Chapter-III

Effects of Couple Stress and Surface Roughness of Finite Journal Bearings in Squeeze Film Lubrication-Rapid-Narang technique
Effects of Couple Stress and Surface Roughness of Finite Journal Bearings in Squeeze Film Lubrication - Rapid- Narang technique

3.1 Introduction:

The phenomenon of two lubricated surfaces approaching each other with a normal velocity is known as squeeze film lubrication. The thin film of lubricant present between the two surfaces acts as a cushion and it prevents the surfaces from making instantaneous contact. The time required to squeeze out the lubricant depends upon surface configuration, fluid properties and the load applied. In general, the relation between the load carrying capacity and the rate of approach is studied in the most squeeze film analysis. Much work is done in this line and the mathematical review on such process was done by various workers [1-32].

behaviour between isotropic rough rectangular plates is studied by Lin [16]. Lin and his co-workers have extensively studied the surface roughness effects on the oscillating squeeze film behaviour of long partial journal bearing [14]. Naduvinamani et al. [19] studied the squeeze film lubrication of a short porous journal bearing with couple stress fluids. Jaya Chandra Reddy et al. [13] studied the analysis of load carrying capacity using rapid technique. Raghavendra [24] studied the effects of couple stresses and surface roughness on roller bearings under lightly loaded conditions. Naduvinamani et al. [20] studied the combined effects of surface roughness and couple stresses on squeeze film lubrication between porous circular plates. The effect of surface roughness on couple stress squeeze film lubrication of long porous partial journal bearings is studied by Bujurke et al. [6].

In this chapter, the generalized Reynolds equation derived by taking into account the effects of surface roughness and additives in the form of couple stresses in chapter-II, is applied to various squeeze film bearings. Now we apply this generalized Reynolds equation to see the effects of couple stresses and surface roughness on the squeeze film of finite journal bearings.

3.2 Basic Equation:

The equations governing the fluid flow in the bearing is given by equation(2.39) as,

\[
\frac{\partial}{\partial x} \left( F \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial z} \left( F \frac{\partial p}{\partial z} \right) = \frac{dH}{dt}
\]  

(3.1)

Where,

\[
F = \frac{M - Tanh(M)}{M'} \left( \frac{2h_c}{2k_\mu} \right) + \frac{1}{4} \left[ \left( h_a - 2h_c \right) + 2h_c Tanh(M) \right]^2 \left( \frac{h_a - 2h_c}{k_\mu(2M)} \right)
\]

\[
+ \frac{1}{12\mu} \left( h_a - 2h_c \right)^2 - 12l^2 (h_a - 2h_c) + 24l^3 Tanh \left( \frac{h_a - 2h_c}{2l} \right)
\]
\[ M = \alpha h_n, \quad \alpha = \left( \frac{1}{k\phi} \right)^{1/2}, \quad l = \left( \frac{\eta}{\mu} \right)^{1/2} \] (3.2)

Here \( h_n \) is the nominal film thickness. \( h_r \) is the mean height of surface asperities, \( \phi \) is the roughness parameter of the bearing surface, \( k \) is the ratio of lubricant viscosities at the rough surface to purely hydrodynamic zone and \( l \) characteristics the couple stress property of the lubricant.

The nominal oil film thickness \( h_n \) is given by
\[ h_n = c(1 + \epsilon \cos \theta) \] (3.3)
\[ \frac{dh_n}{dt} = c \frac{de}{dt} \cos \theta \]

where \( c = r - R \) is the clearance width and \( \epsilon = \frac{e}{c} \) is the eccentricity ratio as shown in Fig. (3.1.1).

