at customers. Jaber and Goyal [70] have investigated the synchronization of order quantities in a three-echelon supply chain. In the first, second and third level of the chain there are multiple buyers, one manufacturer and multiple retailers respectively. They have shown that, when the players agree to coordinate, it is possible to have some of the players benefiting more than others. They have not proposed any coordination contract that resolves channel conflict. Recently Jonrinaldi and Zhang [76] have developed a supply chain that consists of multiple players in different echelons. They have assumed finite production capacity of the suppliers and have determined the optimal decisions over a finite time horizon considering reverse logistics. The present study extends the literature in the following aspects. First, impact of product deterioration has been examined in such a complex supply chain structure. Second, using coordination contract channel conflict is resolved and win-win benefit ranges for all channel members are identified in closed form. Third, to settle for particular benefit splits within the win-win ranges the channel members bargain. In fact the study proposes two nested contract-bargaining processes, viz, backward contract-bargaining and forward contract-bargaining. Optimal decisions in decentralized, semi-centralized and centralized channels are determined by formulating cost functions. Convexities of the cost functions are also verified. Using quantity discount, percentage compensation on deterioration cost and bargaining the channel conflict is resolved and the benefit is divided among the channel members.

The present study differs from the prior works in the following aspects. First, instead of considering channel structure of Jaber and Goyal [70], the present model assumes that the third level of the channel consists of a manufacturer, the second level consists of multiple distributors and in the first level corresponding to each distributor there are multiple retailers. Second, unlike the previous studies, the present study incorporates product deterioration into modeling and analysis. Third, the present model applies a hybrid contract to eliminate channel conflict. The hybrid contract consists of quantity discount that the manufacturer provides to each distributor and percentage compensation on deterioration cost, which each distributor provides to each of it’s retailers. The manufacturer provides different quantity discount to different distributor. Also percentage compensations are different for different retailers. To the best of the authors knowledge, in all the models, which assume multi-echelon, multiple members in different echelons, coordination contracts are not used to find best channel performance. Instead, it is assumed the channel members cooperate and agree for the common replenishment cycle or order quantity, which is found through synchronized decisions in different echelons. That is, the models are analyzed based on centralized decision making process. As a result some channel members are benefited and some are not ([70], [76]). Fourth, the win-win ranges for all the channel members for coordinated decision are identified in closed forms. Fifth, in the channel structure as assumed in this model, the distributors act as the intermediators and play central roles. Optimal decisions and costs of the distributors are dependent on the manufacturer’s and the retailer’s decisions. When coordination contract is applied to resolve channel conflict
and the channel members bargain to divide the benefit, the distributors’ costs depend on their sequence of approaches towards sharing the costs. Because of standard marketing practice, which assumes a channel member does not reveal its benefit achievement from the other echelon members, the study incorporates two procedures, namely backward contract-bargaining process and forward contract-bargaining process. In the former process, each retailer of a particular distributor assumes the distributor does not get anything from the manufacturer. Based on it, first win-win ranges for the retailer and the distributor for compensation on deterioration cost are identified. Within this range they bargain on compensation for benefit split. A particular distributor has multiple retailers and the distributor has different reservations for different retailers. Corresponding to each retailer, the distributor settles benefit share. In the second stage, based on the decentralized cost minus accumulated benefit shares from the retailers, first the distributor and the manufacturer identify the win-win ranges for quantity discount. Finally, they bargain on percentage compensation on deterioration cost for a particular benefit share. Since, the manufacturer also has different reservations for different distributors, it identifies win-win range and benefit share for each distributor independently. Thus, in the backward contract-bargaining process the coordination contracts and bargaining are interdependent. The process is also nested because of the sequence of events as, percentage compensation on deterioration cost for win-win range → bargaining on compensation for benefit share → quantity discount for win-win range. Finally, they bargain on quantity discount for benefit share. In the forward contract-bargaining process, same events in reverse sequence are performed. Based on decentralized cost minus benefit share from the manufacturer, a distributor and each of its retailers determine their win-win ranges for compensation fractions. Finally, they bargain for particular benefit splits. The bargaining game that is used here is based on Nash bargaining product. Moreover, as the inventory decision depends on quantity discount and percent compensation on deterioration cost, these two become key parameters for the best outcome in such a supply chain.

3.2.1 Model formulation and basic analysis

Consider a three-echelon distribution channel consisting of a manufacturer, multiple distributors and multiple retailers of each distributor. The manufacturer produces a perishable product and supply it to n distributors. The jth distributor (j = 1, 2, ..., n) transfers the product to its n_{ij}(i = 1, 2, ..., n) number of retailers and the retailers sell the product to customers and fulfill the channel demand. Thus, there is \( \sum_{j=1}^{n} \sum_{i=1}^{n_{ij}} n_{ij} \) number of retailers in the system. Assume that each of n_{ij} (i = 1, 2, ..., n; j = 1, 2, ..., n) retailers faces deterministic demand D_{ij} (i = 1, 2, ..., n; j = 1, 2, ..., n) per unit time. A particular retailer can place order to a particular distributor, who is associated with it, i.e., for each retailer there is only one available distributor. The product deteriorates at a constant rate \( \theta \). It can’t be reworked and is disposed without any salvage value. Cost of deterioration for a channel member is equal to the purchasing cost of the member. The manufacturer produces the product at a unit cost \( c \) and sales it to the distributors at a wholesale price \( w^m \). Each distributor supplies the product to its retailers at a unit cost \( w^d \). Shortages are not allowed at each stages of the system. The lead times between manufacturer and distributors, distributors and retailers are considered as zero because the demand of the end customer is considered as constant. Under this model setting optimal replenishment and ordering policies for decentralized, semi-centralized and centralized decision making processes are discussed.

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4Fig-2.12 (chapter-2) represents a descriptive diagram of the supply chain.
3.2.2 Decentralized policy

In decentralized structure, members do not pay attention to overall performance of the channel. Each member concentrates on minimizing its own cost function. The upstream channel members (manufacturer & distributors) make their decisions depending on the downstream channel members’ (retailers & distributors) decisions. The downstream channel members have different demands, holding costs, setup costs. When these members optimize their own decisions (order cycle and order quantity), it appears different for different channel members although they are in the same stage.

3.2.2.1 Retailers’ decisions

At the beginning of the replenishment cycle $ij$ ($i = 1, 2, ..., n; j = 1, 2, ..., n$) retailer has $Q_{ij}^r$ ($i = 1, 2, ..., n; j = 1, 2, ..., n$) units of inventory. As time progresses, the inventory level decreases due to constant demand and deterioration. The inventory level reaches zero level after time $T_{ij}^r$ ($i = 1, 2, ..., n; j = 1, 2, ..., n$) then the next replenishment cycle starts. The instantaneous inventory level is governed by the following differential equation

$$\frac{dq_{ij}(t)}{dt} + \theta q_{ij}(t) = -D_{ij}^r,$$  \hspace{1cm} (3.36)

with the initial and terminal conditions $q_{ij}(0) = Q_{ij}^r$ and $q(T_{ij}^r) = 0$ respectively.

Solving the differential equation, the instantaneous inventory level of the $ij$th retailer at time $t$ is found as

$$q_{ij}(t) = \frac{D_{ij}^r}{\theta} (e^{\theta(T_{ij}^r-t)} - 1), \hspace{1cm} i = 1, 2, ..., n; j = 1, 2, ..., n \hspace{1cm} (3.37)$$

Using the terminal condition the order quantity of the $ij$th retailer is found as

$$Q_{ij}^r = \frac{D_{ij}^r}{\theta} (e^{\theta T_{ij}^r} - 1), \hspace{1cm} i = 1, 2, ..., n; j = 1, 2, ..., n \hspace{1cm} (3.38)$$

Average system running cost of the $ij$th retailer ($i = 1, 2, ..., n; j = 1, 2, ..., n$) retailer is the sum of set up cost, holding cost and purchase cost, and is given by

$$C_{ij}^r = \frac{1}{T_{ij}^r} \left( s_{ij}^r + h_{ij}^r \int_0^{T_{ij}^r} q(t) dt + w^d Q_{ij}^r \right)$$

Which on simplification yields

$$C_{ij}^r = D_{ij}^r \left( \frac{h_{ij}^r}{\theta} + w^d \right) e^{\theta T_{ij}^r} - 1 \frac{s_{ij}^r}{\theta T_{ij}^r} + \frac{s_{ij}^r}{T_{ij}^r} - D_{ij}^r \frac{h_{ij}^r}{\theta}, \hspace{1cm} i = 1, 2, ..., n; j = 1, 2, ..., n \hspace{1cm} (3.39)$$

Necessary condition $dC_{ij}^r/dT_{ij}^r = 0$ for the existence of optimal solution yields

$$T_{ij}^{r*} = \frac{2 \sqrt{s_{ij}^r}}{\sqrt{2D_{ij}^r (h_{ij}^r + \theta w^d)} + \theta \sqrt{s_{ij}^r}}, \hspace{1cm} i = 1, 2, ..., n; j = 1, 2, ..., n \hspace{1cm} (3.40)$$
Substituting (3.40) in (3.38) the optimal order quantity of the \( ij \)th retailer \( (i = 1, 2, ..., n; j = 1, 2, ..., n) \) is found as

\[
Q_{ij}^* = \frac{D_{ij}}{\theta} e^{\theta T_{ij}^*} - 1, \quad i = 1, 2, ..., n; j = 1, 2, ..., n
\]  

(3.41)

Furthermore,

\[
\frac{d^2 C_{ij}^*}{dT_{ij}^*} = \frac{2s_{ij}^*}{T_{ij}^*} \left( 1 + \frac{\theta \sqrt{s_{ij}^*}}{\sqrt{2\theta D_{ij}^* (h_{ij}^* + \theta w)}} \right) > 0
\]

Therefore, \( ij \)th retailer’s cost function is convex and the optimal cycle length \( T_{ij}^* \) provides the global minimum.

### 3.2.2.2 Distributers’ decisions

The \( j \)th distributer \( (j = 1, 2, ..., n) \) satisfy orders of its \( n_j \) retailers simultaneously. Since, a retailer can place an order only to it’s upstream distributer, the demand per unit time of the \( j \)th distributer is given by

\[
D_j^d = \sum_{i=1}^{n_j} Q_{ij}^*, \quad j = 1, 2, ..., n
\]

The distributers and their associated retailers operate independently. So, it is not possible to transfer inventories from one channel member to another in the same stage of the channel. Also, it is not possible for a distributer to supply inventories to its non-associated retailer. This assumption is quite justified and consistent with real market. In a conventional distribution channel, channel members are generally situated at geographically dispersed locations. Transferring inventories in such case may incur huge transportation cost to the associated members that may be larger than the shortage cost. Further, it may violate the agreement among the channel members. The retailers under a distributer operate with completely different set of parameters. So, their replenishment cycles will be different and they will place orders discretely. As such, a distributer needs to carry a large stock of the product to satisfy all demands of it’s associated retailers at the same time and product deterioration at it’s own and retailer’s end. Otherwise, it will face stock out and/or late deliveries (27, 28). The \( j \)th distributer’s \( (j = 1, 2, ..., n) \) order quantity is the sum of safety stock, general stock and first order from all its associated retailers. When all the retailers replenish at the same time, the largest possible aggregate order quantity is \( \sum_{i=1}^{n_j} Q_{ij}^* \). Since the product deteriorates at a constant rate \( \theta \), the safety stock of the \( j \)th distributer is \( (1 + \theta + \theta^2 + \theta^3 + ...) \sum_{i=1}^{n_j} Q_{ij}^*/(1 - \theta) \). At the beginning of the order cycle, the distributer satisfies all demands of its associated retailers. Thus, the general stock of the \( j \)th \( (j = 1, 2, ..., n) \) distributer is \( T_j^d \sum_{i=1}^{n_j} Q_{ij}^* \). For deterioration this stock increases to \( (T_j^d D_j^d - \sum_{i=1}^{n_j} Q_{ij}^*)/(1 - \theta) \). Therefore, the order quantity of the \( j \)th distributer for each replenishment cycles is

\[
Q_j^d = \frac{\sum_{i=1}^{n_j} Q_{ij}^*}{1 - \theta} + \frac{T_j^d D_j^d - \sum_{i=1}^{n_j} Q_{ij}^*}{1 - \theta} + \sum_{i=1}^{n_j} Q_{ij}^* = \sum_{i=1}^{n_j} Q_{ij}^* + \frac{T_j^d D_j^d}{1 - \theta}, \quad j = 1, 2, ..., n
\]  

(3.42)
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The average cost function of the j-th distributor is

\[ C^d_j = \frac{s^d_j}{T^d_j} + \left( \frac{h^d_j}{1 - \theta} \right) \sum_{i=1}^{n_j} Q^r_{ij} + \frac{h^d_j[D^d_j - \sum_{i=1}^{n_j} Q^r_{ij}^*]}{2(1 - \theta)} + \frac{w^m}{T^d_j} \left[ \sum_{i=1}^{n_j} Q^r_{ij}^* + \frac{1}{1 - \theta}T^d_j D^d_j \right] \]  (3.43)

The first term is the set up cost per unit time. The second term is the holding cost for a stock to anticipate the orders from the retailers at the same time if the distributor is to have zero stock outs. Third term is the holding costs for the general stock, \( T^d_j D^d_j - Q^D_j \) because at the beginning of order cycle, products at distributors will be immediately used to satisfy the first order from retailers. Last term describes purchasing cost.

Differentiating \( C^d_j \) with respect to \( T^d_j \) and equating to zero the optimal cycle length of the jth \((j = 1, 2, ..., n)\) distributor is found as

\[ T^d*_j = \sqrt{\frac{2(1 - \theta)[s^d_j + w^m \sum_{i=1}^{n_j} Q^r_{ij}^*]}{h^d_j D^d_j}}, \quad j = 1, 2, ..., n \]  (3.44)

Substituting (3.44) in (3.42) and simplifying, the jth \((j = 1, 2, ..., n)\) distributor’s optimal order quantity can be found as

\[ Q^d*_j = \sum_{i=1}^{n_j} Q^r_{ij} + \frac{1}{1 - \theta} \sqrt{\frac{2(1 - \theta)[s^d_j + w^m \sum_{i=1}^{n_j} Q^r_{ij}^*]D^d_j}{h^d_j}} \]  (3.45)

Moreover,

\[ \frac{d^2 C^d_j}{dT^d_j} = \frac{2(s^d_j + w^m Q^d_j)}{(T^d_j)^3} > 0 \]

Thus, jth distributor’s cost function is convex and (3.44) provides global optimum.

3.2.2.3 Manufacturer’s decision

The manufacturer faces demands from all the distributors. The demand per unit time is

\[ D^m = \sum_{j=1}^{n} \frac{Q^d*_j}{T^d*_j} \]

When distributors and the manufacturer operate independently, the manufacturer needs to carry a large stock of the finished products to satisfy all demands at the same time, or the manufacturer will have to suffer stock outs and/or late deliveries. The largest possible aggregate ordering size is \( \sum_{j=1}^{n} Q^d*_j \). Similar to the jth distributor, the manufacturer’s production lot size is

\[ Q^m = \sum_{j=1}^{n} Q^d*_j + \frac{T^m D^m}{1 - \theta} \]  (3.46)
The average cost function of the manufacturer can be expressed as

\[ C_m = \frac{s_m}{T_m} + \left( \frac{h_m \sum_{j=1}^{n} Q_{j}^{d_s}}{1 - \theta} \right) + \frac{h_m [T_m D_m - \sum_{j=1}^{n} Q_{j}^{d_s}]}{2(1 - \theta)} + \frac{c}{T_m} \left[ \sum_{j=1}^{n} Q_{j}^{d_s} + \frac{T_m D_m}{1 - \theta} \right] \]  

(3.47)

Necessary condition, \( dC_m / dT_m = 0 \) yields

\[ T_m^* = \sqrt{\frac{2(1 - \theta)(s_m + c \sum_{j=1}^{n} Q_{j}^{d_s})}{h_m D_m}} \]  

(3.48)

Substituting (3.48) in (3.46) and simplifying the manufacturer’s optimal production lot size can be found as

\[ Q_m^* = \sum_{j=1}^{n} Q_{j}^{d_s} + \frac{1}{1 - \theta} \sqrt{\frac{2(1 - \theta)(s_m + c \sum_{j=1}^{n} Q_{j}^{d_s}) D_m}{h_m}} \]  

(3.49)

Moreover,

\[ \frac{d^2 C_m}{dT_m^2} = \frac{2(s_m + c \sum_{j=1}^{n} Q_{j}^{d_s})}{T_m^3} > 0 \]

Therefore, \( T_m^* \) provides global minimum of the equation(3.47).

### 3.2.3 Semi-centralized policy

Semi-centralized channel refers to the situation, where all the members in a particular echelon of the supply chain opt for integrated decision of that echelon. Since, there is more than one member in the first and second tires of the channel, it is needed to find the optimal decisions of these two stages and to examine how these decisions differ from the decentralized decision.

#### 3.2.3.1 Centralized decision in the first echelon

In the first echelon of the chain, different retailer has different optimal order quantities and different replenishment cycles. Thus, a distributor has to keep buffer stock through out it’s replenishment cycle anticipating any of its associated retailer’s stock out based situation. This incurs additional holding cost in the distributor’s cost. Product deterioration makes the situation worse further. This situation can be avoided if the replenishment cycles of the retailers are synchronized. That is, all the retailers opt for a common replenishment cycle \( T^s \) As such, total average cost of all the retailers is

\[ C^{r-s} = \left[ \frac{e^{\theta T^s} - 1}{\theta T^s} \right] \sum_{j=1}^{n} \sum_{i=1}^{n_j} \left( \frac{h_{ij}^{r}}{\theta} + w^d \right) D_{ij}^r + \frac{\sum_{j=1}^{n} \sum_{i=1}^{n_j} s_{ij}^r}{T^r} - \sum_{j=1}^{n} \sum_{i=1}^{n_j} D_{ij}^r \frac{h_{ij}^{r}}{\theta} \]  

(3.50)
The necessary condition for the existence of optimal solution yields

\[ T^{\ast \ast} = \frac{2\sqrt{\sum_{n=1}^{n} \sum_{i=1}^{n_j} s^r_{ij}}}{\sqrt{2\sum_{j=1}^{n} \sum_{i=1}^{n_j} (h^r_{ij} + \theta w^d) D^r_{ij}} + \theta \sqrt{\sum_{n=1}^{n} \sum_{i=1}^{n_j} s^r_{ij}}^{\frac{1}{2}} \]  \quad (3.51)

The optimal order quantity of the ijth \((i = 1, 2, \ldots, n_j; j = 1, 2, \ldots, n)\) retailer can be found from (3.38) by substituting \(T^{\ast \ast}\) for \(T^r_{ij}\) as

\[ Q^{rs}_{ij} = D^r_{ij} \sqrt{\frac{2\sum_{j=1}^{n} \sum_{i=1}^{n_j} s^r_{ij}}{\sum_{n=1}^{n} \sum_{i=1}^{n_j} (h^r_{ij} + \theta w^d) D^r_{ij}}} \]  \quad (3.52)

Optimal average profit of the ijth retailer can be found from (3.39) by using \(T^{\ast \ast}\). Note that \(T^{\ast \ast} \neq T^r_{ij}\) for all \(i = 1, 2, \ldots, n_j; j = 1, 2, \ldots, n\). Since, for \(T^r_{ij}\), ijth retailer’s cost is minimum, any deviation of the replenishment cycle length, leads to increment of it’s cost. Thus, when the retailers follow a common order cycle in the first echelon, their costs will be higher when compared with decentralized costs.

3.2.3.2 Centralized decision in the second echelon

When there is coordination among all the distributors in the second echelon of the channel, the distributors will accept a common replenishment cycle length that is an integer multiple of, say \(N_\alpha\), of the common replenishment cycle length of the retailers. The average cost function is

\[ C^{ds} = \left[ \frac{\sum_{j=1}^{n} s^d_{ij}}{N_\alpha T^{\ast \ast}} + \frac{(N_\alpha - 1)T^{\ast \ast}}{2(1 - \theta)} \sum_{j=1}^{n} h^d_{ij} D^{ds}_{j} + \frac{(N_\alpha - \theta)w^m}{N_\alpha(1 - \theta)} \sum_{j=1}^{n} D^{ds}_{j} \right] \]  \quad (3.53)

Where \(D^{ds}_{j} = \sum_{i=1}^{n} Q^{rs}_{ij}/T^{\ast \ast}\). Thus, the distributors do not need to keep additional stock as in the case of decentralized decision making because the retailers’ demands are perfectly anticipated by the distributors. Also the distributors can use inventories immediately to satisfy the last order from the retailers in the replenishment cycle. Thus, the distributors maximum stock will be \(D^{ds}_{j}(N_\alpha - 1)T^{\ast \ast}/(1 - \theta)\). The holding cost and deterioration cost of the distributors will be incurred in its average cost for \((N_\alpha - 1)T^{\ast \ast}\), that is less one cycle time of the retailers. So the average system cost of the distributors will be less when compared with the decentralized decision.

3.2.3.3 Manufacturer’s decision

When there are optimal centralized decisions in the first and second echelons of the supply chain, the manufacturer considers a common production cycle length that is an integer multiple of the common replenishment cycle length of the distributors i.e., \(N_\alpha N_\beta T^{\ast \ast}\), where \(N_\beta\) is an integer. The average cost function of the manufacturer is

\[ C^{ms} = \left[ \frac{s^m}{N_\alpha N_\beta T^{\ast \ast}} + \frac{(N_\beta - 1)N_\alpha T^{\ast \ast}}{2(1 - \theta)} h^m D^{ms} + \frac{(N_\beta - \theta)c}{N_\beta(1 - \theta)} D^{ms} \right] \]  \quad (3.54)
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Where $D^{ms} = \sum_{j=1}^{n} Q_{j}^{ms} / N_{a} T^{**}$. As in the case of the distributors, the manufacturer can anticipate the distributors order quantity because of the centralized decisions of the distributors. As a result, the manufacturer does not carry stocks for the last replenishment cycle of the distributors. The maximum stock and production cycle of the manufacturer is $D^{ms} (N_{\beta} - 1) N_{a} T^{**} / (1 - \theta)$. So, the manufacturer cost is less than that of its decentralized cost.

From the above discussion of semi-centralized decision making process the following lemma can be realized.

**Lemma-3.1** When compared with decentralized costs, the semi-centralized costs of the retailers’ are higher, whereas distributors and manufacturers costs are lower.

### 3.2.4 Centralized policy

When all the channel members cooperate and synchronize their decisions, the channel can be treated as a vertical marketing system that minimizes total channel cost. The manufacturer, the distributors and the retailers adopt a common cycle $T$. The manufacturer and the distributor follow lot-for-lot policy. Since the manufacturer and the distributors do not carry any inventory, the stock holding cost and product deterioration cost will be incurred in the retailers’ costs only. Average total cost of the channel is

$$
C = \left[ \frac{e^{\theta T} - 1}{\theta T} \right] \sum_{i=1}^{n} \sum_{j=1}^{n} \left( \frac{h_{ij}^{r}}{\theta} + w^{d} + w^{m} + c \right) D_{ij}^{r} + \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{j=1}^{n} \frac{s_{ij}^{r} + s_{j}^{d} + s_{m}}{T} - \sum_{i=1}^{n} \sum_{j=1}^{n} D_{ij}^{r} \left( \frac{h_{ij}^{r}}{\theta} \right) \right] (3.55)
$$

The necessary condition for the existence of optimal solution yields the common cycle length of the channel as

$$
T^{*} = \frac{2 \sqrt{\sum_{i=1}^{n} \sum_{j=1}^{n} s_{ij}^{r} + \sum_{j=1}^{n} s_{j}^{d} + s_{m}}}{\sqrt{2 \sum_{i=1}^{n} \sum_{j=1}^{n} [h_{ij}^{r} + \theta (w^{d} + w^{m} + c)] D_{ij}^{r} + \theta \sqrt{\sum_{i=1}^{n} \sum_{j=1}^{n} s_{ij}^{r} + \sum_{j=1}^{n} s_{j}^{d} + s_{m}}} \left(2 - \theta T^{*}\right) (3.56)
$$

Moreover,

$$
\frac{d^{2}C}{dT^{2}} = \frac{2(\sum_{i=1}^{n} \sum_{j=1}^{n} s_{ij}^{r} + \sum_{j=1}^{n} s_{j}^{d} + s_{m})}{(T)^{4}} + \frac{4 \theta \sum_{i=1}^{n} \sum_{j=1}^{n} [h_{ij}^{r} + \theta (w^{d} + w^{m} + c)] D_{ij}^{r}}{(2 - \theta T)^{3}} \left(2 - \theta T^{*}\right)
$$

Since, $(2 - \theta T^{*}) > 0$, the left hand side of the above expression is positive hence (3.56) provides the global optimal solution of (3.55).

The optimal order quantities of the ijth retailer, the jth distributer and the manufacturer are
respectively as

\[ Q_{rc}^{\ast} = \frac{D_{r}^{ij} (e^{\theta T^*} - 1)}{\theta}, \quad i = 1, 2, \ldots, n; \quad j = 1, 2, \ldots, n \]  
(3.57)

\[ Q_{dc}^{\ast} = \left( \frac{e^{\theta T^*} - 1}{\theta} \right) \sum_{i=1}^{n_j} D_{r}^{ij}, \quad j = 1, 2, \ldots, n \]  
(3.58)

\[ Q_{mc}^{\ast} = \left( \frac{e^{\theta T^*} - 1}{\theta} \right) \sum_{j=1}^{n} \sum_{i=1}^{n_j} D_{r}^{ij} \]  
(3.59)

Therefore, in the centralized channel optimal costs of retailer, the distributor and the manufacturer are

\[ C_{rc}^{\ast} = \left[ \frac{e^{\theta T^*} - 1}{\theta T^*} \right] \left( \frac{h_{r}^{ij}}{\theta} + w^{d} \right) D_{r}^{ij} + \frac{s_{j}^{ij}}{T^*} - \frac{h_{r}^{ij} D_{r}^{ij}}{\theta}, \quad i = 1, 2, \ldots, n; \quad j = 1, 2, \ldots, n \]  
(3.60)

\[ C_{dc}^{dcs} = \frac{s_{j}^{d}}{T^*} + w^{m} Q_{dc}^{dcs}, \quad j = 1, 2, \ldots, n \]  
(3.61)

\[ C_{mc}^{mcs} = \frac{s_{m}}{T^*} + cQ_{mc}^{mcs} \]  
(3.62)

It is well established ([70], [76], [27], [28]) that optimal total cost of the centralized channel is less when compared with semi-centralized total cost. Since, the manufacturer and the distributors do not have stock holding cost and product deterioration cost, their costs are less than their semi-centralized costs as well as decentralized costs. So the manufacturer and the distributors prefer centralized decisions. But it may be noted that

\[ C_{rc}^{\ast} - C_{rc}^{\ast} < 0, \quad (i = 1, 2, \ldots, n; \quad j = 1, 2, \ldots, n) \]

i.e. the retailer’s cost increases. Thus, the lemma follows.

**Lemma-3.2** When compared with the semi-centralized cost, the centralized costs of retailers’ are higher, whereas the distributors’ and the manufacturer’s costs are lower.

Therefore, from lemma-3.1 and lemma-3.2 the proposition follows.

**Proposition-3.1** In a conventional three-echelon distribution channel, the retailers’ costs are maximum but the distributors’ cost and the manufacturer’s cost are minimum for centralized decision.

### 3.2.5 Channel coordination, ranges of win-win opportunities and benefit sharing

As indicated, earlier studies ([70], [76]) related to multi-echelon, multi-members supply chain consider the centralized decision as the optimal decision for all the channel members. But proposition-3.1 indicates that the retailers have no reason to accept the decision because their average costs increase. The retailers accept the centralized decision when they receive some incentives, which ensure at least their decentralized costs, from their upstream channel mem-
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bers. Because of incentives that the distributors provide to their retailers, the distributors costs may be higher than their decentralized costs. So, to encourage the distributors, the manufacturer provides incentives. Assume that the manufacturer provides quantity discounts to the distributors and the distributors provide a percentage of compensation on deterioration cost as incentives. The next subsections will show that, the hybrid contract mechanism is sufficient for channel coordination.

3.2.5.1 Minimum and maximum contract limits for the retailers and the manufacturer

In a multi member distribution channel the general convention is the upstream channel member analyzes the situation of it’s downstream members in one-to-one basis and takes decision. It is very common in marketing channel that a manufacturer provides more facilities to some distributers than the others because it receives more from those distributers. Similarly, a distributer provides different percentage of compensations for its different retailers independently. The increased cost of the $ij$th ($i = 1, 2, ..., n; j = 1, 2, ..., n$) retailer for accepting $Q_{ij}^{rc}$ instead of $Q_{ij}^{r}$ is $C_{ij}^{rc} - C_{ij}^{r}$. As incentive the $j$th ($j = 1, 2, ..., n$) distributer provides compensation on deterioration cost to its associated ($i = 1, 2, ..., n; j = 1, 2, ..., n$) retailers. If $\lambda_{ij}(i = 1, 2, ..., n; j = 1, 2, ..., n)$ is the minimum percent of compensation that $j$th distributer provides to its $ij$th retailer then

$$C_{ij}^{rc} - C_{ij}^{r} = \lambda_{ij} DC_{ij}(T^*), \quad i = 1, 2, ..., n; j = 1, 2, ..., n \quad (3.63)$$

Where $DC_{ij}(T^*) = (w^d D_{ij}^r/T^*)[(e^{\theta T^*} - 1)/\theta) - 1]$, is the deterioration cost of the $ij$th retailer for its centralized order quantity. Thus, from (3.63) it follows that

$$\lambda_{ij} = \frac{C_{ij}^{rc} - C_{ij}^{r}}{DC_{ij}(T^*)} \quad (3.64)$$

Therefore, $ij$th retailer’s maximum cost for coordinate order quantity is

$$C_{ij}^{rc} = \left[\frac{e^{\theta T^*} - 1}{\theta T^*}\right] \left(\frac{h_{ij}}{\theta} + w^d\right) D_{ij}^r + \frac{s_{ij}^r}{T^*} - \frac{h_{ij} D_{ij}^r}{\theta} - \lambda_{ij} DC_{ij}(T^*), \quad i = 1, 2, ..., n; j = 1, 2, ..., n \quad (3.65)$$

The manufacturer has $n$ distributers. The manufacturer considers it’s cost corresponding to each distributer separately and takes decision. So, the cost component of the manufacturer for $j$th ($j = 1, 2, ..., n$) distributer in centralized channel and decentralized channel are respectively as

$$C_{mc}^j = \frac{Q_{mc}^{de}}{Q_{mc}^{de}} C_{mc}^{r}, \quad i = 1, 2, ..., n; j = 1, 2, ..., n \quad (3.66)$$

$$C_{mc}^j = \frac{1}{Q_{mc}^{de}} \left[Q_{mc}^{de} + \frac{T_{mc}^* Q_{mc}^{de}}{(1-\theta)}\right] C_{mc}^{r}, \quad i = 1, 2, ..., n; j = 1, 2, ..., n \quad (3.67)$$

The manufacturer provides quantity discount to induce a distributer anticipating centralized lot from the distributer. It can provide quantity discount to a particular distributer until it’s
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The cost corresponding to that distributor is not greater than the corresponding decentralized cost. If the manufacturer provides maximum $\rho_j$ discount on wholesale price to the $j$th distributor for accepting its centralized order quantity then

$$(\rho_j w^m)Q^{dc*}_j = C^{m*}_j - C^{mc*}_j$$

i.e,

$$\rho_j = \frac{C^{m*}_j - C^{mc*}_j}{w^m Q^{dc*}_j}, \quad j = 1, 2, ..., n \quad (3.68)$$

Therefore, after providing maximum quantity discount to all the distributors the manufacturer’s cost will be exactly equal to its decentralized cost. The distributors simply act as the mediators, who receive the lots from the manufacturer and distribute among the retailers. By doing so they make profit. Interestingly, for channel coordination the distributors play central role because they actually maintain the lot streaming between the manufacturer and the retailers. Thus, when the coordination contracts resolve channel conflict, there may arise two questions. (i) What is the minimum amount of quantity discount that a distributor accept? ii) What is the maximum percent of compensation on deterioration cost that a distributor can provide to it’s corresponding retailer? These two questions are interrelated and one definitely get different results when approaches from the manufacturer to the retailers and from the retailers to the manufacturer. In the former case, a distributor accepts quantity discount until it’s decentralized cost is ensured. In the later case, the distributor provides compensation as long as it’s decentralized cost is reserved. The next subsection finds closed form solutions to these questions and analyzes how the benefit for channel coordination can be shared among the channel members.

### 3.2.6 Win-win ranges and benefit division

Assume that as an intermediator, a distributor instead of dealing with both the upstream manufacturer and downstream retailers simultaneously, it deals with one at a time. It assumes that there is no other echelon. Once the deal is finalized with one, it deals with the other. A distributor deals with each of it’s retailers independently because it has different reservations for different retailers. How much compensation it can provide to the retailer that depends on how much of it’s cost is saved for that retailer’s centralized order quantity. Also, the manufacturer deals with each distributor independently. Although these assumptions make the model restrictive to some extent, still it is common in marketing practice. Thus, there arises two cases, viz, (i) backward contract-bargaining process and (ii) forward contract-bargaining process. Contract mechanism and bargaining for benefit share are interrelated and the outcome of one is dependent on the other, and vice versa. Moreover, in both the processes, an upstream member deals with each of it’s multiple downstream members independently in a pre-specified manner or randomly. As the dealings are independent, the result will not differ when any one of these two is implemented.
3.2.6.1 Backward contract-bargaining process

In this case the sequence of events applying contract mechanisms and bargaining are as follows.

1. The jth \( (j = 1, 2, \ldots, n) \) distributor and each of it’s \( n_{ij} \) retailers determine the upper limit of percentage compensation on deterioration cost. This limit can be found using the criteria that the jth distributor provides compensation to its ijth \( (j = 1, 2, \ldots, n) \) retailer as long as it’s decentralized cost corresponding to the retailer is reserved.

2. The jth distributor \( \{j = 1, 2, \ldots, n\} \) bargains with each retailer independently for percentage compensation within its range and settles for benefit share. Decentralized cost minus the benefit share is the optimal cost of the ijth retailer.

3. After the process, the distributor’s intermediate cost is the decentralized cost minus cumulative benefit shares, which it acquired from all of it’s retailers. Based on this cost the jth distributor and the manufacturer identifies the lower limit of quantity discount. This limit can be determined by assuming that the distributor accept the quantity discount as long as it’s cost is reserved.

4. The manufacturer and the jth distributor bargain for benefit share. The jth distributor’s optimal cost is decentralized cost plus accumulated benefits from all of it’s retailers minus benefit share from the manufacturer. The manufacturer’s cost is decentralized cost minus accumulated benefits from all the distributors.