The modified Reynolds Equation is
\[ \frac{\partial}{\partial \theta} \left( F \frac{\partial p}{\partial \theta} \right) + \frac{\partial}{\partial z} \left( k \frac{\partial p}{\partial z} \right) = c \frac{de}{dt} \cos \theta \] (3.4)

Introducing the non dimensional variables and parameters
\[ \theta = \frac{x}{R'}, \quad z = \frac{z}{L}, \quad \lambda = \frac{L}{2R'}, \quad \nu = \frac{1}{c'}, \quad \phi = \frac{\phi}{c}, \]
\[ \bar{h}_n = \frac{h_n}{c'}, \quad M = \alpha h_n, \quad \alpha = \left( \frac{1}{k\phi} \right)^{1/2}, \quad \bar{h}_r = \frac{h_r}{c} \] (3.5)

The modified Reynolds equation in a non-dimensional form can be written as:
\[ \frac{\partial}{\partial \theta} \left( F \frac{\partial p}{\partial \theta} \right) + \frac{1}{4\lambda^2} \frac{\partial}{\partial z} \left( F \frac{\partial p}{\partial z} \right) = 12 \frac{\mu R'}{c'} \frac{de}{dt} \cos \theta \] (3.6)
Where,
\[
\bar{F} = 6 \frac{M - \text{Tanh}(M) \left(2\bar{h}_r\right)}{M^2} + 6 \left[\bar{h}_r - 2\bar{h}_r + 2h_i \frac{\text{Tanh}(M)}{M} \right]^2 2h_i \frac{\text{Tanh}(2M)}{k(2M)} + \left(\bar{h}_r - 2\bar{h}_r\right) - 12l^2 (\bar{h}_r - 2\bar{h}_r) + 24l^2 \text{Tanh} \left(\frac{h_i - 2\bar{h}_r}{2l}\right)
\]  
(3.7)

\[M = \alpha h_i, \quad \alpha = \frac{1}{\sqrt{k\phi}}\]

The dimensionless Pressure is given by
\[
\bar{p} = \frac{\rho c^2}{\mu R^2 \frac{dx}{dt}}
\]

The equation (3.6) reduces to
\[
\frac{\partial}{\partial \theta} \left( F \frac{\partial \bar{p}}{\partial \theta} \right) + \frac{1}{4l^2} \frac{\partial}{\partial z} \left( F \frac{\partial \bar{p}}{\partial z} \right) = 12 \cos \theta
\]  
(3.8)

Where \(\bar{F}\) is given in equation (3.7).

The boundary conditions for the equation (3.8) are
\[
\bar{p} = 0 \quad \text{at} \ \theta = \frac{\pi}{2}, \quad \frac{3\pi}{2}
\]
\[
\bar{p} = 0 \quad \text{at} \ \bar{z} = \pm \frac{l}{2}
\]
\[
\frac{d\bar{p}}{dz} = 0 \quad \text{at} \ \bar{z} = 0
\]  
(3.9)

where \(\theta\) is circumferential angle, \(\bar{z}\) is bearing axis parallel to the shaft axis.

3.2.1 Short Bearing Analysis:

If \(\lambda \leq 0.5\) it is called short bearing or narrow bearing [13]. Neglecting pressure variations in the \(x\) direction, the modified Reynolds equation reduces to

\[
\frac{\partial}{\partial \bar{z}} \left( F \frac{\partial \bar{p}}{\partial \bar{z}} \right) = 48\lambda^2 \cos \theta
\]  
(3.10)
Fig. 3.1.1 Squeeze Film Bearing
Integrating the above equation (3.10) twice, the equation becomes,

\[ p = \frac{48\lambda^2 \cos \theta \bar{z}}{F} \left[ \frac{1}{2} z + C_1 \frac{1}{2} z + C_2 \right] \]  

(3.11)

where \( C_1 \) and \( C_2 \) are constants of integration and evaluated using boundary conditions of equation (3.9)

\[ C_1 = 0, \quad C_2 = \frac{48\lambda^2 \cos \theta \bar{z}}{F} \left( \frac{1}{8} \right) \]  

(3.12)

Substituting equation (3.12) in (3.11), the pressure for a short bearing is

\[ p = \frac{24\lambda^2 \cos \theta \bar{z} \left( z \frac{1}{8} \right)}{F} \]  

(3.13)

Pressure at the centre line of the bearing is, i.e., \( \bar{z} = 0 \)

\[ p = -\frac{6\lambda^2 \cos \theta}{F} \]  

(3.14)

Therefore the non-dimensional pressure for short bearing is

\[ P_r = -\frac{6\lambda^2 \cos \theta}{F} \]  

(3.15)

where \( F \) is given in equation (3.7)

The load carrying capacity is

\[ W_r = -2 \int_{0}^{1} p R \cos \theta \, d\theta \, dz \]  