The jth \( (j = 1, 2, \ldots, n) \) distributor’s decentralized and centralized cost corresponding to its ijth \( (i = 1, 2, \ldots, n_{ij}; j = 1, 2, \ldots, n) \) retailer are respectively as

\[
C_{ij}^{dc} = \frac{Q_{ij}^{rcs}}{Q_{j}^{dcs}} C_{j}^{dcs}, \quad i = 1, 2, \ldots, n_{ij}; j = 1, 2, \ldots, n \quad (3.69)
\]

\[
C_{ij}^{d} = \frac{1}{Q_{j}^{dcs}} \left[ Q_{ij}^{rcs} + \frac{T_{j}^{d} Q_{ij}^{rcs}}{(1 - \theta) T_{ij}^{rcs}} \right] C_{j}^{d}, \quad i = 1, 2, \ldots, n_{ij}; j = 1, 2, \ldots, n \quad (3.70)
\]

If the jth distributor provides maximum \( \lambda_{ij}^{b} \) percent compensation on deterioration cost to its ijth retailer then

\[
\overline{\lambda_{ij}^{b}} DC_{ij}(T^{*}) = C_{ij}^{d} - C_{ij}^{dc}
\]

i.e,

\[
\overline{\lambda_{ij}^{b}} = \frac{C_{ij}^{d} - C_{ij}^{dc}}{DC_{ij}(T^{*})}, \quad i = 1, 2, \ldots, n_{ij}; j = 1, 2, \ldots, n \quad (3.71)
\]

After providing the compensation, the jth distributor’s and ijth \( (i = 1, 2, \ldots, n_{ij}; j = 1, 2, \ldots, n) \) retailer’s costs are

\[
C_{ij}^{db} = \frac{S_{j}^{d}}{I^{*}} + w^{m} Q_{ij}^{dcs} + \sum_{i=1}^{n_{ij}} \lambda_{ij}^{b} DC_{ij}(T^{*}), \quad i = 1, 2, \ldots, n_{ij}; j = 1, 2, \ldots, n \quad (3.72)
\]
\[
C_{rc*}^{ij} = \left[ \frac{e^{\theta T^*} - 1}{\theta T^*} \right] \left( \frac{h_i^r}{T^*} + w_i^d \right) D_{ij}^r + \frac{h_i^r D_{ij}^r}{\theta} - \lambda_{ij}^{\text{b}} C_{i_j}^r(T^*) \right), \quad i = 1, 2, ..., n_j; j = 1, 2, ..., n \tag{3.73}
\]

Therefore, any \( \lambda_{ij} \in (\lambda_{ij}, \lambda_{ij}^{\text{b}}) \), \( (i = 1, 2, ..., n_j; j = 1, 2, ..., n) \) ensures that the jth distributor’s cost for ith retailer and the jth retailer’s cost are win-win. Now within this range the jth distributor and the jth retailer bargain for a particular compensation fraction that effectively divides the benefit.

The Nash bargaining product is used to divide benefit. The Nash bargaining model that has been used in various contexts, is an axiomatic derivation of bargaining solution. The axiomatic derivation leaves out the actual process of negotiations while focusing on the expected outcome based on pre-specified solution procedures. Also the axioms do not reflect the rationale of the agents or the process in which the agreement is reached. One of the important characteristics of the Nash solution concept is that the outcome is random because it depends on the participating players negotiation powers. In Nash bargaining model the objective function is the product of the players benefit from cooperation and it must be minimized. Each players benefit is the difference between the negotiated profit and profit under decentralized decision making.

Since, any \( \lambda_{ij} \in (\lambda_{ij}, \lambda_{ij}^{\text{b}}) \) resolves channel conflict, the jth distributor and the jth retailer bargain to determine an acceptable \( \lambda_{ij} \in (\lambda_{ij}, \lambda_{ij}^{\text{b}}) \) that will minimize the Nash bargaining product. The Nash bargaining product can be found as

\[
\min_{\lambda_{ij} \leq \lambda_{ij} \leq \lambda_{ij}^{\text{b}}} [C_{rc*}^{ij} - C_{r*}^{ij} - \lambda_{ij} D C_{i_j}^r(T^*)][C_{d}^{ij} - C_{dc}^{ij} - \lambda_{ij} D C_{i_j}^c(T^*)], \quad i = 1, 2, ..., n_j; j = 1, 2, ..., n \tag{3.74}
\]

Optimal value of \( \lambda_{ij} \) is

\[
\lambda_{ij}^{\text{b}} = \frac{C_{rc*}^{ij} - C_{r*}^{ij} + C_{d}^{ij} - C_{dc}^{ij}}{2DC_{i_j}^r(T^*)}, \quad i = 1, 2, ..., n_j; j = 1, 2, ..., n \tag{3.75}
\]

Thus, the jth retailer’s optimal cost after bargaining can be found from (3.65) by replacing \( \lambda_{ij} \) by \( \lambda_{ij}^{\text{b}} \) of (3.75).

The jth distributor’s intermediate cost is

\[
C_{j}^{db} = \frac{s_j^d}{T^*} + \sum_{i=1}^{n_j} \lambda_{ij}^{\text{b}} D C_{i_j}^r(T^*), \quad j = 1, 2, ..., n \tag{3.76}
\]

As soon as the jth distributor find optimal cumulative benefit shares from its \( n_j \) retailers, it determines the minimum quantity discount that it can accept from the manufacturer. If the manufacturer provides minimum \( \rho_j^b \) discount that ensures the jth distributor’s cost after the first bargaining then

\[
\rho_j^{\text{b}} = \frac{C_{j}^{db} - C_{j}^d}{w_m D C_{i_j}^r T^*}, \quad j = 1, 2, ..., n \tag{3.77}
\]
Any \( \rho_j \in (\rho^b_j, \rho^r_j) \) ensures win-win costs for the manufacturer and the jth distributor. Now, if the jth distributor bargains independently with the manufacturer for a \( \rho_j \in (\rho^b_j, \rho^r_j) \) that will minimize their cost then the Nash bargaining product can be found as

\[
\min_{\rho^b_j \leq \rho_j \leq \rho^r_j} [C_{db}^j - C_{ds}^j - \rho_j w^m Q_{dj}^j] [C_{m}^j - C_{mc}^j - \rho_j w^m Q_{dj}^j], \quad j = 1, 2, \ldots, n
\]  

(3.78)

Optimal value of \( \rho_j \) is

\[
\rho^b_j = \frac{C_{db}^j - C_{ds}^j + C_{m}^j - C_{mc}^j}{2w^m Q_{dj}^j}, \quad j = 1, 2, \ldots, n
\]  

(3.79)

Therefore, optimal costs of the retailers, the distributors and the manufacturer after the contract-bargaining process are respectively as

\[
C_{rb}^{bs} = C_{rc}^{rs} - \frac{1}{2} \left[ C_{ij}^d - C_{ij}^{rcs} - C_{ij}^{dc} + C_{ij}^{rs} \right], \quad i = 1, 2, \ldots, n_j; \quad j = 1, 2, \ldots, n
\]  

(3.80)

\[
C_{db}^{bs} = C_{ds}^j - \frac{1}{4} \left[ 2(C_{mc}^j - C_{mc}^{rs}) + (C_{ds}^j - C_{ds}^{dc}) - \sum_{i=1}^{n_j} (C_{ij}^{rc} - C_{ij}^{rs}) \right], \quad j = 1, 2, \ldots, n
\]  

(3.81)

\[
C_{mb}^{bs} = C_{ms}^* - \frac{1}{4} \left[ 2(C_{ms}^* - C_{mc}^{rs}) + \sum_{j=1}^{n} (C_{j}^{ds} - C_{j}^{dc}) - \sum_{j=1}^{n} \sum_{i=1}^{n_j} (C_{ij}^{rc} - C_{ij}^{rs}) \right]
\]  

(3.82)

From the above bargaining process following results can be realized. First, unlike the centralized decision, the ranges of the contract parameters ensure win-win opportunities for all the channel members. Moreover, the channel is coordinated and maximum channel benefit is distributed among the channel members. Second, the manufacturer shares the cost benefit for channel coordination with the jth \((j = 1, 2, \ldots, n)\) distributor equitably. Third, the jth distributor and its ith \((i = 1, 2, \ldots, n_j; j = 1, 2, \ldots, n)\) retailer share the benefit equitably.

### 3.2.6.2 Forward contract-bargaining process

Unlike the previous case, the approach here will start from the manufacturer and ends to the retailers through the distributors, i.e., the decision making starts from the manufacturer and ends at the retailers. The sequence of events are as follows.

1. The manufacturer and all the distributors determine lower limits of the quantity discounts in one-to-one basis.
2. Within the range, the manufacturer bargains with each distributor independently for quantity discount that effectively determines benefit share. The decentralized cost minus cumulative benefit shares from all the distributors is the manufacturer’s optimal cost. The decentralized cost minus the benefit share is the distributor’s intermediate cost.
3. Based on this cost a distributor identifies the upper limits of percentage compensation on deterioration cost corresponding to each of it’s retailers independently.
4. Within this range a distributor bargains with each of it’s retailers independently for com-
penetration fraction that determines benefit share. The decentralized cost minus benefit share is a retailer’s optimal cost. Cost of step 2 minus cumulative benefit share is the distributor’s optimal cost.

If the jth \((j = 1, 2, ..., n)\) distributor demands minimum \(\rho^f_j\) \((0 \leq \rho^f_j \leq 1)\) quantity discount from the manufacturer then \(\rho^f_j\) can be found as

\[
\rho^f_j = \frac{C^d_{dcj} - C^d_{sj}}{w^m Q^d_{dcj}}, \quad j = 1, 2, ..., n
\]

Thus, at this point for any \(\rho_j \in (\rho^f_j, \bar{\rho}_j)\), the jth \((j = 1, 2, ..., n)\) distributor’s cost and the manufacturer’s cost corresponding to the distributor are win-win. Now, if the manufacturer bargains with the jth \((j = 1, 2, ..., n)\) distributor then the Nash bargaining product is

\[
\min_{\rho^f_j \leq \rho_j \leq \bar{\rho}_j} \left[ C^d_{dcj} - C^d_{sj} - \rho_j w^m Q^d_{dcj} \right] \left[ C^m_{j} - C^{mc}_{j} - \rho_j w^m Q^d_{dcj} \right], \quad j = 1, 2, ..., n
\]

The optimal value of \(\rho_j\) \((j = 1, 2, ..., n)\) can be found as

\[
\rho^*_j = \frac{C^d_{dcj} - C^d_{sj} + C^m_{j} - C^{mc}_{j}}{2w^m Q^d_{dcj}}, \quad j = 1, 2, ..., n
\]

After settling benefit share with the manufacturer, cost of the jth \((j = 1, 2, ..., n)\) distributor becomes

\[
C^d_{fj} = \frac{s_d}{T^*} + (1 - \rho^*_{fj}) w^m Q^d_{dcj}, \quad j = 1, 2, ..., n
\]

Cost component of the jth distributor \((j = 1, 2, ..., n)\) corresponding to the ijth retailer is

\[
C^d_{ij} = \frac{Q^c_{rcij}}{Q^d_{dcj}} C^d_{fj}, \quad i = 1, 2, ..., n; j = 1, 2, ..., n
\]

Now, if the jth \((j = 1, 2, ..., n)\) distributor provides maximum \(\lambda^f_{ij}\) \((i = 1, 2, ..., n; j = 1, 2, ..., n)\) compensation on deterioration cost to the ijth retailer then \(\lambda^f_{ij}\) can be found as

\[
\lambda^f_{ij} = \frac{C^d_{ij} - C^d_{ij}}{DC^d_{ij}(T^*)}, \quad i = 1, 2, ..., n; j = 1, 2, ..., n
\]

Thus, for any \(\lambda_{ij} \in (\lambda^f_{ij}, \lambda^i_{ij})\) the jth distributor’s cost and ijth retailer’s cost are win-win. Now, if the distributor and the retailer bargain for a particular \(\lambda_{ij}\) within this range then the Nash bargaining product can be found as
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\[ \min_{\lambda_{ij} \leq \lambda_{ij} \leq \lambda_{ij}} [C_{ij} - C_{ij} - \lambda_{ij}DC_i(T^*)] \] (3.89)

Optimal value of \( \lambda_{ij} \) is

\[ \lambda_{ij}^f = \frac{C_{ij} - C_{ij} - C_{ij}}{2DC_i(T^*)}, i = 1, 2, ..., n \] (3.90)

Therefore, optimal costs of the retailers, the distributors and the manufacturer after the contract-bargaining process are

\[ C_{rf}^* = C_{ij} - \frac{1}{2} \left[ \left( C_{ij} - C_{ij} - C_{ij} \right), i = 1, 2, ..., n \right] \] (3.91)

\[ C_{df}^* = C_{df} - \frac{1}{4} \left[ (C_{ij} - C_{ij}) + (C_{ij} - C_{ij}) - 2 \sum_{i=1}^{n} (C_{ij} - C_{ij}) \right], j = 1, 2, ..., n \] (3.92)

\[ C_{mf}^* = C_{mf} - \frac{1}{2} \left[ (C_{ij} - C_{ij}) + \sum_{j=1}^{n} (C_{ij} - C_{ij}) \right] \] (3.93)

Note that, the jth distributor’s average cost in decentralized decision is higher when compared with the cost in centralized decision, i.e., \( C_{df}^* < C_{df}^* \), i.e., \( C_{df}^* - C_{df}^* < 0, j = 1, 2, ..., n \). Then, from (3.83) it follows that \( \rho_{ij}^f < 0 \), since \( w^mQ_{ij}^* > 0 \). Similarly, from (3.86) it follows that \( \rho_{ij}^f < 0 \), because \( C_{df}^* < C_{df}^* \). Thus, the proposition follows.

Proposition 3.2 In both contract-bargaining processes, minimum of quantity discounts demanded by each distributor are negative.

Thus, without providing any quantity discount to the distributors, it is possible to coordinate a channel though the retailers need some incentives. Moreover, in this case the distributors will be in win-win stage unless the lower limits are positive. The intuitive reason is straightforward. As an intermediary, a distributor bargains with the manufacturer as well as with each of it’s retailers for benefit share. When it does not get any quantity discount, it’s benefit share from the manufacturer is zero. But, by providing percentage compensation on deterioration cost it receives benefit from the retailers, which are sufficient for it’s win-win opportunity and efficient for channel coordination. Thus, the distributors have the power to resolve channel conflict without receiving quantity discounts from the manufacturer.

3.2.6.3 Comparison of two contract-bargaining processes

From the above bargaining procedures two lower limits of quantity discount and two upper limits of percentage compensation on deterioration cost are identified. Note from (3.76) and
(3.83) that

$$\rho_{bj} - \rho_{fj} = \frac{C_{db}^j - C_{dc^*}^j}{w^m Q_{dc}^*}, j = 1, 2, \ldots, n$$

But $C_{db}^j > C_{dc^*}^j$. Thus, $\rho_{bj} - \rho_{fj} > 0$, since $w^m Q_{dc}^* > 0$ i.e., $\rho_{bj} > \rho_{fj}$.

Thus, minimum quantity discount demanded by the $j$th ($j = 1, 2, \ldots, n$) distributor in backward process is higher when compared with minimum quantity discount of forward process. Similarly, $\lambda_{ij}^b - \lambda_{ij}^f < 0$, i.e, the minimum compensation provided by the $j$th ($j = 1, 2, \ldots, n$) distributor to its $ij$th ($i = 1, 2, \ldots, n_j; j = 1, 2, \ldots, n$) retailer in backward process is higher than that of forward. So, the proposition follows.

**Proposition 3.3** (i) $\rho_{bj}^f < \rho_{bj}^b, j = 1, 2, \ldots, n$ (ii) $\lambda_{ij}^f > \lambda_{ij}^b, i = 1, 2, \ldots, n_j; j = 1, 2, \ldots, n$

Moreover,

$$C_i^{rf^*} - C_i^{rb^*} = \frac{1}{2} [C_{ij}^{df} - C_{ij}^{dc}] = \frac{Q_{rc}^*}{2Q_{dc}^*} \left[ C_{ij}^{df} - C_{ij}^{dc^*} \right] < 0$$

$$C_j^{df^*} - C_j^{db^*} = \frac{1}{4} \left[ (C_{mj}^m - C_{mj}^{mc}) + \sum_{i=1}^{n_j} (C_{ri}^{rc^*} - C_{ri}^{r^*}) \right] > 0$$

$$C_{mj}^{mf^*} - C_{mj}^{mb^*} = -\frac{1}{4} \left[ \sum_{j=1}^{n} (C_{j}^{ds} - C_{j}^{dc^*}) + \sum_{j=1}^{n} \sum_{i=1}^{n_j} (C_{ij}^{rc^*} - C_{ij}^{r^*}) \right] < 0$$

Thus, the proposition follows.

**Proposition 3.4** The distributors prefer the backward contract-bargaining process, whereas the manufacturer and the retailers prefer the forward contract-bargaining process.

In both the processes, the manufacturer shares the benefit equitably with each distributor. In forward process, for minimum quantity discount a distributor’s cost is equal to its decentralized cost and the manufacturer’s cost is maximum. When backward process is used a distributor demands at least decentralized cost minus the share of benefit, which it received through the bargaining with all of its retailers. The manufacturer satisfies this demand through minimum quantity discount. Obviously, the demand for quantity discount amount is higher in the later, i.e., $\rho_{bj}^f < \rho_{bij}^b$. As a result the monetary value in backward process over which the manufacturer bargains with a distributor is larger when compared with that of forward process. Since the monetary value is divided equitably between the manufacturer and a distributor, a distributor’s benefit is higher, whereas the manufacturer’s profit is lower in backward contract-bargaining process. Using the same argument it can be explained that each retailer prefer forward bargaining to the other.
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3.2.7 Numerical illustration

Assume that in the supply chain network there are one manufacturer two distributors and five retailers. Distributer-1 supplies the product to retailer-1 and retailer-2 and Distributer-2 supplies the product to retailer-3, retailer-4 and retailer-5. The parameter values of the manufacturer are $s^m = 10000$, $h^m = 2$, $c = 20$, $w^m = 25$. The product deteriorates at a constant rate $\theta = 0.1$ per unit per unit time and wholesale price of a distributer is $w^d = 30$. Other parameter values are shown in Table-3.1.

<table>
<thead>
<tr>
<th>Retailer</th>
<th>$s^r_{ij}$ ($/order$)</th>
<th>$h^r_{ij}$ ($/unit/Year$)</th>
<th>$D^r_{ij}$ (items/Year)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{11}$</td>
<td>300</td>
<td>3</td>
<td>500</td>
</tr>
<tr>
<td>$R_{21}$</td>
<td>310</td>
<td>3</td>
<td>150</td>
</tr>
<tr>
<td>$R_{12}$</td>
<td>350</td>
<td>3</td>
<td>450</td>
</tr>
<tr>
<td>$R_{22}$</td>
<td>400</td>
<td>3</td>
<td>300</td>
</tr>
<tr>
<td>$R_{32}$</td>
<td>275</td>
<td>3</td>
<td>100</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Distributer</th>
<th>$s^d_{ij}$ ($/order$)</th>
<th>$h^d_{ij}$ ($/unit/Year$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_1$</td>
<td>1200</td>
<td>2.5</td>
</tr>
<tr>
<td>$D_2$</td>
<td>1500</td>
<td>2.5</td>
</tr>
</tbody>
</table>

Table 3.1: Input parameters of retailers(R) and distributers(D).

Using the solution procedure of appendix 3.1, the computational results are presented in tables 3.2, 3.3, 3.4 and 3.5.

<table>
<thead>
<tr>
<th>Decision variables</th>
<th>Decentralized</th>
<th>Semi-centralized</th>
<th>Centralized</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$</td>
<td>-</td>
<td>0.585136</td>
<td>1.26393</td>
</tr>
<tr>
<td>$N_\alpha$</td>
<td>-</td>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td>$N_\beta$</td>
<td>-</td>
<td>4</td>
<td>-</td>
</tr>
<tr>
<td>Retailers’ total cost</td>
<td>50292</td>
<td>50480</td>
<td>52229</td>
</tr>
<tr>
<td>Distributors’ total cost</td>
<td>58570</td>
<td>43233</td>
<td>42109</td>
</tr>
<tr>
<td>Manufacturer’s total cost</td>
<td>86699</td>
<td>40755</td>
<td>39890</td>
</tr>
<tr>
<td>Channel cost</td>
<td>195562</td>
<td>134468</td>
<td>134228</td>
</tr>
</tbody>
</table>

Table 3.2: Computational results for decentralized, semi-centralized and centralized policies

From table 3.2 note that total cost of the channel is maximum in decentralized decision, whereas it is least in centralized decision. Optimal channel cost is reduced by 31.36% in centralized decision. The manufacturer’s cost and the distributors’ costs are decreased by 53.99% and 28.10% respectively but the retailers’ costs are increased by 3.85%, when compared with decentralized decisions (see table-3.2). In table-3.3 computation results for the channel members are provided in explicit form. It indicates that the manufacturer and the distributers are benefited from the centralized decision though the retailers are not. Thus, the centralized decision is not acceptable to the retailers unless they get some incentives from the distributers.

Table-3.4 and table-3.5 indicate that all channel members' costs in backward and forward
<table>
<thead>
<tr>
<th>Member</th>
<th>Decentralized</th>
<th>Semi-centralized</th>
<th>Centralized</th>
<th>centralized cost change(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time</td>
<td>Q</td>
<td>Cost</td>
<td>Time</td>
<td>Q</td>
</tr>
<tr>
<td>R_{11}</td>
<td>0.4374</td>
<td>223.57</td>
<td>16351.6</td>
<td>0.5851</td>
</tr>
<tr>
<td>R_{21}</td>
<td>0.7969</td>
<td>124.43</td>
<td>5257.3</td>
<td>0.5851</td>
</tr>
<tr>
<td>R_{12}</td>
<td>0.4965</td>
<td>229.08</td>
<td>14886.4</td>
<td>0.5851</td>
</tr>
<tr>
<td>R_{22}</td>
<td>0.6452</td>
<td>199.93</td>
<td>10213.3</td>
<td>0.5851</td>
</tr>
<tr>
<td>R_{32}</td>
<td>0.9137</td>
<td>95.67</td>
<td>3583.6</td>
<td>0.5851</td>
</tr>
<tr>
<td>D_{1}</td>
<td>3.2685</td>
<td>2771.16</td>
<td>25075.6</td>
<td>0.5851</td>
</tr>
<tr>
<td>D_{2}</td>
<td>3.4662</td>
<td>3898.30</td>
<td>33495.0</td>
<td>0.5851</td>
</tr>
<tr>
<td>M</td>
<td>8.0885</td>
<td>24396.9</td>
<td>86698.8</td>
<td>2.3304</td>
</tr>
</tbody>
</table>

**Table 3.3:** Computational result for individuals in decentralized, semi-centralized and centralized policies
contract-bargaining processes are significantly reduced when compared with decentralized decision. Moreover, optimal total channel costs in both the processes are 134228, same as centralized channel. Thus, using any one of these processes it’s possible to resolve channel conflict.

The ranges of the contract parameters in backward and forward contract-bargaining processes ensure win-win opportunities for the channel members. The ranges of the contract parameters for the two bargaining processes are different. For example, the ranges of percentage compensation on deterioration cost of the retailer-21 are (0.3469, 8.6911) and (0.3469, 13.6820) for backward contract-bargaining and forward contract-bargaining processes respectively. The upper limits are significantly different. Thus, the ranges of quantity discount for the distributor-1 in two processes are (-0.0669, 0.4486) and (-0.1554, 0.4486), i.e., the lower limits are different. The range of percentage compensation in backward contract-bargaining is wider when compared with forward. So, the monetary space over which distributor-1 and retailer-11 bargain is larger. Since, the benefit is divided equitably, the distributor-1 gets more cost benefit. The distributor’s-1 intermediate cost is lower in backward process than the forward and it demands more quantity discount from the manufacturer. As a result the lower limit is higher and the win-win range in backward process is shorter than the forward, i.e., the monetary value for which they bargain is lower in backward process. Finally, after bargaining optimal cost
of the distributor-1 will be lower in backward contract-bargaining than the forward. So, the distributors’ always prefer the backward bargaining process, whereas the retailer’s and the manufacturer prefer the other. Justification of this may be observed from table-3.4 and table-3.5 and fig-3.3. That is, percentage decrease in the distributor’s cost in backward contract-bargaining is higher than forward contract-bargaining process. Reverse trend is observed for the retailers’ and the manufacturer.

From table-3.4 and table-3.5 note that the lower limits of win-win ranges of quantity discount are negative. It indicates that, if the manufacturer does not provide quantity discounts then also the distributors have the powers to coordinate the channel by providing percentage compensations to the retailers.

The upper limits of percentage compensation on deterioration cost are more than one. Thus, if the distributors provide more than deterioration cost to the retailers then also they can achieve win-win opportunities. This happens because of smaller holding cost when compared with ben-
Sensitivities of the optimal bargaining cost functions with respect to demand and deterioration are presented in fig-3.3. The cost functions are highly sensitive to the error in estimating demand and are moderately sensitive with respect to change in deterioration. Interestingly, the manufacturer’s cost increases but the retailers’ and the distributers’ costs decreases when \( \theta \) increases, whereas all channel members costs increase when \( \theta \) increases. The intuition is straightforward. In decentralized decision, the manufacturer and the distributers keep safety stocks. So, for deterioration their average cost will increase. Similarly, for depletion of on hand inventory, the retailers’ cost will increase. In centralized decision, deterioration does not have direct impact on the manufacturer’s and the distributers’ costs because they do not keep safety stock and follow lot-for-lot policies. It has impact only on the retailers’ costs. In the contract-bargaining processes the retailers get compensations on deterioration cost from the distributers and the distributers receive quantity discounts from the manufacturer. As a result the manufacturer gets less benefit share in comparison to other channel members when \( \theta \) increases. This is also justified in fig-3.4, which represents the coordination contracts sensitivities with respect to deterioration. Sensitivity of coordination contract bounds are presented in fig-3.4. It indicates that the ranges are moderately sensitive for changes in parameters \( D \) and \( \theta \).

### Table 3.4: Computational result for backward contract-bargaining process

<table>
<thead>
<tr>
<th>Member</th>
<th>( \lambda_{ij} )</th>
<th>( \lambda_{ij}^b )</th>
<th>( \lambda_{ij}^f )</th>
<th>( \rho_i^b )</th>
<th>( \rho_j^b )</th>
<th>( \rho_j^f )</th>
<th>Cost</th>
<th>Cost decrease(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_{11} )</td>
<td>1.1041</td>
<td>3.5956</td>
<td>6.0872</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>14401.7</td>
<td>11.92</td>
</tr>
<tr>
<td>( R_{21} )</td>
<td>0.3469</td>
<td>4.5190</td>
<td>8.6911</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>4277.8</td>
<td>18.63</td>
</tr>
<tr>
<td>( R_{12} )</td>
<td>0.9526</td>
<td>3.7525</td>
<td>6.5523</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>12914.3</td>
<td>13.25</td>
</tr>
<tr>
<td>( R_{22} )</td>
<td>0.6179</td>
<td>4.0986</td>
<td>7.5793</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>8578.9</td>
<td>16.00</td>
</tr>
<tr>
<td>( R_{32} )</td>
<td>0.1892</td>
<td>4.8306</td>
<td>9.4720</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>2857.1</td>
<td>20.27</td>
</tr>
</tbody>
</table>

| \( D_1 \) | | | | -0.0669 | 0.3816 | 0.4486 | 15535.3 | 38.05 |
| \( D_2 \) | | | | -0.0757 | 0.3988 | 0.4745 | 20128.5 | 39.91 |
| \( M \) | | | | - | - | - | 55534 | 35.95 |

### Table 3.5: Computational result for forward contract-bargaining process

<table>
<thead>
<tr>
<th>Member</th>
<th>( \lambda_{ij} )</th>
<th>( \lambda_{ij}^b )</th>
<th>( \lambda_{ij}^f )</th>
<th>( \rho_i^b )</th>
<th>( \rho_j^b )</th>
<th>( \rho_j^f )</th>
<th>Cost</th>
<th>Cost decrease(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_{11} )</td>
<td>1.1041</td>
<td>6.0911</td>
<td>11.0781</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>12448.7</td>
<td>23.87</td>
</tr>
<tr>
<td>( R_{21} )</td>
<td>0.3469</td>
<td>7.0144</td>
<td>13.6820</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>3601.9</td>
<td>29.78</td>
</tr>
<tr>
<td>( R_{12} )</td>
<td>0.9526</td>
<td>6.2479</td>
<td>11.5432</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>11156.6</td>
<td>25.05</td>
</tr>
<tr>
<td>( R_{22} )</td>
<td>0.6179</td>
<td>5.9491</td>
<td>12.5703</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>7407.1</td>
<td>27.48</td>
</tr>
<tr>
<td>( R_{32} )</td>
<td>0.1892</td>
<td>7.3261</td>
<td>14.4629</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>2466.5</td>
<td>31.17</td>
</tr>
</tbody>
</table>

| \( D_1 \) | | | | -0.1554 | 0.2931 | 0.4486 | 19607.2 | 21.81 |
| \( D_2 \) | | | | -0.1686 | 0.3058 | 0.4745 | 25554.3 | 23.70 |
| \( M \) | | | | - | - | - | 51895.3 | 40.14 |

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3.3 Coordinating three-echelon duopoly supply chain with imperfect quality products

This section considers a three-echelon supply chain with duopolistic retailers in the downstream and the retailers may play Cournot, Collusion and Stackelberg games. In practice, there exist various forms of competition between the two retailers in the downstream supply chain. Cournot behavior of duopolistic retailers is a common phenomenon in industry. Wal-Mart and Tesco, GOME and Suning, Carrefour and Auchan, etc., are the examples of the retailers who follow Cournot. Collusion is another realistic behavior of the duopolistic retailers where they do not compete with each other. In Collusion behavior, retailers perform secretly with a confidential agreement among them. Although, Collusion is illegal as the retailers play it tactically that is common in Chinese market. In another way, the duopolistic retailers may compete with each other following Stackelberg. In Stackelberg game, one retailer act as leader and the other as follower. The manufacturer supplies a product in a single lot to the distributor that contains a random proportion of defective items. After receiving the lot, distributor separates the imperfect items by screening process and sells in a secondary market with a discounted price. At the end of the screening process, the distributor satisfies the demand of two retailers with perfect quality products only. The purpose of the study is to (i) examine the effect of imperfect quality product on the optimal decisions, (ii) verify whether the hybrid contract mechanism resolves the channel conflict or not and (iii) demonstrates how a nested bargaining process depicts win-win profits for the channel members after channel coordination. Also, the study presents a preference analysis to highlight the channel members game and bargaining process preference.

The research reported in the third model differs from the prior works in many aspects. Firstly, unlike Yang and Zhou, this model assumes three-echelon supply chain with duopolistic retailers. The manufacturer supplies the product in a single lot that contains random proportion of defective items and the distributor differentiates perfect and imperfect quality products. Assumption of random proportion of imperfect items in a lot is well-established and well-studied (Salemah and Jaber; Sana; Khan et al.) in inventory literature. The model discusses the effect of imperfect quality items on optimal decisions. Secondly, all unit quantity discounts with franchise fee is used as the coordination mechanism to cut out channel conflict in all the three games, which the retailers play. Although the contract resolves channel conflict, it does not ensure win-win profits for the channel members. Thirdly, to determine win-win profits of all the channel members the study applies a sequential bargaining process that consists of two Nash bargaining and determination of two limits of franchise fee. Entire sequential bargaining process is nested, i.e., outcome of one limit of franchise fee and bargaining have direct influences on the other two. It is assumed that in the multi-member supply chain only two channel members at a time participate in the process. As such, the process may flow either from the manufacturer to the retailers, i.e., forward or from the retailers to the manufacturer, i.e., backward. The section discusses these two cases separately and presents a comparative study for acceptability of the process by the channel members. Fourthly, for win-win coordinated profits the section depicts which channel member prefers which game of the retailers and which bargaining process.

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5 This section is based on the paper “Three-echelon supply chain coordination considering duopolistic retailers with perfect quality products.” published in International Journal of Production Economics, Article in Press, (2015).
3.3.1 Model formulation and basic analysis

Assume that a manufacturer produces a single type of products and delivers in a single lot to the distributor. The lot contains a random proportion \( r \) \((1 \leq r \leq 1)\) imperfect quality product that follows a probability distribution. After receiving the lot, the distributor performs a screening process to separate perfect and imperfect quality products. The distributor supplies perfect quality product to two retailers to satisfy their demand. Demand at retailer \( i \), \((i = 1, 2)\) is downward sloping and in the form \( D_i = \alpha_i - a_ip_i + \beta p_{3-i} \) \((i = 1, 2)\) where \( \alpha_i > 0 \) is the market potential of the retailer-\( i \), \( a_i \) denotes the measure of sensitivity of retailer-\( i \)'s sales to changes of the retailer-\( i \)'s price. \( \beta \) is the degree of substitutability between retailers, which reflects the impact of the marketing mix decision of the retailers on customer demand. The distributor sells imperfect quality product at a lower price in a secondary market in a single lot. Two retailers compete on price and set their prices independently. Assume that the manufacturer acts as the Stackelberg leader and the distributor is its immediate follower. The distributor supplies the perfect quality product to two retailers. Under these assumptions the model first finds the decentralized decisions.

3.3.2 Decentralized policy

In decentralized channel the expected profit functions of the manufacturer, the distributor and the retailers can be represented as follows:

\[
\pi^m = (1 + E(r))(D_1 + D_2)(w^m - c) \quad (3.94)
\]

\[
\pi^d = [w_{dp} + E(r)w_{dt} - (1 + E(r))(s + c^d + w^m)](D_1 + D_2) \quad (3.95)
\]

\[
\pi_{ri} = (p_i - w_{dp} - c_{ri})(\alpha_i - a_ip_i + \beta p_{3-i}), \quad i = 1, 2 \quad (3.96)
\]

where \( E(.) \) is the expectation corresponding to the pdf \( f(.) \) of the imperfect quality product.

In the manufacturer Stackelberg setting, this is a three-stage game. In first stage, the manufacturer announces wholesale price of the product. Based on this, in second stage game, the distributor determines wholesale price of perfect quality products. In third stage, following the distributor’s wholesale price, two retailers make their further sales decisions. Two retailers in the channel may behave in three ways, (i) Cournot i.e., the retailers compete, (ii) Collusion, i.e., the retailers jointly take decision and (iii) Stackelberg, i.e., one of the retailers takes decision and based on that the other retailers makes decision. Next sub-sections shall discuss how the members of the decentralized three-echelon supply make their decisions under these three situations.