(3.16)

\[ W_r = \frac{\mu R^3 \cos \theta}{c^2} \int_{0}^{1} \frac{6\lambda^2 \cos^2 \theta}{F} \, d\theta \]  

(3.17)
and the dimensionless squeeze load is given by

\[
\bar{W}_s = \frac{W_c e^2}{\mu R \frac{d\varepsilon}{dt} L} = 6A^2 \int_0^{\pi} \frac{\cos \theta}{F} d\theta
\]  

(3.18)

Where,

\[
\bar{F} = 6 \left( \frac{M - \tanh(M)}{M^3} \right) \left( \frac{2h_r}{k} \right)^2 + 6 \left( \frac{h_n - 2h_r}{2h_r} \right) + 2h_r \frac{\tanh(M)}{M} \left( \frac{2h_r \tanh(2M)}{k(2M)} \right) 
\]

\[
+ \left( \frac{h_n - 2h_r}{2h_r} \right) + 12l^2 \left( \frac{h_n - 2h_r}{2l} \right) + 24l^3 \tanh \left( \frac{h_n - 2h_r}{2l} \right)
\]

3.2.2 Long Bearing Analysis:

If \( \lambda > 2 \), it is called long bearing. That is the bearing is infinitely long in axial direction and the pressure is constant in that direction. Thus neglecting the \( \left( \frac{\partial p}{\partial z} \right) \), the modified Reynolds equation reduces to

\[
\frac{\partial}{\partial \theta} \left( \bar{F} \frac{\partial \bar{p}}{\partial \theta} \right) = 12 \cos \theta
\]  

(3.19)

Integrating with respect to \( \theta \)

\[
\bar{F} \frac{\partial \bar{p}}{\partial \theta} = 12 \cos \theta + B
\]  

(3.20)

where \( B \) is the integral constant.

Apply boundary conditions \( \frac{\partial \bar{p}}{\partial \theta} = 0 \) at \( \theta = \pi \)

The constant \( B = 0 \)

\[
\frac{\partial \bar{p}}{\partial \theta} = \frac{12 \sin \theta}{F}
\]  

(3.21)

again integrating and applying the boundary conditions
\[
\tilde{p} = 0 \quad \text{at } \theta = \frac{\pi}{2}, \frac{3\pi}{2} \\
\tilde{p} = 0 \quad \text{at } z = \frac{1}{2} \\
\frac{d \tilde{p}}{dz} = 0 \quad \text{at } z = 0
\]

and the dimensionless pressure is

\[
\tilde{p}_r = 12 \int_0^1 \frac{\sin \theta}{F} d\theta
\]  
(3.22)

Squeeze load capacity \( W_s \) for the long bearing is

\[
W_s = 2L \int_0^\pi \rho R \cos \theta d\theta
\]

\[
W_s = 2L \int_0^\pi \frac{\mu R^3 \frac{dc}{dt} \int_0^1 \frac{24 \sin^2 \theta}{F} d\theta}{c^i}
\]  
(3.23)

and the dimensionless load capacity is

\[
\tilde{W}_s = \frac{W_s c^i}{\mu R^3 \frac{dc}{dt} L} = -24 \int_0^1 \frac{\sin^2 \theta}{F} d\theta
\]  
(3.24)

Where,

\[
\tilde{F} = 6 \frac{M - \tanh(M)}{M^3} \left(\frac{2h_i}{k}\right) + 6 \left[\left(\tilde{h}_s - 2\tilde{h}_i\right) + 2\tilde{h}_s \frac{\tanh(M)}{M}\right] \frac{2h_i}{k(2M)} \\
+ \left[\left(\tilde{h}_s - 2\tilde{h}_i\right) - 12i (\tilde{h}_s - 2\tilde{h}_i) + 24i \frac{\tanh}{2i}\right]
\]
3.2.3 Finite Bearing Analysis:

For finite bearings, the two dimensional Reynolds equation is solved using Rapid technique. If $p, p', p_i$ are the pressures in finite, narrow and long bearings respectively, then the relationship between them is [28]

$$\frac{1}{p} = \frac{1}{p'} + \frac{1}{p_i} \tag{3.25}$$

The finite bearing pressure is

$$p = \frac{p' p_i}{p' + p_i} \tag{3.26}$$

Load carrying capacity:

As the load is proportional to the pressure, the load carrying capacity for the finite bearing is

$$\frac{1}{W_f} = \frac{1}{W'} + \frac{1}{W_i}$$

$$W_f = \frac{W' W_i}{W' + W_i} \tag{3.27}$$

Substituting the short and long bearing load equations (3.18) and (3.24) in the above equation (3.27) the finite bearing load carrying capacity in non-dimensional form is

$$\frac{W}{W} = \frac{W_f c^2}{\mu R^2 \frac{d\delta}{dt} L} = \frac{W' W_i}{W' + W_i} \tag{3.28}$$
Equation (3.28) is solved numerically. Graphs have been plotted for $N$ for various $h, \phi$ and $l$ in the Figures 3.1 to 3.7.

3.3 Results and Discussions:

Couple Stress Parameter

In couple stress fluids, the dimensionless parameter, $I$ defined by \( I = \left( \frac{\eta}{\mu} \right)^{\frac{1}{2}} \), characterizes the couple stress property of the fluid and also distinguishes it from the Newtonian fluid. This parameter $I$ has dimensions of length and can be identified with a property which depends on the size of the fluid molecule of a polar additive in a non-polar lubricant. Thus the parameter $I$ can be considered as a characterization of the interaction of the fluid with the bearing geometry. Therefore the parameter $I$ provides a mechanism which might be helpful in explaining some of the rheological abnormalities that are commonly observed in certain additive containing fluids when the flows are confined to narrow passages.

It is expected that the effect of couple stresses on various bearing characteristics would be prominent when $I$ is large. A large value of $I$ corresponds either to an additive lubricant molecule with long chain length or to a very small film thickness of the lubricant. When the lubricant flows through a narrow passage couple stresses would become significant. Thus the larger $I$, the more pronounced are the effects due to couple stresses. A couple stresses which arise as a consequence of the intrinsic motion of the lubricant of additive molecule when confined to a narrow passage is not significant when $I$ is small. A small value of $I$ corresponds to a short molecular length and a large clearance width. When $I \to 0$ the lubricant becomes Newtonian in character. This situation happens when $\eta \to 0$, which in turn implies that the additive molecule has vanishing chain length.
In Fig. 3.1 and 3.2 the load carrying capacity $\overline{W}$ are plotted with $\overline{h}$, the mean height of roughness asperities for different couple stress parameter $\overline{\phi}$. As the mean height of roughness parameter $\overline{h}$ increases, the load carrying capacity $\overline{W}$ increases with the increase in the value of couple stress parameter $\overline{\phi}$. However from the graphs it may be noted that as the mean height of roughness parameter $\overline{h}$ increases, the load carrying capacity $\overline{W}$ increases, for low values of $\overline{\phi}$ and decreases for high values of $\overline{\phi}$.

In the above the load capacity of the bearing increases for low values of $\overline{\phi}$, i.e., for transverse roughness and for higher values of $\overline{\phi}$, i.e., for longitudinal roughness, the load support characteristics decreases however as the couple stress parameter increases, i.e., as the chain length of the additive molecule increases, the load capacity increases.

It is seen from the Fig. 3.3 and 3.4 the load capacity increases for $k>1$ as $\overline{h}$ increases when there is no roughness and decreases for $k<1$ i.e., the load capacity increases as the thickness of the peripheral layer increases when the peripheral layer viscosity more than the middle layer viscosity. On the other hand the load capacity decreases as the peripheral layer increases when the peripheral layer viscosity less than the middle layer viscosity in the case of no roughness.