3.3.2.1 Retailers play the Cournot game

Suppose duopolistic retailers pursue the Cournot solution, i.e., each retailer independently sets the retail price by assuming its rival’s selling price as a parameter. To find the subgame perfect equilibrium of this three-stage game backward induction is used. First the retailers simultaneously determine the optimal selling prices given \( w_{dp} \). Based on these two prices the distributor determines the optimal selling price of the perfect quality product. Finally the manufacturer determines the optimal wholesale price based on the distributor’s wholesale price.
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...are calculated and are presented in table-3.6. Using the optimal values in (3.94), (3.95) and (3.96) the optimal profits of the channel members of this manufacturer-Stackelberg game can be found (see appendix) and is presented in table-3.6. (3.95). Using backward induction as in the previous case the subgame perfect equilibrium of the game is found (see appendix) and is presented in table-3.6. The expected profit functions of the manufacturer and the distributor are same as in (3.94) and (3.96). Reorganizing the terms of B as

\[ B = \left[ \frac{(2a_1a_2 - \beta^2)(a_1 + a_2) - 2a_1a_2\beta}{4a_1a_2 - \beta^2} \right]. \]

Substituting \( p_1 \) and \( p_2 \) in \( \pi^d \) and solving \( dp_i/dw_{dp} = 0 \) gives

\[ w_{dp} = \frac{1}{2} B + \frac{1}{2} (1 + E(r)) w^m + \frac{(1 + E(r))(s + c) - w_{dl} E(r)}{2} \]

(3.99)

Substitution of \( w_{dp} \) in \( \pi^m \) provides

\[ \pi^m = \frac{1}{2} (1 + E(r))[A - B(1 + E(r)) w^m - B((1 + E(r))(s + c) - w_{dl} E(r))](w^m - c) \]

(3.100)

and

\[ \frac{d^2 \pi^m}{dw_{dp}^2} = -(1 + E(r))^2 B \]

Reorganizing the terms of B as \( B = \left[ \frac{(a_1a_2 - \beta^2)(a_1 + a_2) + a_1a_2(a_1 + a_2 - 2\beta)}{4a_1a_2 - \beta^2} \right]. \)

Since \( 0 < \beta < a_i \) (i = 1, 2), it is clear that B is positive. It means \( \pi^m \) is a concave function of \( w^m \). Therefore, the solution of \( d\pi^m/dw^m = 0 \) is an optimal wholesale price that is set by the manufacturer and is presented in table-3.6. The optimal values of the other decision variables are calculated and using these values the optimal profits of the channel members are calculated and are also presented in table-3.6.

3.3.2.2 Retailers play the collusion game

Assume that the duopolistic retailers recognize their interdependence and agree to act in union in order to maximize the total profit in the downstream retail market. The total expected profit of the downstream retail market is in such case is

\[ \pi_r = \sum_{i=1}^{2} \pi_{ri} = \sum_{i=1}^{2} (p_i - w_{dp} - c_r)(\alpha_i - a_ip_i + \beta p_{3-i}) \]

(3.101)

The expected profit functions of the manufacturer and the distributor are same as in (3.94) and (3.95). Using backward induction as in the previous case the subgame perfect equilibrium of this manufacturer-Stackelberg game can be found (see appendix) and is presented in table-3.6. Using the optimal values in (3.94), (3.95) and (3.96) the optimal profits of the channel members are calculated and are presented in table-3.6.
<table>
<thead>
<tr>
<th></th>
<th>Optimal</th>
<th>Decentralized Decision</th>
</tr>
</thead>
<tbody>
<tr>
<td>( w^j_m )</td>
<td>( c_m + \frac{(A/B-X_1)}{2(1+E(r))} )</td>
<td>( c_m + \frac{[A_1]}{2(a_1+a_2-2\beta)} - \frac{X_1}{2} )</td>
</tr>
<tr>
<td>( w^j_{dp} )</td>
<td>( \frac{1}{4} \left( \frac{3A}{B} + X_1 \right) )</td>
<td>( \frac{3A_1}{2(a_1+a_2-2\beta)} + \frac{X_1}{4} )</td>
</tr>
<tr>
<td>( p^j_{ri} )</td>
<td>( \frac{4[2a_3-(a_1+a_2c_{ri})+\beta(a_3-a_3c_{ri})]}{4[2a_3-(a_1+a_2c_{ri})+\beta(a_3-a_3c_{ri})]} + \frac{a_3-a_3c_{ri}}{4[2a_3-(a_1+a_2c_{ri})+\beta(a_3-a_3c_{ri})]} )</td>
<td>( \frac{a_3-a_3c_{ri}}{4[2a_3-(a_1+a_2c_{ri})+\beta(a_3-a_3c_{ri})]} + \frac{X_1}{8} )</td>
</tr>
<tr>
<td>( \pi^j_m )</td>
<td>( \frac{B}{8} \left( \frac{A}{B} - X_1 \right)^2 )</td>
<td>( \frac{16}{8} \left( \frac{2a_1}{a_1+a_2-2\beta} - X_1 \right)^2 )</td>
</tr>
<tr>
<td>( \pi^j_d )</td>
<td>( \frac{B}{16} \left( \frac{A}{B} - X_1 \right)^2 )</td>
<td>( \frac{32}{16} \left( \frac{2a_1}{a_1+a_2-2\beta} - X_1 \right)^2 )</td>
</tr>
<tr>
<td>( \pi_{ri}^j )</td>
<td>( (p^e_i - w^e_{dp} - c_{ri})D^e_i )</td>
<td>( (p^e_i - w^e_{dp} - c_{ri})D^e_i )</td>
</tr>
</tbody>
</table>

Centralized Decision

- \( p^e_i \) | \( \frac{a_3-a_1+\beta(a_3-a_3c_{ri}+\beta c_{ri}+X_1)}{2(a_3-a_3c_{ri}+\beta c_{ri}+X_1)} \) |
- \( D^e_i \) | \( \frac{a_3-a_1+\beta c_{ri}+\beta c_{ri}+X_1}{2} \) |
- \( \pi_{ci} \) | \( \sum_{i=1}^{2} \frac{1}{2} \frac{(a_3-a_1+\beta(a_3-a_3c_{ri}+\beta c_{ri}+X_1))}{2} \left[ \frac{a_1-a_1c_{ri}+\beta c_{ri}+X_1}{2} \right] \) |

Table 3.6: The optimal solutions for the centralized decision and three games of decentralized decision

Where, \( X_1 = (1 + E(r))(s + d + e) - E(r)w_{df} \); \( F_1 = \left[ 2a_2c_{ri} + \beta a_{ri} + (2a_1a_2 - \beta^2)c_{ri} + a_2b_{2c_{ri}} / [2(2a_1a_2 - \beta^2)] \right] ; \)

- \( F_2 = [2a_2+\beta(a_1+a_2c_{ri})+\beta(a_3-a_3c_{ri})]c_{ri}+X_1 \) \( / [4a_2+\beta(a_1+a_2c_{ri})+\beta(a_3-a_3c_{ri})] \) ;
- \( F_3 = [2a_2a_2 - \beta^2 + a_2^2] / [2(2a_1a_2 - \beta^2)] ; \)
- \( F_4 = [2a_2c_{ri} + \beta a_{ri} + (2a_1a_2 - \beta^2)c_{ri} + a_2b_{2c_{ri}} / [2(2a_1a_2 - \beta^2)] ; \)
- \( F = a_1 + a_2 - (a_1 - \beta)F_1 - (a_2 - \beta)F_2 / (a_2 - \beta)F_3 ; \)
- \( F_5 = (a_1 - \beta)F_3 + (a_2 - \beta)F_4 ; \)
- \( A_1 = a_1 + a_2 - (a_1 - \beta)(a_2a_1 + \beta a_{ri}) / (a_1a_2 - \beta^2) + c_{ri} / 2 - (a_2 - \beta)(a_1a_2 + \beta a_{ri}) / (a_1a_2 - \beta^2) + c_{ri} / 2 ; \)
3.3.2.3 Retailers play Stackelberg game

Under manufacturer-Stackelberg game setting at the final level assume that one of the two retailers, say, retailer-1 acts as the Stackelberg leader and the other i.e., retailer-2 acts as the Stackelberg follower. That is, to find the subgame perfect equilibrium of the entire three-stage game, using backward induction one first need to find the Stackelberg equilibrium between the retailers. Thus, for given \( w_{dp} \) and \( p_1 \), retailer-2 maximizes its profit. Based on it retailer-1 finds optimal \( p_1 \) given \( w_{dp} \). The rest of the game can be approached same as the previous two cases. The optimal values of the decision variables (see appendix) are presented in table-1. Consequently, using these values the optimal profits of the channel members are calculated and are also presented in table-3.6.

3.3.2.4 Comparison of optimal decisions

Comparing the wholesale price of the manufacturer and the distributor under three scenarios, the proposition follows.

**Proposition 3.5** Wholesale price of the manufacturer and the distributor have similar properties and are as follows

- (i) \( w_{mcn} > w_{msg} > w_{mct} \) if \( \Psi < 0 \) and \( (2a_1a_2 - 2a_2^2 + a_2\beta - \beta^2) < 0 \).
- (ii) \( w_{msg} > w_{mct} > w_{mcn} \) if \( \Psi < 0 \), \( (2a_1a_2 - 2a_2^2 + a_2\beta - \beta^2) > 0 \) and \( a_2 < a_1 \).
- (iii) \( w_{msg} > w_{mcn} > w_{mct} \) if \( \Psi < 0 \), \( (2a_1a_2 - 2a_2^2 + a_2\beta - \beta^2) > 0 \) and \( a_2 > a_1 \).
- (iv) \( w_{mct} > w_{mcn} > w_{msg} \) if \( \Psi > 0 \), \( (2a_1a_2 - 2a_2^2 + a_2\beta - \beta^2) > 0 \) and \( a_2 < a_1 \).
- (v) \( w_{mct} > w_{mnc} > w_{msg} \) if \( \Psi > 0 \), \( (2a_1a_2 - 2a_2^2 + a_2\beta - \beta^2) > 0 \) and \( a_2 < a_1 \).
- (vi) \( w_{mct} = w_{msg} = w_{mnc} \) if \( \beta = 0 \) and/or \( \Psi = 0 \).

Where \( \Psi = [(a_2 - \beta)\alpha_1 - (a_1 - \beta)\alpha_2 - (a_1a_2 - \beta^2)(c_{r1} - c_{r2})] \).

**Proof** See Appendix

Wholesale prices of the distributor for the retailers three behaviours are same as the manufacturer and hence the proof is omitted. Proposition 3.5 indicates that based on \( \Psi \) and the retailers price sensitivity factors superiority of wholesale prices of the manufacturer and distributor can be identified. Note that, when \( a_1 = a_2 \), \( \alpha_1 = \alpha_2 \) and \( c_{r1} = c_{r2} = 0 \) then \( \Psi = 0 \) and it gives same outcome of special case of Yang and Zhou [151].

Generally, \( \pi_{ct}^m > \pi_{sg}^m > \pi_{cn}^m \) but when \( \beta = 0 \) then \( \pi_{ct}^m = \pi_{sg}^m = \pi_{cn}^m \) and graphical representation of the manufacturer profits are given in figure-3.5. From fig-3.5 one can observe that, the differences of the optimal profit of the manufacturer among the three cases increases with increasing \( \beta \).

From table-3.6 one may note that profits of the distributor in all cases are half of the manufacturer’s profits. So, profits of the distributor posses same behavior as of profits of the manufacturer. Graphical representation of comparison of the retailers profits are presented in
CHAPTER 3. PERISHABLE AND IMPERFECT QUALITY PRODUCTS IN SC

Figure 3.5: Optimal profit of the manufacturer under three scenario figure-3.6(a) while figure-3.6(b) represents the comparison of retail prices.

Figure 3.6: (a) Optimal profit (b) optimal selling price of the ith retailer (i=1,2) under three scenario

Fig-3.6 depict that the differences of the prices and profits of the retailers in three games decrease as $\beta$ decreases. When $\beta = 0$, the outcomes of Cournot, Collusion and Stackelberg games are invariant. Thus, cross price effect of the downstream retailers has significant impact on the channel members profits. Also, note from fig-3.6 that the retailers profits are maximum when they play Collusion. This is quite obvious because at the downstream the retailers cooperate. As a result the upstream channel members profits are minimum in that case. That is why, Collusion game is downstream market is illegal in some countries. The next subsection develops decisions of centralized channel.

3.3.3 Centralized policy

Suppose that all the channel members of the decentralized three-echelon supply chain are willing to form a vertically integrated system. Then, the decision maker in this system sets prices $p_1^c$ and $p_2^c$ at its two retail outlets. Let $\pi_c$ denotes the expected profit of the integrated system,
then

\[ \pi_c = \sum_{i=1}^{2} [p_i^c - c_{ri} - X_1]D_i^c \]  \hspace{1cm} (3.102)

Taking the second-order partial derivatives of \( \pi_c \) with respect to \( p_1 \) and \( p_2 \), provides \( \partial^2 \pi_c / \partial p_1^2 = -2a_1 \), \( \partial^2 \pi_c / \partial p_2^2 = -2a_2 \) and \( \partial^2 \pi_c / \partial p_1 \partial p_2 = \partial^2 \pi_c / \partial p_2 \partial p_1 = 2\beta \). Let \( \Delta_1 \) and \( \Delta_2 \) denote respectively the first and second-order principal minors of Hessian matrix of the total channel profit, \( \pi_c \). Clearly, \( \Delta_1 = -2a_1 < 0 \) and \( \Delta_2 = 4a_1a_2 - 4\beta^2 > 0 \). Hence, the Hessian matrix of the total channel profit, is negative definite. So, the profit function of centralized channel is a concave function of \( p_1^c \) and \( p_2^c \). Optimal selling prices of the duopolistic retailers can be obtained by solving the necessary conditions \( \partial \pi_c / \partial p_1 = 0 \) and \( \partial \pi_c / \partial p_2 = 0 \) and are displayed in table-3.6. Centralized demand in two retail outlets and centralized channel profit are also given in table-3.6.

### 3.3.4 Effect of imperfect quality product

Generally speaking, it is common in marketing that selling price of imperfect quality product is lower than its production cost and so it is assumed that \( s + c^d + c > w_d \). Differentiation of wholesale prices, selling prices, profits of the channel members are given in appendix. From appendix observe that wholesale price of the manufacturer in all three cases decreases with increasing expected rate of imperfect quality product while wholesale price of the distributer and selling price of the retailers increase with increasing expected rate of imperfect quality product.

![Figure 3.7: Effect of expected rate of imperfect quality on wholesale prices under three scenario](image)

This is quite obvious because the distributer is bound to increase its wholesale price for perfect quality product when the average rate of imperfect quality product increases. By increasing the wholesale price of the perfect quality product, the distributer minimizes its loss that is
generated through lower price of the imperfect quality product. Obviously in that case the retailers will increase the retail prices. Fig-3.7 represents the behaviour of wholesale prices of the channel members. Profits of all the channel members in all three cases decrease with increasing expected rate of imperfect quality product. Thus, the proposition follows.

**Proposition 3.6** When selling price of imperfect quality product is lower than its production cost then wholesale price of the manufacturer, profit function of all channel members decrease but wholesale price of the distributer increase with increasing expected rate of imperfect quality product in decentralized channel for all three behaviors of the duopolistic retailers.

The conclusion of proposition 3.6 will be opposite if the distributer can sale the imperfect quality product more than its production cost i.e., if \( s + c^d + c < w_{dl} \). Effect of expected rate of imperfect quality product in centralized decision can be found by differentiating optimal selling prices, demand and centralized profit with respect to \( E(r) \) as

\[
\frac{dp^c_i}{dE(r)} = \frac{(s + c^d + c - w_{dl})}{2}, \quad i = 1, 2 \tag{1.103}
\]

\[
\frac{dD^c_i}{dE(r)} = -\frac{(a_i - \beta)(s + c^d + c - w_{dl})}{2}, \quad i = 1, 2 \tag{1.104}
\]

\[
\frac{d\pi^c_i}{dE(r)} = -\sum_{i=1}^{2} [D^c_i + (p^c_i - c_{ri} - X_1)(a_i - \beta)] \frac{(s + c^d + c - w_{dl})}{2} \tag{1.105}
\]

That is, selling price of the product in centralized scenario increases while demand and profit decreases with increasing expected rate of imperfect quality product. Thus, the proposition follows.

**Proposition 3.7** Selling price of the product in centralized scenario increases while demand and profit decreases with increasing expected rate of imperfect quality product if \( s + c^d + c > w_{dl} \) otherwise the effect will be reversed.

### 3.3.5 Channel coordination using all unit quantity discount with franchise fee

In the three-echelon supply chain for all the competitive behaviors of the retailers the channel is not coordinated. For channel coordination, assume that an upstream channel member provides all unit quantity discount to its immediate downstream channel member and charge a franchise fee. All unit quantity discount has been well studied and well applied to resolve channel conflict in supply chain literature. When all unit quantity discount is applied, a channel member provides discounts on the wholesale price to the other member anticipating centralized quantity to be ordered. Franchise fee is another contract that is used for cutting out channel conflict, where the upstream channel member supplies the product to the downstream channel member at its own marginal cost and charge a franchise fee for profit enhancement. The subsection applies these two coordination contracts jointly for channel coordination in Cournot, Collusion.
and Stackelberg games of the retailers separately.