In Fig. 3.5 the load carrying capacity $\overline{W}$ are plotted with $\overline{h}$, the mean height of roughness asperities for various $\overline{\phi}$, the roughness parameter. The load capacity $\overline{W}$ increases for low values of $\overline{\phi}$ and decreases for high values of $\overline{\phi}$. Comparing these results with stochastic models, it may be noted that lower values of $\overline{\phi}$ may represent the case of transverse roughness where as higher values of $\overline{\phi}$ may represent longitudinal roughness. The effects of roughness are more pronounced in case of transverse roughness as compared to the case of longitudinal roughness.
The dimensionless load carrying capacity $\bar{W}$ versus the couple stress parameter $\bar{l}$ at different eccentricity ratio $\varepsilon$ is depicted in Fig. 3.6. The effects of couple stresses result in a higher film pressure as compared with the case of Newtonian lubricant; the integrated load carrying capacity is similarly affected. The bearing lubricated with couple stress fluids shows a larger load carrying capacity of journal bearing, especially for high values of $\varepsilon$. Meanwhile, no matter what the type of roughness is longitudinal or transverse one, the roughness effects boost the load capacity more than that of the smooth case.

In Fig. 3.7 the load carrying capacity $\bar{W}$ is plotted against the couple stress parameter $\bar{l}$ for various $k$, it is the ratio of peripheral viscosity near the surface to the viscosity in the middle of the lubricant. As $\bar{l}$ increases, the load capacity $\bar{W}$ increases as $k$ increases. The enhanced viscosity near the surfaces and the large chain length of the molecule in the lubricant increases the bearing life considerably.
Fig. 3.1: Dimensionless Load Vs $\bar{h}$, for different $l$

c = 0.4  
$\lambda = 0.2  
$\phi = 0.001  
k = 2  
$l = 0.14$

Fig. 3.2: Dimensionless Load Vs $\bar{h}$, for different $l$

c = 0.4  
$\lambda = 0.2  
$\phi = 100  
$l = 0.14$

Fig. 3.3: Dimensionless Load Vs $\tilde{h}$, for different $l$

Fig. 3.4: Dimensionless Load Vs $\tilde{h}$, for different $l$
Fig. 3.5: Dimensionless Load Vs \( \overline{h} \) for different \( \overline{\phi} \)

Fig. 3.6: Dimensionless Load Vs \( \overline{\theta} \) for different \( \varepsilon \)
Fig. 3.7: Dimensionless Load Vs / for different $k$
3.4 Summary:

In this chapter the generalized Reynolds equation with additives and surface roughness is applied to finite journal bearing. The cases of short and long journal bearings is analyzed and is applied to study finite journal bearing using Rapid - Narang technique.

It is found that as the chain length of additive molecules increases i.e., as the couple stress parameter increases, the load carrying capacity of finite journal bearing increases. Also as the mean height of roughness asperities increases, the load carrying capacity increases for low values of roughness parameter i.e., transversal roughness pattern and decreases for high values of roughness parameter i.e., longitudinal roughness pattern.
### NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$a$</td>
<td>The mean height surface asperities in the symmetric roughness case</td>
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<tr>
<td>$h_0$</td>
<td>Nominal film thickness</td>
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<tr>
<td>$h_r$</td>
<td>Mean height of roughness asperities</td>
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<tr>
<td>$h$</td>
<td>Film thickness</td>
</tr>
<tr>
<td>$k$</td>
<td>Ratio of the viscosities near the surface to the purely hydrodynamic zone</td>
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<tr>
<td>$l$</td>
<td>Couple stress parameter</td>
</tr>
<tr>
<td>$c$</td>
<td>Radial clearance</td>
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<tr>
<td>$e$</td>
<td>Eccentricity</td>
</tr>
<tr>
<td>$e_c$</td>
<td>Eccentricity ratio</td>
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<tr>
<td>$\mu$</td>
<td>Viscosity of the base lubricant</td>
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<tr>
<td>$\phi$</td>
<td>Roughness parameter</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Material coefficient of the couple stress fluid</td>
</tr>
<tr>
<td>$\nu_i$</td>
<td>Velocity of the surfaces in the case of one-dimensional form</td>
</tr>
<tr>
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<td>Cartesian coordinates</td>
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<tr>
<td>$W$</td>
<td>Load carrying capacity</td>
</tr>
<tr>
<td>$\bar{W}$</td>
<td>Dimensionless Load carrying capacity</td>
</tr>
<tr>
<td>$L$</td>
<td>Length of the bearing</td>
</tr>
<tr>
<td>$R$</td>
<td>Radius of the shaft</td>
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<tr>
<td>$r$</td>
<td>Radius of the bearing</td>
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REFERENCES