### 3.3.5.1 Retailers play Cournot game

Assuming that by providing discount, the manufacturer charges wholesale price $\rho mw^{mct}$ to distributor and claims franchise fee $f_i^{ct}$ from the distributor. The distributor supplies the product to the $i$th retailer $(i=1, 2)$ at a wholesale price $\rho_iw_i^{ct}$ and charges $f_i^{ct}$ franchise fee from the $i$th retailer. Then the expected profit function of retailers, distributor and manufacturer are as

$$\pi_{ri}^{q/d/ct} = (\rho_i - \rho_iw_i^{ct} - c_{ri})(\alpha_i - a_ip_i + \beta p_{3-i}) - f_i^{ct}, \quad (i = 1, 2)$$

(1.106)

$$\pi_{d}^{q/d/ct} = \sum_{i=1}^{2} (\rho_iw_i^{ct} + E(r)w_{dI} - (1 + E(r))(s + c^d + \rho_mw^{mct})(\alpha_i - a_ip_i + \beta p_{3-i}) + f_i^{ct}) - f_m^{ct}$$

(1.107)

$$\pi_{m}^{q/d/ct} = (1 + E(r))(\rho_mw^{mct} - c)(\sum_{i=1}^{2} D_i) + f_m^{ct}$$

(1.108)

Under Cournot behavior of the retailers the necessary conditions $d\pi_{ri}^{q/d/ct}/dp_i = 0, \quad (i = 1, 2)$ yields

$$p_i^{q/d/ct} = \frac{2a_i\alpha_3 - c_{ri} + 2a_i\alpha_3 - c r_3 - i + \alpha_3 - i \beta + \alpha_3 - i \beta + 2a_i a_{3-i} w_i^{ct} p_i + a_{3-i} w_i^{ct} \beta p_{3-i}}{4a_i a_{3-i} - \beta^2}, \quad (i = 1, 2)$$

(1.109)

The proposed coordination contract resolves channel conflict only when $p_i^{q/d/ct} = p_i^{eq}, \quad (i = 1, 2)$ i.e.,

$$p_i^{q/d/ct} = \frac{a_3 - i \alpha_i + a_3 - i \beta}{2w_i^{ct} (a_i a_{3-i} - \beta^2)} + \frac{(2a_i - \beta)X_2 - a_i - c_{ri} - i \beta}{2a_i w_i^{ct}}$$

(1.110)

On the other hand, based on the retailers decision the distributor will maximize its profit function by determining the discount factors, which are different for different retailer. This assumption is quite reasonable and common in marketing practice. Generally, an upstream channel member will provide how much discount to its different downstream channel members that depends on how much it receives from those members. That is, the amount of quantity discount is proportionate to the order quantity and hence profit. As such, the necessary conditions for the existence of the optimal solution yields

$$\rho_{i/ct} = \frac{a_{3-i} \alpha_i + a_3 - i \beta + (a_i a_{3-i} - \beta^2)(X_2 - c_{ri} + (1 + E(r))w^{mct} p_m)}{2w_i^{ct} (a_i a_{3-i} - \beta^2)}, \quad (i = 1, 2)$$

(1.111)

Where $X_2 = (c^d + s)(1 + E(r)) - E(r)w_{dI}$

Obviously for channel coordination $\rho_{i/ct} = \rho_i^{q/d/ct}, \quad (i = 1, 2)$, which on simplification suggests

$$\rho_{mri/ct} = -\frac{\alpha_1 + (c_{r3-i} + X_1) \beta - a_1(c_{ri} + (1 + E(r))c_m + X_1)}{a_i(1 + E(r))w^{mct}}, \quad (i = 1, 2)$$

(1.112)

The result of (1.112) is quite interesting. Firstly, $\rho_{mri/ct}, \quad (i = 1, 2)$ may be negative i.e.,
initially the manufacturer has to provide some money to the distributor for channel coordination but finally it makes up its loss and gains through franchise fee. This scenario shows that for channel coordination the intermediator plays major role, even it initially demands money from upstream channel members to implement integrated decision. Secondly, it suggests that the manufacturer provides different discounts to different retailers under the same distributor. But, in the decision making context it is not possible as well as feasible. Assume that the manufacturer sets the discount factor for the distributor as the weighted mean of the discount factors of its corresponding retailers. Then, the discount factor of the manufacturer for the distributor is

\[ \rho_{\text{qd/c}}^{\text{m}/i} = \frac{\sum_{i=1}^{2} \rho_{\text{r/mri}} D_i^c}{\sum_{i=1}^{2} D_i^c} \]  

(1.113)

Thus, the manufacturer sets discount factor \( \rho_{\text{qd/c}}^{\text{m}/i} \) on its wholesale price. In response the distributor provides \( \rho_{\text{qd/c}}^{\text{d}/i} \) (\( i = 1, 2 \)) discount on ith (i=1,2) retailer’s wholesale price and the ith retailer sets the centralized retail price.

3.3.5.2 Retailers play collusion game

For Collusion game the retailers jointly optimize their total profit. Total profit of the retailers under the all unit quantity discount with franchise fee is

\[ \pi_{\text{qd/c}}^{\text{r}/i} = \sum_{i=1}^{2} (p_i - p_i w_{dp}^{cn} - c_{ri})(\alpha_i - a_i p_i + \beta p_{1-i}) - f_i \]  

(1.114)

Solving the necessary conditions for maximizing \( \pi_{\text{qd/c}}^{\text{r}/i} \) i.e., \( \frac{\partial \pi_{\text{qd/c}}^{\text{r}/i}}{\partial p_i} = 0 \), \( i = 1, 2 \) gives

\[ p_{\text{qd/c}}^{\text{r}/i} = \frac{a_{3-i}(\alpha_i + a_i(c_{ri} + w_{dp}^{cn} \rho_i)) + \beta(\alpha_{3-i} - \beta(c_{ri} + w_{dp}^{cn} \rho_i))}{2(a_i a_{3-i} - \beta^2)}, \quad (i = 1, 2) \]  

(1.115)

Proceeding similar way as in Cournot scenario, the amount quantity discount which will coordinate the channel are as follows

\[ \rho_{\text{qd/c}}^{\text{r}/i} = \frac{X_1}{w_{dp}^{cn}}, \quad (i = 1, 2) \]  

(1.116)

and

\[ \rho_{\text{qd/c}}^{\text{m}/i} = \frac{\rho_{\text{mri}}^{cn} D_2^c + \rho_{\text{mr}}^{cn} D_2^c}{D_1^c + D_2^c} \]  

(1.117)

Where

\[ \rho_{\text{mri/cn}} = \frac{c_{ri} + c(1 + E(r)) X_1 - (a_{3-i} \alpha_i + \alpha_{3-i} \beta)/a_i a_{3-i} - \beta^2}{(1 + E(r)) w_{mcn}^{cn}}, \quad (i = 1, 2) \]  

(1.118)
### 3.3.5.3 Retailers play Stackelberg game

Similar to decentralized scenario, the retailer-2 (follower) first optimize its profit then the retailer-1 (leader) optimize its profit function. The optimal quantity discount for coordination when the retailers play stackelberg game are found as

\[
\rho_{1}^{q_{d}/s_{g}} = \frac{X_{1} - c_{r_{1}}}{2w_{dp}^{sg}} + \frac{a_{2}a_{1} + a_{2}\beta}{2w_{dp}^{sg}(a_{1}a_{2} - \beta^{2})} + \frac{2a_{2}(a_{1}c_{r_{1}} + X_{1}) - \alpha_{1} - (c_{r_{2}} + X_{1})\beta}{2w_{dp}^{sg}(a_{1}a_{2} - \beta^{2})} \quad (1.119)
\]

\[
\rho_{2}^{q_{d}/s_{g}} = \frac{a_{1}a_{2} + a_{1}\beta}{2w_{dp}^{sg}(a_{1}a_{2} - \beta^{2})} - \frac{a_{2} - 2a_{2}X_{1} - \beta(c_{r_{1}} + X_{1})}{2w_{dp}^{sg}a_{2}} \quad (1.120)
\]

and

\[
\rho_{m}^{q_{d}/s_{g}} = \frac{\rho_{mr1}/s_{g}D_{1}^{c} + \rho_{mr2}/s_{g}D_{2}^{c}}{D_{1}^{c} + D_{2}^{c}} \quad (1.121)
\]

Where,

\[
\rho_{mr1}/s_{g} = \frac{2a_{1}a_{2}(c_{r_{1}} + c(1 + E(r)) + X_{1}) - c(1 + E(r))\beta^{2} - 2a_{2}(a_{1} + (c_{r_{2}} + X_{1})\beta)}{(1 + E(r))m_{ssg}(a_{1}a_{2} - \beta^{2})} \quad (1.122)
\]

\[
\rho_{mr2}/s_{g} = \frac{a_{2}(c_{r_{2}} + c(1 + E(r)) + X_{1}) - a_{2} - (c_{r_{1}} + X_{1})\beta}{a_{2}(1 + E(r))m_{ssg}} \quad (1.123)
\]

Now, the expected profits of the channel members for jth (j=ct, cn, sg) game are respectively as

\[
\pi_{ri}^{q_{d}/j} = (p_{i}^{c} - \rho_{i}^{q_{d}/j} w_{dp}^{j} - c_{r_{i}})D_{i}^{c} - f_{i}^{j}, \quad (i = 1, 2; j = ct, cn, sg) \quad (1.124)
\]

\[
\pi_{d}^{q_{d}/j} = -f_{i}^{j} + \sum_{i=1}^{2} \left( [\rho_{i}^{q_{d}/j} w_{dp}^{j} + E(r)w_{dl}^{j} - (1 + E(r))(s + c_{d}^{j} + \rho_{m}^{q_{d}/j} w_{mj})]D_{i}^{c} + f_{i}^{j} \right), \quad (j = ct, cn, sg) \quad (1.125)
\]

\[
\pi_{m}^{q_{d}/j} = (1 + E(r))(D_{1}^{c} + D_{2}^{c})(\rho_{m}^{q_{d}/j} w_{mj} - c) + f_{m}^{j}, \quad (j = ct, cn, sg) \quad (1.126)
\]

Observe that \(\pi_{ri}^{q_{d}/j} + \pi_{d}^{q_{d}/j} + \sum_{i=1}^{2} \pi_{ri}^{q_{d}/j} = \pi_{c}, \quad (j = ct, cn, sg)\) i.e., the channel is coordinated and the proposition follows.

**Proposition 3.8** All unit quantity discount with franchise fee coordinates the three-echelon supply chain under the duopolistic retailers Cournot, Collusion and Stackelberg behaviors.

Now, considering the situation without franchise fee i.e., before settlement of franchise fee

\[
(p_{1}^{c} - \rho_{1}^{q_{d}/j} w_{dp}^{j} - c_{r_{1}})D_{1}^{c} - (p_{1}^{c} - \rho_{1}^{q_{d}/j} w_{dp}^{j} - c_{r_{1}})D_{1}^{c} = \frac{D_{c}^{2}}{2} \left[ \beta(a_{1}a_{2} + \beta a_{1}) - (a_{1}a_{2} - \beta^{2})(c_{r_{2}} + X_{1}) \right] > 0
\]
(p_2 - \rho_2 \rho_{2/d/cn} w_{dp}^c - c_r)D_2^2 - (p_2 - \rho_2 \rho_{2/d/c} w_{dp}^c - c_r)D_2^2 = \frac{D_2^2}{2} \left[ \frac{\beta(a_2a_1 + \beta a_2) - (a_2a_1 - \beta^2)(c_{r1} + X_1)}{a_2a_1 - \beta^2} \right] > 0

That is, before the settlement of franchise fee both the retailers get more money in Collusion game than Cournot game. Again,

(p_1 - \rho_1 \rho_{1/d/sg} w_{dp}^c - c_r)D_1^c - (p_1 - \rho_1 \rho_{1/d/ct} w_{dp}^c - c_r)D_1^c = \frac{D_1^c}{4} \left[ \frac{\beta(a_1a_2(2a_2 + (c_{r1} + X_1)) - 2a_2(c_{r2} + X_1)))}{2a_1a_2 - \beta^2} (a_1a_2 - \beta^2) \right] > 0

And

(p_2 - \rho_2 \rho_{2/d/sg} w_{dp}^c - c_r)D_2^2 - (p_2 - \rho_2 \rho_{2/d/ct} w_{dp}^c - c_r)D_2^2 = 0

The retailers get maximum wealth when the play collusion game and minimum in Cournot game. Since total profit of the channel members is equal to the centralized profit, it is obvious that first preference of the manufacturer will be Cournot and last is Collusion behavior of the duopolistic retailers. Thus, the proposition follows.

**Proposition 3.9** Under the all unit quantity discount before the settlement of franchise fee retailers preference sequence are (i) Collusion (ii) Stackelberg and (iii) Cournot, while the manufacturer prefers oppositely.

### 3.3.5.4 Some limits of franchise fee

Although the proposed all unit quantity discount with franchise fee resolves channel conflict, it is acceptable to the channel members only when they receive win-win outcomes. The win-win outcomes of the channel members are ensured if they receive at least equal to their respective decentralized profits i.e., \( \pi_{d/ij} \geq \pi_{d/ij}^j \), \( (i = 1, 2; j = ct, cn, sg) \), \( \pi_{d/ij} \geq \pi_{d/ij}^j \), \( (j = ct, cn, sg) \) and \( \pi_{d/ij} \geq \pi_{d/ij}^j \), \( (j = ct, cn, sg) \), which on simplification yields

\[
\begin{align*}
&f_1^j \leq (p_1 - \rho_1 \rho_{1/d/ct} w_{dp}^c - c_r)D_1^c - \pi_{d/ij} = \bar{f}_1^j, \quad (i = 1, 2; j = ct, cn, sg) \quad (1.127) \\
&f_1^m \geq \pi_{d/ij} - (1 + E(r))(D_1^d + D_2^d)(\rho_{1/d/ct} w_{mi}^j - c) = \bar{f}_1^m, \quad (j = ct, cn, sg) \quad (1.128) \\
&f_1^j + f_1^m \geq \pi_{d/ij} - \sum_{i=1}^2 \left( (p_1 - \rho_{1/d} w_{dp}^c + E(r)w_{dl} - (1 + E(r))(s + c^d)(\rho_{1/d/ct} w_{mj}^j)D_1^c) \right), (j = ct, cn, sg) \quad (1.129)
\end{align*}
\]

Now the following subsection determines the other limits of franchise fee and particular surplus
3.3.6 Surplus profit division

Although the inequalities indicate the upper limits of the franchise fees that the retailers provide to the distributor and lower limit of franchise fee that the distributor provides the manufacturer, still it does not ensure win-win profits for the channel members. Thus, how the surplus profit generated through coordination will be distributed among the channel members given all will achieve win-win profits. To determine this one first need to determine the ranges of franchise fees, that provide win-win outcomes. The best possible approach in this direction is to apply Nash bargaining product. In this context note that the distributors acts as the mediator, who receives the lot from the manufacturer and distributes among the retailers. By doing so it makes profit. Interestingly, for channel coordination the distributor plays central role because it actually maintains the lot streaming between the manufacturer and the retailers. Thus, when the coordination contracts resolve channel conflict, there may arise two questions. (i) What is the minimum amount of franchise fee that the distributor accept? (ii) What is the maximum percent of franchise fee that the distributor can provide to it’s manufacturer? These two questions are interrelated and one definitely get different results when approaches from the manufacturer to the retailers and from the retailers to the manufacturer i.e., backward sequential bargaining and forward sequential bargaining. The intuitive reason behind this instead of considering multi player Nash bargaining is quite reasonable and common in marketing practice. In marketing deal on implementation of contract issues and then division of surplus...
generated through contract is done between two players. But how one can use the bargaining to divide the surplus if the amount of surplus due to coordination, i.e., the ranges of franchise fees are not explicitly identified. Thus, the study applies the process that first identifies the range of franchise fee and then determine the profit split. It is done in to steps by considering two immediate next or immediate previous channel members. Also, it can be done either by approaching from forward or from backward. The next subsections apply these two processes and analyze the results.

3.3.6.1 Backward-sequential-bargaining for surplus profit division

In this process the optimal decisions for the ranges of franchise fees and surplus profit division are made in one-to-one basis. First the distributor and the retailers interact and then the distributor and the manufacturer finds the decisions. The objectives of all channel members are to determine the shares of surplus profit that is generated through channel coordination. The sequence of events in this process are as follows.

1. Each retailer interacts with the distributor separately and determines the lower limit of franchise fee.
2. The distributor bargains with each retailer independently to determine the franchise fee within its range. Decentralized profit plus the surplus share is the optimal profit of the ith (i=1, 2) retailer. The distributor’s intermediate profit is the decentralized profit plus cumulative surplus shares, which it acquired from all of it’s retailers.
3. Based on the intermediate profit the distributor and the manufacturer identify the upper limit of franchise fee. This limit can be determined by assuming that the distributor accept the franchise fee as long as it’s decentralized profit is reserved.
4. The manufacturer and the distributor bargain for surplus share. The distributor’s optimal profit is decentralized profit plus accumulated surplus from all of it’s retailers minus surplus share to the manufacturer. The manufacturer’s profit is decentralized profit plus surplus share that it receives from the distributor.

The distributor deals with each of it’s retailers independently because it has different reservations for different retailers. How much profit it can share to the retailer that depends on how much of it’s profit increased for that retailer’s centralized order quantity. The distributor’s decentralized and centralized profit corresponding to its ith (i = 1, 2) retailer are respectively as

\[ \pi_{d^2}^j = \frac{D^j_i}{D^1_i + D^2_i} \pi_{d^2}^j, \quad i = 1, 2; j = ct, cn, sg \]  

\[ \pi_{d^2}^{q^d/j} = [\rho_i^{q^d/j} w_{dl} + E(r)w_{dl} - (1 + E(r))(s + c^d + \rho_m^{q^d/j} w_{ml})]D^1_i + f^j_i - (D^1_i f_{m}^j)/(D^1_i + D^2_i), \quad (i = 1, 2; j = ct, cn, sg) \]  

6The process of the backward-sequential-bargaining is similar to section 3.2 but here it is applied with another coordination mechanism for different channel structure with some modifications.
CHAPTER 3. PERISHABLE AND IMPERFECT QUALITY PRODUCTS IN SC

Where $D^j_i$ denotes demand of the $i$th ($i=1,2$) retailer in decentralized Cournot behavioral scenario. The minimum amount of franchise fee that the distributor will accept from $i$th ($i=1,2$) retailer if $\pi^{q/d/j}_{di} \geq \pi^{d/i}_{di}$, $j=ct, cn, sg$ i.e.,

$$f^j_i = \pi^j_{di} - [\rho^{q/d/j}_{di} w^j_{dp} + E(r)w_{dt} - (1 + E(r))(s + c^d + \rho^{q/d/j}_{m} w^{mj})]D^c_i$$
$$+ (D^c_i f^j_m)/(D^c_i + D^c_2), \quad (i = 1, 2; j = ct, cn, sg) \quad (1.132)$$

Clearly, $f^j_i$ can not be determined uniquely until $f^j_m$ is determined. But least amount of franchise fee is acceptable to the distributor from the $i$th retailer only when it pays minimum amount of franchise fee $f^j_m$ to the manufacturer, i.e.,

$$f^{j/min}_i = \pi^j_{di} - [\rho^{q/d/j}_{di} w^j_{dp} + E(r)w_{dt} - (1 + E(r))(s + c^d + \rho^{q/d/j}_{m} w^{mj})]D^c_i$$
$$+ (D^c_i f^{j/min}_m)/(D^c_i + D^c_2), \quad (i = 1, 2; j = ct, cn, sg) \quad (1.133)$$

Interestingly profit of the manufacturer is equal to its decentralized profit when the distributor gives $f^j_m$ amount of franchise fee. But it is reasonable to assume that, the distributor have to share a reasonable portion of surplus profit with the manufacturer i.e., $f^j_m > f^j_i$ otherwise the manufacturer has no reason to accept the contract. Suppose the distributor predetermines a target that it will pay $f^{j/I}_m$ ($f^{j/I}_m > f^j_m$) amount of franchise fee to the manufacturer. The intuitive reason is straightforward. As a mediator, the distributor first settles the franchise fee without knowing about how much franchise fee it will give the manufacturer. Since the players operate under symmetric information, the distributor takes a risk by supplying the information to the retailers that it has to pay $f^{j/I}_m$ to the manufacturer and settles the franchise fees that it will take from the retailers. Later when the distributor determines the franchise fee, that it will give to the manufacturer through negotiation, may be higher or lower than $f^{j/I}_m$. That is, the risk which the distributor takes on franchise fee during the entire negotiation process is sometimes beneficial and sometimes not. This feature is quite common in marketing, where the players has the opportunity to take decisions freely. Thus, minimum amount of franchise fee that the distributor will accept from $i$th ($i=1,2$) retailer is

$$f^{j/min}_i = \pi^j_{di} - [\rho^{q/d/j}_{di} w^j_{dp} + E(r)w_{dt} - (1 + E(r))(s + c^d + \rho^{q/d/j}_{m} w^{mj})]D^c_i$$
$$+ (D^c_i f^{j/I}_m)/(D^c_i + D^c_2), \quad (i = 1, 2; j = ct, cn, sg) \quad (1.134)$$

Now, the Nash bargaining product [104] is used to find equilibrium solution. The Nash bargaining model that has been used in various contexts, is an axiomatic derivation of bargaining solution. The axiomatic derivation leaves out the actual process of negotiations while focusing on the expected outcome based on pre-specified solution procedures. Also the axioms do not reflect the rationale of the agents or the process in which the agreement is reached. One of the important characteristics of the Nash solution concept is that the outcome is random because it depends on the participating players negotiation powers. In Nash bargaining model the objective function is the product of the players surplus from cooperation and it must be maximized. Each players surplus profit is the difference between the negotiated profit and profit under decentralized decision making.

Since, any $f^j_i \in (f^{j/I}_i, f^j_i)$ resolves channel conflict, the distributor and the $i$th retailer bargain
to determine an acceptable \( f^i_j \) \((f^{i/j}, f^j_i)\) that will maximize the Nash bargaining product. The Nash bargaining product can be found as

\[
\max_{f^{i/j} \leq f^i_j \leq f^j_i} [\Delta^i_d - (D^c f^i_j / (D^c + D^e)) + f^j_i], \quad i = 1, 2; \quad j = ct, cn, sg
\]  

(1.135)

Where \( \Delta^i_d = (p^i - \rho^d w^d_j - c_{ri})D^c_i - \pi^i_d \) and \( \Delta^j_d = (\rho^d w^d_j + E(r)w_{dt} - (1 + E(r))(s + c^d + \rho^d_{mj} w^m_j)]D^c_i - \pi^j_d \). The necessary conditions for the existence of optimal solution yields

\[
f^{b/j}_i = \frac{\Delta^i_d - \Delta^j_d + (D^c f^{i/j}_m / (D^c + D^e))}{2}, \quad i = 1, 2; \quad j = ct, cn, sg
\]  

(1.136)

Thus, the \( j \)th retailer’s optimal profit after bargaining can be found from (1.124) by replacing \( f^j_i \) by \( f^{b/j}_i \) of (1.136). As a consequence, the distributor’s intermediate profit is

\[
\pi^{q/d/j}_d = \sum_{i=1}^{2} \left( [\rho^d w^d_j + E(r)w_{dt} - (1 + E(r))(s + c^d + \rho^d_{mj} w^m_j)]D^c_i + f^{b/j}_i \right) \]

\[
- f^j_i \quad (j = ct, cn, sg)
\]  

(1.137)

As soon as the distributor finds optimal cumulative surplus shares from its retailers, it determines the maximum amount of franchise fee that it can provide to the manufacturer. If the distributor provides maximum \( f^j_i \) amount of franchise fee that ensures the its decentralized profit after the first bargaining then

\[
\overline{f^j_i} = \sum_{i=1}^{2} \left( [\rho^d w^d_j + E(r)w_{dt} - (1 + E(r))(s + c^d + \rho^d_{mj} w^m_j)]D^c_i + f^{b/j}_i \right) - \pi^j_d, \quad j = ct, cn, sg
\]  

(1.138)

Any \( f^j_i \in (f^{j}_m, \overline{f}^j_m) \) ensures win-win profits for the manufacturer and the distributor. The feasible region of win-win outcome of all channel members in backward bargaining process for Cournot behavior of the retailers displayed in figure-3.9.

Now, if the distributor bargains with the manufacturer for \( f^j_i \in (f^j_m, \overline{f}^j_m) \) that will maximize their profit then the Nash bargaining product can be found as

\[
\max_{f^j_m \leq f^j_i \leq \overline{f}^j_m} [\Delta^j_d + f^{b/j}_i + f^{b/j}_j - f^j_i][\Delta^j_m + f^j_m], \quad j = ct, cn, sg
\]  

(1.139)

Where \( \Delta^j_d = \sum_{i=1}^{2} [\rho^d w^d_j + E(r)w_{dt} - (1 + E(r))(s + c^d + \rho^d_{mj} w^m_j)]D^c_i - \pi^j_d \) and \( \Delta^j_m = (1 + E(r))(D^c_i + D^e)\rho^d_{mj} w^m_j - c) - \pi^j_m \). Optimal value of \( f^j_m \) is

\[
f^{b/j}_m = \frac{\Delta^j_d + f^{b/j}_i + f^{b/j}_j - \Delta^j_m}{2}, \quad j = ct, cn, sg
\]  

(1.140)

Therefore, optimal profits of the retailers, the distributor and the manufacturer after the
sequential-bargaining process are respectively as

\[
\pi_{rb/j}^{i} = \pi_{ri}^{j} + \frac{\Delta_{1}^{j} + \Delta_{dt}^{j} - (D_{c}^{j} f_{m}^{j}/(D_{c}^{j} + D_{r}^{j}))}{2}, \quad i = 1, 2; \quad j = ct, cn, sg \quad (1.141)
\]

\[
\pi_{db/j}^{j} = \pi_{d}^{j} + \frac{2\Delta_{t}^{j} + \Delta_{d}^{j} + \Delta_{r1}^{j} + \Delta_{r2}^{j} + f_{m}^{j}}{4}, \quad j = ct, cn, sg \quad (1.142)
\]

\[
\pi_{mb/j}^{j} = \pi_{m}^{j} + \frac{2\Delta_{m}^{j} + \Delta_{d}^{j} + \Delta_{r1}^{j} + \Delta_{r2}^{j} + f_{m}^{j}}{4}, \quad j = ct, cn, sg \quad (1.143)
\]

Note that, \(d\pi_{rb/j}^{i}/df_{m}^{j} = -(D_{c}^{j}/2(D_{c}^{j} + D_{r}^{j})) < 0 \) (j=ct, cn, sg; i=1,2); \(d\pi_{db/j}^{j}/df_{m}^{j} = d\pi_{mb/j}^{j}/df_{m}^{j} = 1/4 > 0 \) (j=ct, cn, sg), that is the bargaining profit of the channel members depends on \(f_{m}^{j}\) and when it increases retailers’ profit decrease but profit of the manufacturer and the distributor increase. From the above bargaining process following results can be realized. Firstly, unlike the centralized decision, the ranges of the contract parameters ensure win-win opportunities for all the channel members. Moreover, the channel is coordinated and maximum channel benefit is distributed among the channel members. Secondly, retailers share the surplus profit equitably with the distributor. Thirdly, the distributor shares the surplus profit for channel coordination with the manufacturer equitably. From the above discussion the proposition follows

**Proposition 3.10**  All unit quantity discount with franchise fee coordinates the channel
and the channel members get win-win profit through backward-sequential-bargaining for \( f^j_i \in (f^{i/j}_i, f^j_i) \) (i=1, 2; j=ct, cn, sg) and \( f^j_m \in (f^j_m, f^{j/m}_m) \) and agreed on franchise fee \( f^{p/j}_i, f^{p/j}_m \) (i=1, 2, j=ct, cn, sg).

### 3.3.6.2 Forward-sequential-bargaining process

In this case also the win-win franchise fee ranges and bargaining take place in one-to-one basis but from upstream channel members to downwards. The sequence of events and bargaining are as follows.

1. The distributor and manufacturer determine the upper limit of franchise fee depending on a minimum target amount of franchise fee that the distributor can receive from the retailers.
2. The distributor bargains with the manufacturer for determination of franchise fee within its range. Decentralized profit plus the surplus share is the optimal profit of the manufacturer. After the process, the distributor’s intermediate profit is the decentralized profit plus surplus shares, which is depending on minimum target amount of franchise fee from the retailers.
3. Based on this profit the distributor deals with each of the retailers independently to identify the lower limit of franchise fee. This limit can be determined by assuming that the distributor accepts the franchise fee as long as it’s decentralized profit is reserved.
4. The distributor bargains with each of its retailers to determine equilibrium amount of surplus share. The distributor’s optimal profit is decentralized profit plus accumulated surplus from all of its retailers minus surplus share to the manufacturer. Each of the retailer’s profit is its decentralized profit plus surplus share from the distributor.

Using the same logic as the previous case assume that the minimum target amount of franchise fee that the distributor can achieve from the ith (i=1, 2) retailer is \( f^{t/j}_i \). Based on the cumulative \( f^{t/j}_i \), (i=1,2) the distributor determines the maximum amount of franchise fee that it can provide to the manufacturer. If the distributor provides maximum \( f^{t/j}_m \) amount of franchise fee that ensures its decentralized profit then

\[
\bar{f}^{t/j}_m = \sum_{i=1}^{2} \left(\rho^{g/j}_i w^{d/j}_i + E(r)w^{d/j}_i - (1 + E(r))(s + c^{d/j} + \rho^{g/j}_m w^{m/j}_m)\right) \pi_{d,j}, j = ct, cn, sg
\]

(1.144)

Any \( f^j_m \in (f^{j}_m, f^{t/j}_m) \) ensures win-win profits for the manufacturer and the distributor. Note that, first stage of forward-sequential-bargaining process will be possible only when \( \bar{f}^{t/j}_m > f^j_m \) i.e., if \( f^{t/j}_1 + f^{t/j}_2 \geq f^j_m - \Delta_d \). The forward-sequential-bargaining process depends on distributor’s managing capability and decision making power. When the distributor assured that the cumulative amount of franchise fee from retailers will exceed the amount \( f^j_m - \Delta_d \) then it will participate in bargaining with the manufacturer for a \( f^j_m \in (f^{j}_m, f^{t/j}_m) \) that will maximize

7The process of the forward-sequential-bargaining is similar to section 3.2 but here it is applied with another coordination mechanism for different channel structure with some modifications.
Figure 3.10: Graphical representation of feasible region of win-win profit of the channel members in forward sequential bargaining for Cournot behavior of the retailers.

their profits. In such case the Nash bargaining product can be found as

$$\max_{f_j^m \leq f_j^m \leq f_j^m} [f_j^m + f_j^m + f_j^m - f_j^m][\Delta_j^m + f_j^m], \quad j = ct, cn, sg$$

(1.145)

Optimal value of $f_j^m$ is

$$f_j^m = \frac{\Delta_j^m + f_j^m + f_j^m - \Delta_j^m}{2}, \quad j = ct, cn, sg$$

(1.146)

After bargaining the distributor's intermediate profit is

$$\pi_{qd/j}^{\text{d}} = \sum_{i=1}^{2} \left( (\rho_t^{qd/j} w_{dpi} + E(r)w_{dt} - (1 + E(r))(s + c^d + \rho_t^{qd/j} w^{mj})D_i + f_i^j \right)$$

$$- f_j^{f/j}, \quad (j = ct, cn, sg)$$

(1.147)

As soon as the distributor find its intermediate profit, it will demand the franchise fee and bargain with the retailers independently because it has different reservations for different retailers. Applying similar approach as in the backward sequential bargaining, the minimum amount of
franchise fee that the distributor will accept from ith (i=1,2) retailer can be found as
\[
f^j_i = \pi^j_{di} - \left[ p^{qd/j} w^{dp} + E(r) w_{dI} - (1+E(r))(s+c^d + \rho^{gd} w^{mj}) \right] D^c_i + \frac{D^c_i f^j_m}{D^c_i + D^c_2}, i = 1, 2; j = ct, cn, sg
\] (1.148)

Figure 3.11: Effect of distributor’s target franchise fee to the manufacturer in channel members bargaining profit for backward sequential bargaining.

Since, any \( f^j_i \in (f^j_i, \bar{f}^j_i) \quad i = 1, 2 \) resolves channel conflict, the distributor and the ith retailer bargain to determine an acceptable \( f^j_i \in (f^j_i, \bar{f}^j_i) \) that will maximize the Nash bargaining product. The Nash bargaining product can be found as
\[
\max_{\bar{f}^j_i \leq f^j_i \leq f^j_i} \left[ \Delta_{ri}^j - f^j_i \right] \left[ \Delta_{di}^j - (D^c_i f^j_m/(D^c_1 + D^c_2)) + f^j_i \right], \quad i = 1, 2; j = ct, cn, sg
\] (1.149)

Optimal value of \( f^j_i \) is
\[
f^{f/j}_i = \frac{\Delta_{ri}^j - \Delta_{di}^j + (D^c_i f^j_m/(D^c_1 + D^c_2))}{2}, \quad i = 1, 2; j = ct, cn, sg
\] (1.150)

Thus, the ith retailer’s optimal profit after bargaining can be found from (1.124) by replacing \( f^j_i \) by \( f^{f/j}_i \) of (1.150). The feasible region of win-win outcome of all channel members in forward bargaining process for Cournot behavior of the retailers displayed in figure-3.10.

Therefore, optimal profits of the retailers, the distributor and the manufacturer after the sequential-bargaining process are respectively as
\[
\pi^{rf/j}_i = \pi^j_{ri} + \frac{\Delta_{ri}^j + \Delta_{di}^j}{2} - \frac{D^c_i (\Delta_{ri}^j - \Delta_{di}^j + f^{f/j}_1 + f^{f/j}_2)}{4(D^c_1 + D^c_2)}, \quad i = 1, 2; j = ct, cn, sg
\] (1.151)
\[
\pi^{df/j}_d = \pi^d + \frac{\Delta_{d}^j + \Delta_{m}^j + 2(\Delta_{r1}^j + \Delta_{r2}^j) - f^{f/j}_1 - f^{f/j}_2}{4}, \quad j = ct, cn, sg
\] (1.152)
\[ \pi^{mf/j} = \pi^j_m + \frac{\Delta^j_1 + f^{1/j}_1 + f^{1/j}_2 + \Delta^j_m}{2}, j = ct, cn, sg \]  

(1.153)

Note that, 
\[ \frac{d\pi^{rb/j}_i}{df^{t/j}_i} = -\left(\frac{D^c_i}{4(D^c_1 + D^c_2)}\right) < 0 \]  
\(j = \text{ct, cn, sg}\); 
\[ \frac{d\pi^{mb/j}_i}{df^{t/j}_i} = -\frac{1}{4} < 0 \]  
\(j = \text{ct, cn, sg}\) and 
\[ \frac{d\pi^{mb/j}_m}{df^{t/j}_m} = 1/2 > 0 \]  
\(j = \text{ct, cn, sg}\), that is the bargaining profit of the retailers and the distributor decrease but profit of the manufacturer increase with increasing 
\(f^{t/j}_i\).

**Figure 3.12:** Effect of distributors target franchise fee from the ith retailer in channel members bargaining profit for forward sequential bargaining.

From the above discussion the proposition follows.

**Proposition 3.11** All unit quantity discount with franchise fee coordinates the channel and the channel members get win-win profit through forward-sequential-bargaining when 
\[ f^{t/j}_i \in (f^{t/j}_1, f^{t/j}_2) \]  
\(i=1, 2; j = \text{ct, cn, sg}\) and 
\[ f^{t/j}_m \in (f^{t/j}_m, f^{t/j}_m) \]  
\(i=1, 2; j = \text{ct, cn, sg}\) and agreed on franchise fee 
\[ \frac{df^{t/j}_i}{df^{t/j}_i} \]  
\(j = ct, cn, sg\).

### 3.3.6.3 Comparison of two bargaining processes

Comparison of optimal profits of the retailers, the distributor and the manufacturer obtained in the sequential-bargaining process provides

\[ \pi^{mb/j} - \pi^{mf/j} = \frac{\Delta^j_1 + \Delta^j_2 - \Delta^j_m + f^{t/j}_m - 2(f^{t/j}_1 + f^{t/j}_2)}{4}, j = ct, cn, sg \]  

(1.154)

\[ \pi^{db/j} - \pi^{df/j} = \frac{\Delta^j_m - (\Delta^j_1 + \Delta^j_2) + f^{t/j}_m + (f^{t/j}_1 + f^{t/j}_2)}{4}, j = ct, cn, sg \]  

(1.155)

\[ \pi^{rb/j} - \pi^{rf/j} = \frac{D^c_i(\Delta^j_1 - \Delta^j_m + f^{t/j}_1 + f^{t/j}_2 - 2f^{t}_m)}{4(D^c_1 + D^c_2)}, i = 1, 2; j = ct, cn, sg \]  

(1.156)

Thus, the proposition follows.
Proposition 3.12 The manufacturer will prefer the backward-sequential-bargaining if \( f^m_t > 2(f^{t/j}_1 + f^{t/j}_2) - \Delta^j_{r_1} - \Delta^j_{r_2} + \Delta^j_d \quad (j = ct, cn, sg) \). The distributor will prefer the backward-sequential-bargaining if \( f^m_t > \Delta^j_{r_1} + \Delta^j_{r_2} - (f^{t/j}_1 + f^{t/j}_2) - \Delta^j_m \quad (j = ct, cn, sg) \). While retailers will prefer the forward-sequential-bargaining if \( f^m_t > (f^{t/j}_1 + f^{t/j}_2 + \Delta^j_d - \Delta^j_m)/2 \).

3.3.7 Numerical illustration

In this section, numerical study is carried out to demonstrate the mathematical behavior of the proposed models and gain some insights of the problem which are being studied. The numerical works have been done using MATHEMATICA-7. Let the potential market demand of the retailer-1 and the retailer-2 are respectively \( \alpha_1 = 190 \) and \( \alpha_2 = 200 \). Price sensitivity parameters are \( a_1 = 0.9 \), \( a_2 = 1.0 \); degree of substitutability between retailers is \( \beta = 0.4 \); and marginal costs are \( c = 10 \), \( c^d = 2 \), \( c_{r_1} = 1.5 \), \( c_{r_2} = 1.5 \). Average wholesale price of the imperfect quality product is \( w_{dI} = 5 \). Cost of screening a product is \( s = 0.15 \).

Suppose the random amount of imperfect quality item, \( r \), is uniformly distributed between 0.02 to 0.20. Hence,

\[
E(r) = (0.02 + 0.20)/2 = 0.11.
\]

Optimal prices and optimal individual expected profits and total channel profit in Cournot, Collusion, Stackelberg game are presented in table-3.7. The pricing decisions and channel profit is also included in the table-3.7.

<table>
<thead>
<tr>
<th>Game</th>
<th>( w^m )</th>
<th>( w_{dp} )</th>
<th>( p_{r_1} )</th>
<th>( p_{r_2} )</th>
<th>( \pi_m )</th>
<th>( \pi_d )</th>
<th>( \pi_{r_1} )</th>
<th>( \pi_{r_2} )</th>
<th>Channel profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ct</td>
<td>163.30</td>
<td>268.18</td>
<td>306.19</td>
<td>296.08</td>
<td>10083.6</td>
<td>5041.8</td>
<td>1199.8</td>
<td>696.9</td>
<td>17022.1</td>
</tr>
<tr>
<td>Cn</td>
<td>163.20</td>
<td>268.02</td>
<td>317.19</td>
<td>307.73</td>
<td>7952.6</td>
<td>3976.3</td>
<td>1316.8</td>
<td>731.6</td>
<td>13977.3</td>
</tr>
<tr>
<td>Sg</td>
<td>163.18</td>
<td>267.18</td>
<td>307.85</td>
<td>296.31</td>
<td>9910.8</td>
<td>4955.4</td>
<td>1207.0</td>
<td>719.7</td>
<td>16792.9</td>
</tr>
<tr>
<td>Ce</td>
<td>–</td>
<td>–</td>
<td>189.6</td>
<td>180.2</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>31870.6</td>
</tr>
</tbody>
</table>

Table 3.7: The optimal solutions of the decentralized (under Cournot(Ct), Collusion (Cn) and Stackelberg (Sg) behavior of duopolistic retailers) and centralized (Ce) model.

The table-3.7 varifies that retail price of the product is maximum in Collusion behavior of the duopolistic retailers. The retailers also get maximum profit in Collusion game, e.g. retailer-1 get 8.36% more profit in Collusion game compared to Cournot game. The manufacturer and the distributor get maximum profit in the duopolistic retailers’ Cournot behavior. Total channel profit is maximum in Cournot and minimum in Collusion behavior of the duopolistic retailers. Centralized channel profit is 87% more than total channel profit of the Cournot game. Thus channel coordination is essential to achieve integrated channel profit. Table-3.8 numerically illustrates ranges and bargaining outcome of franchise fees in backward and forward sequential
<table>
<thead>
<tr>
<th>Bargaining process</th>
<th>Game</th>
<th>$(f_1, \overline{f}_1)$</th>
<th>$f_1$</th>
<th>$(f_2, \overline{f}_2)$</th>
<th>$f_2$</th>
<th>$(f_m, \overline{f}_m)$</th>
<th>$f_m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Backward-Sequential-Bargaining</td>
<td>Cournot Collusion Stackelberg</td>
<td>(5550, 8080)</td>
<td>6815</td>
<td>(5053, 8456)</td>
<td>6808</td>
<td>(46949, 58885)</td>
<td>52917</td>
</tr>
<tr>
<td></td>
<td>(11208, 14696)</td>
<td>12952</td>
<td>7508</td>
<td>(10902, 15126)</td>
<td>13014</td>
<td>(71694, 85731)</td>
<td>78712</td>
</tr>
<tr>
<td></td>
<td>(6037, 8979)</td>
<td></td>
<td></td>
<td>(5196, 8433)</td>
<td>6814</td>
<td>(48587, 60576)</td>
<td>54581</td>
</tr>
<tr>
<td>Forward-Sequential-Bargaining</td>
<td>Cournot Collusion Stackelberg</td>
<td>(4461, 8080)</td>
<td>6271</td>
<td>(3913, 8456)</td>
<td>6238</td>
<td>(46949, 60322)</td>
<td>53636</td>
</tr>
<tr>
<td></td>
<td>(9964, 14696)</td>
<td>12330</td>
<td>6964</td>
<td>(9600, 15126)</td>
<td>12363</td>
<td>(71694, 86965)</td>
<td>79329</td>
</tr>
<tr>
<td></td>
<td>(4950, 8979)</td>
<td></td>
<td></td>
<td>(4058, 8433)</td>
<td>6245</td>
<td>(48587, 61935)</td>
<td>55261</td>
</tr>
</tbody>
</table>

Table 3.8: Ranges and bargaining outcome of franchise fees in backward and forward sequential bargaining
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Bargaining for coordination and win-win outcome.

From table-3.7 and table-3.8 one may note that, before the settle of franchise fee the retailer-1 gets 9280, 10186, 16013 while the retailer-2 gets 9153, 9153 and 15857 respectively in Cournot, Stackelberg and Collusion game (these are found by adding $\pi_{r1}^j$ and $f_{t/j}^i$ (i=1,2; j=ct, sg, cn)). That is the retailers mostly prefers Collusion game instead of other two. To develop the numerical illustration of two bargaining processes, it is supposed that the distributer assumes the target franchise fee that it will pay to the manufacturer is $f_{t/j}^i = f_{j}^i + (\pi_c - \pi_{r1}^j - \pi_{r2}^j - \pi_{d}^j - \pi_{m}^j)(\pi_{r1}^j / (\pi_{r1}^j + \pi_{r2}^j + \pi_d^j + \pi_m^j))$ in backward sequential bargaining. On the other hand, in forward sequential bargaining the distributer assumes the target franchise fee that it can collect from the ith (i=1, 2) retailers is $f_{t/j}^i = f_{j}^i - (\pi_c - \pi_{r1}^j - \pi_{r2}^j - \pi_d^j - \pi_m^j)(\pi_{r1}^j / (\pi_{r1}^j + \pi_{r2}^j + \pi_d^j + \pi_m^j))$. Based on these assumptions $f_{t/j}^i$ (i=1,2; j=ct, sg, cn) and $f_{t/j}^m$ (j=ct, sg, cn) are calculated and are presented in table-3.8. For example, for backward sequential bargaining $f_{t/ct}^{l/j} = 55864.6$ and for backward sequential bargaining $f_{t/ct}^{l/ct} = 7019.8$ and $f_{t/ct}^{l/ct} = 8040.7$. Note that, the range of franchise fee of the retailer-1 under Cournot game are respectively (5550, 8080) and (4461, 8080) for backward and forward sequential bargaining. That is, the range of retailers’ franchise fee is more flexible for the retailers in forward sequential bargaining and so the retailers prefer forward sequential bargaining instead of backward sequential bargaining. Similarly, the manufacturer also prefers forward sequential bargaining because upper limit of franchise fee that it gets from the distributer is less in backward sequential bargaining than forward sequential bargaining. This scenario may be change because the preference of bargaining process depend on the decisions of the distributer on target franchise fee outline of it are given in proposition-3.12.

Table-3.9 shows the amount of quantity discount that the upstream channel members provide to its downstream members. Interestingly, $\rho_{m}^{qd} < 0$, i.e., for channel cooordination the manufacturer initially has to pay to the distributer but through settlement franchise fee it makes win-win profit.

<table>
<thead>
<tr>
<th>Bargaining process</th>
<th>Game</th>
<th>$\rho_{1}^{qd}$</th>
<th>$\rho_{2}^{qd}$</th>
<th>$\rho_{m}^{qd}$</th>
<th>$\pi_{m}$</th>
<th>$\pi_{d}$</th>
<th>$\pi_{r1}$</th>
<th>$\pi_{r2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Backward-Sequential Bargaining</td>
<td>Cournot</td>
<td>0.323</td>
<td>0.310</td>
<td>-1.026</td>
<td>16051</td>
<td>11009</td>
<td>2465</td>
<td>2345</td>
</tr>
<tr>
<td></td>
<td>Collusion</td>
<td>0.048</td>
<td>0.048</td>
<td>-1.820</td>
<td>14971</td>
<td>10995</td>
<td>3061</td>
<td>2844</td>
</tr>
<tr>
<td></td>
<td>Stackelberg</td>
<td>0.286</td>
<td>0.310</td>
<td>-1.080</td>
<td>15905</td>
<td>10949</td>
<td>2678</td>
<td>2338</td>
</tr>
<tr>
<td>Forward-Sequential Bargaining</td>
<td>Cournot</td>
<td>0.323</td>
<td>0.310</td>
<td>-1.026</td>
<td>16770</td>
<td>9176</td>
<td>3009</td>
<td>2915</td>
</tr>
<tr>
<td></td>
<td>Collusion</td>
<td>0.048</td>
<td>0.048</td>
<td>-1.820</td>
<td>15588</td>
<td>9105</td>
<td>3683</td>
<td>3494</td>
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<tr>
<td></td>
<td>Stackelberg</td>
<td>0.286</td>
<td>0.310</td>
<td>-1.080</td>
<td>16585</td>
<td>9157</td>
<td>3221</td>
<td>2907</td>
</tr>
</tbody>
</table>

Table 3.9: Value of coordination parameters and optimal profits in backward and forward sequential bargaining.
3.4 Managerial implications and concluding remarks

This chapter considers three different three-echelon supply chain structures and proposes methods to uplift channel performance under deterministic demand environments. The first model has single member in each echelon. The second model has one manufacturer, multiple distributors and multiple retailers associated with each distributors. The third one is consisting of one manufacturer, one distributor and duopolistic retailers. The first two models deal with deteriorating products while the last one considers imperfect quality product. The chapter demonstrates that it is always possible to enhance channel performance significantly by applying suitable coordination mechanisms. The chapter highlighted following managerial insights.

The first model of the chapter proposes a manufacturer-distributor-retailer supply chain. The manufacturer produces a perishable product and supplies it to the retailer through the distributor in a single lot. The deteriorated product cannot be reworked and is disposed by the retailer without any salvage value. For channel optimal profit the manufacturer and the distributor form a coalition that provides a percentage of the disposal cost to the retailer as compensation. The channel members divide the surplus profit that is generated for channel coordination, through a nested bargaining. First, the retailer and the manufacturer-distributor coalition bargain by using their negotiation powers. Based on the bargaining result the manufacturer and the distributor bargain for surplus sharing using their bargaining powers within the coalition. Number of interesting results are obtained, which are summarized below. First, the simple side-payment contract- compensation on disposal cost of deteriorated product, cuts out channel conflict. Second, the compensation fraction may be greater than one. That is, if the manufacturer-distributor coalition offers more than full disposal cost as compensation to the retailer then also the channel is coordinated and win-win result for all channel members are ensured. Third, when the compensation fraction increases within its range, the retailers profit increases but the coalition’s profit decreases. If the coalition’s profit decreases then it does not infer that the manufacturer’s profit as well as the distributor’s profit decrease. The profits of the manufacturer and the distributor depend on how they share the retailer’s disposal cost within the coalition. If $k$ increases within its range then the distributor’s profit decreases but the manufacturer’s profit increases. However, when $\lambda$ attends its maximum value $\lambda_{\text{max}}$, the coalition’s profit is least and the manufacturer and the distributor settle for their decentralized profits. Fourth, profits of all the channel members depend heavily on the set up cost of the manufacturer and the ordering costs of the retailer and the distributor respectively. Thus, the compensation fraction’s range depends on these parameters. In fact, for the retailer’s high ordering cost the range of the compensation fraction is low, whereas it is high for high set up cost and/or higher ordering cost of the manufacturer and/or the distributor. Fifth, the compensation cost sharing fraction $k$ within the manufacturer-distributor coalition determines the profit split of the manufacturer and the distributor. Thus, $k$ plays an important role in the bargaining process within the coalition.

The second model of the chapter discusses about channel coordination and benefit sharing in a three-echelon distribution channel. The existing literature on supply chain considers only
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The centralized approach through synchronized order quantity or replenishment cycle for profit maximization or cost minimization. As a result some channel members are benefited from the optimal decision and some are not. Moreover, product deterioration has not been taken into account though perishability has significant impact on the channel optimal decision. The model yields following insights. First, the centralized decision provides best integrated channel performance though it gives advantages to the distributors and the manufacturer, and disadvantages to the retailers. Thus, to make centralized decision acceptable for all the channel members some incentive is needed to change the disadvantages suffered by the players. As incentives the manufacturer provides quantity discounts independently to each of its downstream distributors and distributors give percentage compensation on deterioration cost to each retailers separately. As such, the disadvantages are eliminated and all channel members obtain win-win opportunities. Second, as far as the implementation of the coordination contract and benefit division is concerned, applying any one of two nested contract-bargaining processes it is possible to fix the benefit shares among the channel members. Third, the ranges of win-win opportunities of the channel members are different in two bargaining processes. Fourth, the distributors prefer the backward contract-bargaining process, whereas the manufacturer and the retailers prefer the other. Fifth, in both the contract-bargaining processes minimum quantity discounts demanded by the distributor are negative. That is, without receiving any quantity discount from the manufacturer, the distributors are able to resolve channel conflict by providing percentage compensation on deterioration cost and still able to avail win-win opportunities. Sixth, if the distributors provide more than deterioration cost to the retailers as compensation, then also they obtain win-win opportunities. The present study may be extended in several directions. The model developed here is only for the first production cycle over a finite time horizon. A model may be developed over a finite time horizon in the line of Jonrinaldi and Zhang [76]. Instead of using quantity discount and percentage compensation on deterioration cost as coordination contracts one may use some other well-known coordination contracts for resolving channel conflict and compare the result with this model. In the bargainings, it is assumed that the channel members have equal negotiation powers. The model may be further extended by considering different negotiation powers of the bargaining parties.

The third model considers a three-echelon supply chain, where in the downstream two retailers may play Cournot, Stackelberg or Collusion game. Also, it assumes that the manufacturer supplies the product to the distributor in a single lot that contains random proportion of defective product. The model yields following insights. First, expected imperfect quality product has considerable impact on pricing. In fact, when the distributor sells the product below its marginal system cost, the manufacturer reduces it wholesale price whereas the distributor increases its wholesale price. The intuitive reason is straightforward. The manufacturer helps the distributor to reduce its loss and the distributor does the same by increasing wholesale price. Reverse trend may be observed when the selling price of the defective items is higher. Second, all unit quantity discount with franchise fee coordinates the channel though unable to provide win-win profits for all the channel members. Third, it is found that the manufacturers preference of retailers games is in the order Cournot, Stackelberg or Collusion, whereas the retailers prefer the reverse order because in Collusion game the retailers cooperate and control the market by setting prices. Fourth, the bargaining processes are used to determine the limits of the franchise fee and to split the surplus profit through negotiation between two consecutive stages of the supply chain. As such, as a mediator the distributor plays the central role by setting a target profit in each of the two processes at the first stage bargaining by assuming that
it can make up the target in the next stage of bargaining. This is quite common in marketing practice that for the best output of the channel, the mediator makes a commitment in the first stage of negotiation and tries to overcome the loss due to the commitment in the next stage of negotiation. Fifth, preference analysis of the bargaining processes for the channel members indicates that the manufacturer and the distributor prefer backward bargaining, whereas the retailers prefer the other.

Managing channel coordination and profit distribution are two major findings of this chapter. All the models in this chapter have a stage of intermediate role (distributor). It makes channel coordination more difficult. Moreover in second and third model there are multiple members in some stages. The problem further intensifies as it addresses product deterioration/imperfect quality. In such a complex situations the chapter proposes suitable mechanisms to coordinate the supply chains. Although these models are robust and dynamic, still they have some limitations and can be extended in several ways. For example, the channel members may have some private information that is, incomplete information. Demand at the retailers end may be consider as stochastic or different types price sensitive and/or inventory level sensitive and/or sales effort sensitive etc. Although in second and third models provide the concept of bargaining processes. But it is essential to verify whether the managerial insights can be generalized or not. The chapter uses sequential bargaining that has two stages. More investigation is needed to verify the applicability of the process in general case. That is, whether the process can be applied in an more then three echelons supply chain or not. Then only it can be concluded that the proposed process is dynamic for surplus profit division among a supply chain members.